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MATERIALITY AND THE PREHISTORY OF NUMBER

Karenleigh A. Overmann

INTRODUCTION

Materiality is important to numbers: Consider the relative difficulty of computing with and without material forms—fingers, pencil-and-paper, or calculator. For almost everyone, the former is far easier and permits greater accuracy, speed, and complexity of calculation than the latter. Visualizing and remembering numbers are similarly facilitated, as material devices (aside from the fingers) present and retain numerical information over longer spans of time and in greater volumes. But material devices for counting and computing are more than external, ancillary support to the brain: When we manipulate numbers, whether on our fingers, as beads on an abacus, or in the form of equations with written notations, thinking and doing with numbers are intertwined to a degree that it is difficult, and perhaps meaningless, to try to separate them.

The centrality of material forms to numbers makes them components of a cognitive system in which brains, bodies, and materiality interact to produce systemic outcomes greater than what any part can achieve in isolation (Malafouris, 2013). The ability to recruit and incorporate materiality is the trait that particularly distinguishes human cognition from that of other species. In this view, cognition is *extended* (i.e., includes materiality as a constitutive component) and *enacted* (our interactions with materiality are “transformational and not merely informational”; di Paolo, Rhohde, & de Jaegher, 2010, p. 39). As a cognitive component, materiality has agency and semiotic function (Malafouris, 2013): As we engage them or the world through them, material forms shape our behavioral and psychological responses, and over generations of such interactivity, our material forms become increasingly fine-tuned to elicit specific psychological and behavioral responses. Materiality acquires semiotic value through this interactivity: What it is and what we do with it give it meaning (Malafouris, 2013). The latter is particularly important in numerical cognition, as materially based meaning precedes language for numbers, and change in material form can alter how numbers are conceptualized and structured.

Since numbers involve material forms and such artifacts and behavioral traces may be all there is for reconstructing past ways of thinking, archaeology, as the science of materiality past and present, is the discipline best equipped to discern where and when numbers originated. Certainly, the material records of Middle Stone Age (MSA) Africa and Upper Paleolithic (UP) Europe provide tantalizing hints of possible counting devices: stringed beads from Blombos Cave, South Africa (75,000 years ago; d’Errico, Henshilwood, Vanhaeren, & van Niekerk, 2005), a notched bone from

Border Cave in the Lebombo Mountains, South Africa (42,000 years; d’Errico et al., 2012), and stenciled handprints in Cosquer Cave, France (Clottes, Courtin, & Vanrell, 2005; Overmann, 2014; Rouillon, 2006). Determining whether such artifacts might have represented counting, however, can be difficult to achieve through archaeological analysis.

Prehistoric devices may look like modern ones for counting, but resemblance is an unreliable criterion, subject to typological imposition. Prehistoric devices are often isolated in time and space, so they cannot signify unambiguous meaning or change over time. Manufactural characteristics (e.g., surface disposition and the use of same or different tools in creating marks; d’Errico, 1991, 1998) and use-wear patterns are often ambiguous regarding the possible use of prehistoric devices for counting, especially when there are other, quite plausible social purposes (e.g., beads are ornaments, notches can decorate, and handprints have ritualistic intentions we may never fully grasp). Further, counting in traditional societies generally involves materials unlikely to leave an archaeological trace—fingers, gestures, notched sticks, knotted strings, torn leaves, body paint, marks on the ground, pebbles, grains, and so on—implying that the archaeological record may underestimate the time depth for counting, perhaps significantly so, especially on timescales and in environments where materials readily and reliably decay.

EVOLUTIONARY PRECURSORS OF THE HUMAN CAPACITY FOR NUMBERS

Besides the material record, we can consider the other components in the cognitive system for numbers (e.g., brain function and form, physical anatomy, and behaviors). As material forms are recruited and incorporated, the other components interact with them to produce number systems with striking cross-cultural similarities (e.g., they start with subitizable quantities and often use the hands to represent numbers; behaviors like pairing and one-to-one correspondence can scaffold numerical concepts, thus obviating the need for pre-existing numerical lexicons). The ability to recruit and incorporate materiality into numerical cognition has several evolutionary precursors, including quantity perception, language, concept formation and manipulation, parietal encephalization, and categorization. Many of these precursors have implications for the timeline on which the human capacity for numbers likely developed. Others bear on what numbers are as concepts, or they inform the materiality, behaviors, and psychological functioning used in numbers, including those of the past. Still others are integral to the way humans think about numbers, and influence the material forms used to represent and manipulate them, just as the material forms influence structure and other characteristics in numerical concepts.¹

¹ While the discussion here focuses on how number systems elaborate, it is not assumed that they necessarily do so. Absent the requisite conditions (e.g., size and internal complexity of groups; contact between groups) that motivate elaboration, number systems may be stable over long periods of time or possibly decline through disuse, rather than grow and elaborate.

Quantity Perception

Objects have quantity, and organisms have the ability to appreciate quantity, the perceptual ability known as *numerosity* (Piazza & Izard, 2009). Besides humans, a range of other species has demonstrated the ability to appreciate quantity—not just other primates but mammals generally, as well as birds, amphibians, fish, and perhaps even insects. Thus, it is reasonable to assume the hominin species ancestral to humans appreciated quantity too, and that the ability would have been a function of the parietal lobe, as it is in contemporary primate species (Orban et al., 2006). Yet despite demonstrating the ability to appreciate quantity, no other species has, or is suspected of having, number concepts. Something other than the ability to subitize (i.e., rapidly and unambiguously distinguish small quantities up to three or four) and appreciate magnitude (discern bigger from smaller in quantities above the subitizing range if the difference is above a noticeability threshold) is needed to yield numbers from numerosity.

Language

Linguists have speculated that the ability needed to crystalize concepts of discrete quantity from the perceptual experience of quantity is language, or a computational capability that underlies both language and numbers to which language provides access (Chomsky, 2004; Hurford, 1987, 2007). However, cross-linguistic patterning in numerical language suggests the influence of language-external mechanisms, rather than rules inherent to language alone. For example, the first lexical numbers to emerge across languages share a characteristic “one, two, three, many” patterning (e.g., Menninger, 1992)², as does grammatical number (the differentiation of singulars from plurals) if and when languages develop this subsequent feature (Overmann, 2015). As this patterning is consistent with the perceptual experience of quantity, it suggests that the structure of language is influenced by the perceptual experience of quantity. Another language-external mechanism is suggested by the way words for numbers are compounded. Grouping emerges because it becomes increasingly difficult to distinguish quantities above about four, known as the subitization constraint. Such grouping is often done by amounts that imply the fingers, “ten-ness” that language absorbs in producing lexical terms for higher quantities. That is, there is no inherent “ten-ness” to language; rather, grouping reflects the use of material forms (i.e., the fingers), implying linguistic structure is influenced by the material forms used for numbers.

Language is undoubtedly key in explicating, expressing, and conventionalizing numerical concepts (e.g., Saxe, 2012). However, the assumption that humans have

² Menninger’s work, originally published in 1934, is dated but provides an example of the myriad observations that the first numbers to emerge are consistent with the perceptual experience of quantity (e.g., individuated names for subitizable quantities and “many” for unindividuable non-subitizable quantities). There is in fact significant variability in how these emerge: for example, in whether “one” emerges before “two,” “two” before “one,” or both together as defined against one another (e.g., Closs, 1993), and, if there is a “three,” whether it is formed as “two and one,” “one and two,” “one and one and one,” or is uniquely named. See discussion in Epps (2006) and the reply to Coolidge’s commentary on Overmann (2015).

numbers because they have language ignores the semiotic meaningfulness of material quantity to perceptual experience. Further, while it is necessary, language is also insufficient for developing numbers, since all human societies have language but some lack numbers (most famously, the Pirahã of Amazonian Brazil). Numerical cognition is also highly dependent on manuovisual access to the material forms that make numerical concepts tangible, manipulable, and representable (Malafouris, 2010). Indeed, language has been called a “slow and hesitant” means of accessing numerical intuitions (Sfard & Linchevski, 1994, p. 198). Further, the prevalence of alinguistic numerical representation by means of fingers, gesture, and material devices (attested ethnographically) suggests that number concepts develop through material engagement in advance of the development and availability of an associated vocabulary. This in turn suggests the ability to manipulate materiality—indeed, the ability to incorporate materiality into the cognitive system for numbers as a constitutive component (Malafouris, 2013)—is the critical piece that yields number concepts from numerosity.

Concept Formation and Manipulation

When extant brains perform mathematical tasks, they recruit neurological circuits involved in planning and executing motor movements (Andres, Seron, & Olivier, 2007; Heimann, Umiltà, & Gallese, 2013; Penner-Wilger et al., 2007; Tschentscher, Hauk, Fischer, & Pulvermüller, 2012). The temporal roots of recruiting motor circuits to manipulate concepts may be quite deep. About 2 million years ago, the habilines demonstrated the earliest unambiguous sign of a new relationship with materiality: They began to retain and reuse the stones they had fashioned and used, rather than discarding them and creating new ones on subsequent occasions. This separation of an object from the process for which it had been made and used created what may have been the first concept of “tool” as an object with an existence separate from both process and maker, a concept that likely recruited the motor circuits involved in tool manipulation (Coolidge & Wynn, 2018).

The use of “neural muscles” in manipulating both objects and numbers is demonstrated by the mental abacus, a physical procedure in which practitioners use their fingers to operate an imaginary device to perform mathematical calculations with remarkable complexity, accuracy, and speed. Mental abacus performance is uninterrupted when practitioners keep their hands motionless but disrupted if the hands perform a different task, implying that motor circuit planning but not execution is essential (Brooks, Barner, Frank, & Goldin-Meadow, 2014; Frank & Barner, 2012). However, we need not suspect the habilines of developing number concepts, but merely recognize that the ability to leverage motor circuit planning to manipulate both tools and concepts likely began with them, which highlights the temporally deep roots of incorporating materiality into human cognition.

Parietal Encephalization

Anatomically modern humans (AMH) appeared in the fossil record about 200,000 years ago (Shea, Fleagle, & Assefa, 2007).³ One of the traits distinguishing

³ More recent evidence appears to push this date back to 300,000 years ago (Richter et al., 2017).

AMH from earlier species is the characteristic globular shape of the skull, influenced by the encephalization of the parietal lobe (Bruner, 2004, 2010, 2014; Bruner, de la Cuétara, & Holloway, 2011; Bruner et al., 2004; Bruner, Manzi, & Arsuaga, 2003). This volumetric increase is pertinent to numbers because the parietal lobe is the region of the brain most closely associated with them, including quantity perception (Piazza & Izard, 2009); numerical reasoning (Amalric & Dehaene, 2016); finger-counting (Kringinger et al., 2011); losing the abilities to finger-count and reason numerically (Roux, Boetto, Sacko, Chollet, & Trémoulet, 2003); manipulating numbers in verbal form (Dehaene, Piazza, Pinel, & Cohen, 2003); retrieving arithmetic facts from memory (Grabner, Ansari, et al., 2009); mathematical competence (Grabner, Ischebeck, et al., 2009); abstraction (Harvey, Klein, Petridou, & Dumoulin, 2013); metaphorizing to express abstract concepts like numbers (Lakoff & Núñez, 2000); inner speech (Geva et al., 2011); and the recursive “merge” function responsible for building compound words for higher numbers (Hung et al., 2015).

Globularity would have maximized inter- and intra-regional connectivity and avoided some of the evolutionary trade-offs associated with encephalization (Gibson, 1991; Kaas, 2000; Rilling & Insel, 1999), while parietal expansion would have co-located regions involved in number with those supporting control of the fingers, tool use, and language functions like metaphorizing and inner speech, facilitating their interaction, which seems critical to expressing numerical intuitions linguistically (Coolidge & Overmann, 2012). The general relation between a brain region's size and the adaptive value of its psychological function (Rehkämper, Frahm, & Mann, 1995) relates parietal expansion to important aspects of the human ecological niche. However, the extent to which gross morphological change can index concomitant or subsequent neural reorganization or change in psychological function is unclear. Certainly, the idea that there may have been a lag, possibly as much as tens of thousands of years, between the appearance of the modern skull and the onset of behavioral modernity suggests that anatomical modernity may do little more than signal neuroanatomical developments later supporting parietal functions important to numbers. The lag certainly reflects matters like the limits on radiocarbon dating materials older than 50,000 years and has been strongly challenged on definitional and preconceptual grounds (e.g., McBrearty & Brooks, 2000). Minimally, however, the change in gross morphology associated with AMH suggests the requisite neuroanatomy for numbers may have been available as early as 200,000 years, with the requisite functionality for numbers likely being realized at some subsequent time.

Categorization

As concepts, numbers involve categorical judgments of *relations*, or sameness and difference in multiple elements (e.g., AA and BB share the quality of containing identical elements, while AB and AD do not; Christie & Gentner, 2007). Numbers start with the judgment that two sets of objects share or differ in quantity (the property of *cardinality*, the number of objects in a set), subsequently represented by one of the sets in the comparison (often the hands, setting up the basic patterning

by fives and tens). Such comparisons take the form of pairing and one-to-one correspondence, behaviors that manipulate sets of objects into arrangements whereby their cardinality can be compared (and for which language for numbers is not required). Like humans, other species can form categorical judgments of *identity* (i.e., A is identical to A but differs from B). Non-human great apes can also form categories of relations, but for them it is neither intuitive nor effortless (like it is for human children), requires specific training (while it is spontaneous in humans), and even with training never appears to become as powerful (Christie & Gentner, 2007; Langer, 1986; Thompson & Oden, 2000; Thompson, Oden, & Boysen, 1997). Non-human great apes also apparently find it difficult to distinguish cross-dimensional relations (e.g., recognizing that aaaaaaaa and BB are both composed of identical elements, aaaaaaaa and cde share small letter size, and BBB and %^&, which share neither elements nor size or kind, are both trios; Gentner & Colhoun, 2010).

Human categorization exceeds that of even closely related species, partly because of our larger brains (cortical volume, adjusted for body size, correlates with intelligence; Falk, 2011) and partly because of our ability to exploit materiality, behavioral complexity that is increasingly attested after 100,000 years. Our use of material culture is not merely motivating (i.e., things to count and reasons to count them); rather, our embodied interactivity with materiality has the potential to generate concepts (e.g., as categorizations of shared quantity are opportunized by behaviors that rearrange sets of objects). Material structures provide anchoring and stability for categorizations (Hutchins, 2005), a continuity that the brain cannot achieve on its own (Fauconnier & Turner, 1998), and a means of representing and manipulating them (Malafouris, 2010). Thus, incorporating material culture into the human cognitive system may be why our categorization has become more complex than that of other primates. Timeline-wise, in signaling social identity, the Blombos beads almost certainly index categorical judgments of social relations, along with the potential for forming other types of relational categories at 75,000 years.

Ordinality

Given concepts of *how many*, their ordering by increasing magnitude is critically important, giving numbers “most of their mathematical properties” (Russell, 1920, p. 29). Sequence, the ability to order, informs a wide variety of cognitive functions, including language, perception, executive functioning, organization and timing, learning, and movement (Bapi, Pammi, Miyapuram, & Ahmed, 2005; Salthouse, 2005). In numbers, sequence informs the ordering of counting numbers from low to high. There is evidence suggesting that sequencing involves dedicated neural circuitry: The condition known as synesthesia implies “cross-talk” between neural networks (e.g., an ordinal category experienced in color implies that neural activity related to the latter intrudes into neural activity related to the former) (Cohen Kadosh, Gertner, & Terhune, 2012; Gertner, Henik, Reznik, & Cohen Kadosh, 2013). Given possible dedicated neural circuitry and the fact that what humans sequence are cultural items (e.g., beads, days of the week, alphabets), the ability is likely to have co-evolved with and through our engagement of material culture (Wynn, Overmann, Coolidge, & Janulis, 2017).

Demographic Density

However, just as we did not suspect the habilines of numbers, we need not assume archaic humans were necessarily capable of using even restricted numbers⁴ as recently as 75,000 years ago, despite their modern anatomy and behavior. In addition to numerosity, “neural muscles” for concept formation and manipulation, enhanced parietal functionality, and the expanded categorizing associated with material culture, yet another critical piece was needed: demographic density. There is an oft-noted (though poorly explained) correlation between the size of and contact between social groups and number system complexity (e.g., Epps, Bowerin, Hansen, Hill, & Zentz, 2012). Small groups living in relative isolation are more likely to have restricted numbers. Currently classified as restricted, for example, is the Pirahã number system, which has been variously described as having no numbers at all or counting only to one or two (Dryer & Haspelmath, 2013; Everett, 2005; Gordon, 2004). Simply, small or isolated groups may be able to manage their resources quite well without counting; groups that become larger and/or less isolated have increased resource management needs that they respond to by developing or extending counting.

For prehistoric peoples, the archaeological evidence suggests that for much of prehistory (i.e., prior to the Neolithic⁵), small groups lived in contact situations ranging from relative isolation to at least some contact with other groups, and which could also include more extensive contact, suggested by exotic materials being found at significant distances from their origin points. This broad scenario, were we talking about extant rather than prehistoric peoples, would be consistent with restricted numbers while admitting the possibility of some systems with numbers higher than 20 (i.e., in contexts with greater contact, etc.). If the “but archaic and extant brains aren’t identical” objection is set aside, a reasonable inference based on the correlation between number system complexity and social demographics is that with the evolutionary prerequisites in place, prehistoric peoples could have had restricted numbers 75,000 years ago, well before the Neolithic. In fact, this inference fits with the artifactual evidence: The African beads, African bone, and Cosquer Cave handprints accumulated quantities consistent with restricted numbers.

INSIGHT INTO RECENT NUMERICAL PREHISTORY

For additional insight into the prehistory of numbers, we can come at the problem from the other direction, starting not with the earliest possible artifacts but the first unambiguous numbers and working backward to their likely precursors. The earliest archaeologically attested numbers come from the Ancient Near East (ANE), southern Mesopotamia in the Neolithic (8300–4500 BC). The cultural groups most closely

⁴ Restricted numbers count no higher than 20 (Comrie, 2013) but often no higher than much smaller numbers, like 2 or 3.

⁵ During the Neolithic, group sizes significantly increased, from dispersed hunting and gathering groups to small settlements and villages, to larger settlements, and ultimately to urban city-states. While this demographic change occurred at different times in different parts of the world, it relates directly to the requisite conditions for numerical elaboration and was presumably attended by it.

associated with these earliest numbers, and with the invention of writing as well, are the Sumerians (the first urban civilization in what is now southern Iraq), Akkadians (the first Semitic-speaking Mesopotamian empire in northern Iraq and Syria), and Elamites (an ancient cultural group in southwestern Iran). By the mid-fourth millennium BC, Mesopotamia was characterized by unprecedentedly large populations living in dense urban centers with intensive agriculture, massive construction projects, and intense periodic warfare, matters requiring extensive workforces and bureaucracies. The earliest unambiguous numbers took the form of numerical impressions on clay tablets. These are understood as an intermediate form between the clay tokens used for numbers in the Neolithic and the proto-cuneiform and cuneiform numbers used in later periods (into the Old Babylonian period, 1900–1600 BC): They are temporally intermediate, and they also bridge the earlier and later technologies through correspondences of shapes and sizes (Amiet, 1972; Schmandt-Besserat, 1992a). The correspondences warrant interpreting at least some of the Neolithic clay tokens as having numerical purpose (Englund, 2004a; Høyrup, 2002).

Objects purported to have been numerical tokens have been found as early as the 10th millennium BC (Moore, 2000). Various problems challenge interpreting such objects as numerical earlier than the fourth millennium (i.e., when their shapes and sizes unambiguously resemble numerical impressions). For example, numerical purpose is difficult to construe from the dimensions of clay objects; archaeological context is often ambiguous or undocumented; tokens may quite plausibly have been games, toys, or funerary or ritualistic offerings; and there are large disparities between the archaeological prevalence of tokens and the commodities they are purported to have represented (Englund, 1998a, 1998b; Friberg, 1994; Shendge, 1983; Zimansky, 1993). Further, even at the point where their numerical interpretation is most secure, tokens did not comprise a single system for representing numbers but several, with shape and size conventions varying by the commodity enumerated, city of use, and time (Friberg, 1994; Nissen, Damerow, & Englund, 1993). Conservatively, numerical notations and corresponding tokens are unambiguous in the mid-to-late fourth millennium; numerical representation prior to notations (i.e., by means of tokens only) grows increasingly ambiguous the further back in time being considered, despite the archaeological record of token finds.

In whatever period they are reasonably construed to have represented counting, tokens are often discussed as if they were the first material technology used for ANE counting. However, there are several reasons to think they were not. First, token shapes and sizes encoded metrological relations (i.e., relations between things like containers of grain or jars of oil, similar to the idea that four quarts equals a gallon; Nissen et al., 1993). This “bundling” meant that one token of a higher value was equivalent to some number of tokens of a lesser value: as few as 2 but, more often, as many as 6 or 10. Importantly, quantities like 6 and 10 are not subitizable. Non-subitizable quantities must be counted to be meaningful, implying that the people who used them would have been able to count beyond the subitizable range.⁶ Indeed, when numerical tokens

⁶ Alternatives include either an inefficient procedure to place tokens into one-to-one correspondence with a known standard or an inexact estimation of the (non-subitizable) quantity represented by tokens. Either would have represented a pressure toward achieving greater efficiency and accuracy.

Figure 20.1. Exponential dimensions of counting devices. (Left) In a tally, the potential relations between numbers are one-dimensional (intra-exponential only) and include greater than, less than, not equal to, falls between, and plus-one. Total value is achieved by accumulating value along the single dimension. (Right) In the archaic ANE notational system for counting objects (shown: sexagesimal system S for counting most objects; Nissen et al., 1993), the relations between numbers are explicit and include those of an ordinal counting sequence (intra-exponential, the horizontal axis), as well as the relations implicit in bundling (inter-exponential, the vertical axis). Total value is achieved by accumulating value along both dimensions. The two-dimensional complexity of tokens implies one or more one-dimensional precursor (e.g., tallies, fingers). Image by the author. Developed and included as Figure 8.10 in Overmann (2016b), *Materiality in numerical cognition: Material engagement theory and the counting technologies of the Ancient Near East*, doctoral thesis, University of Oxford.

and impressions overlap in the late fourth millennium, they attest to a number system that had become capable of representing not only small non-subitizable numbers but ones in the hundreds and thousands. For example, the Cuneiform Digital Library documents at least nine clay artifacts with impressions that include a sign interpreted as 3,600 (“Cuneiform Digital Library,” 2015), implying a number system that had become fairly well elaborated, as consistent with demographic factors of group size (extremely large) and inter-group contact (extensive).⁷ Bundling also meant that tokens were related not only to the commodity they counted (a single dimension, which fingers and tallies instantiate) but to each other as well (a second exponential dimension), making tokens a relatively complex technology (Chrisomalis, 2010 and Figure 20.1). Two-dimensionality suggests that at least one and possibly several one-dimensional technologies necessarily preceded the two-dimensional tokens. And indeed, there is evidence of one-dimensional precursors in the region: tallies and finger-counting.

Worked bones that may have been used as tallies have been found in the Levant and dated to the Late UP (Davis, 1974; Reese, 2002; Tixier, 1974). Few similar artifacts have been found in Mesopotamia in the Neolithic, Chalcolithic, or later Bronze Age, a circumstance consistent with both non-use and failure to preserve. The latter possibility is quite likely, as organic materials like bone are known to preserve

⁷ This elaboration would not have occurred in the absence of numerical concepts or words, even if the words themselves remain unknown because there was no writing system capable of expressing them at the time.

poorly in the region's climatic conditions (Coinman, 1996). Tallies are mentioned in second-millennium texts (e.g., "The debate between grain and sheep," 2005, lines 130–133). Broadly, however, tallies and tokens were used by distinct cultural groups inhabiting neighboring regions at different times. There is strong genetic differentiation (Lazaridis et al., 2016) between the Levantine hunter-gatherers of the Late UP and the Zagros Mountain groups who expanded into southern Mesopotamia during the Neolithic transition. These groups presumably represented separate numerical traditions, suggested by use of distinct bases and lexical terms by the Sumerians, Akkadians, and Elamites.

Two circumstances support the possibility of contact diffusion between these groups. First, in rejecting the idea that farmers displaced the earlier Levantine and Iranian hunter-gatherers during the Neolithic transition, "the spread of ideas and farming technology moved faster than the spread of people, as . . . determine[d] from the fact that the population structure of the Near East was maintained throughout the transition to agriculture" (Lazaridis et al., 2016). As agriculture spread through contact, numerical ideas were likely to as well, since numbers are extremely likely to diffuse between groups (attested by the prevalence of numerical loan words and similar phenomena in extant languages and number systems; Comrie, 1999, 2005). Even infrequent inter-group contact has the potential to spread numerical words, concepts, and technologies, especially given the impetus of interactions like trade; further, it need not be assumed that spreading was the only effect, since contact would have helped motivate numerical development (i.e., for purposes like ensuring reciprocity in trade).

Second, the correlation between social demographics and numerical elaboration suggests that the small, nomadic groups in periodic contact with one another in Late UP Levant could well have counted within the range of restricted numbers, given that possible tallies bearing notches within the restricted range have been found (Reese, 2002). Further, whether they were influenced by contact with Levantine groups or developed an independent numerical tradition, the Zagros populations expanding into southern Mesopotamia would have been under pressure to develop and elaborate higher numbers to support the resource management needs associated with increased group size and sedentism (15,000–10,000 BC), animal husbandry and small-scale cereal cultivation and storage (by 7000 BC), and cattle domestication (after 6500 BC) (Robson, 2008). Given the level of numerical elaboration attained by the late fourth millennium, a numerical prehistory concurrent and consistent with the demographic changes of the Neolithic transition is more plausible than the idea that numbers emerged suddenly and with two-dimensional complexity in the late fourth millennium.

Fingers are another possible one-dimensional precursor for the Neolithic tokens. Finger-counting is attested by a characteristic organization of numerical representations in groupings of 5's, 10's, and 20's, both as notations and words. For example, when the Elamites adopted the written Sumerian numerical representations whose cycles were based on tokens and metrological relations, they reorganized them as a decimal system (Englund, 2004b). Then, as cuneiform became capable of representing not just semantic value but phonetic information by the mid-third millennium, it showed that Sumerian had number-words organized by 5's, 10's, and 20's: The words for "six" through "nine" were "five-plus" compounds (e.g., the word for

“six” was “five-plus-one”); the word for “30” meant “three 10’s” and “40” meant “two 20’s”; Blažek, 1999). Further, Akkadian (a Semitic language) was a decimal system, as numbers in Semitic languages tend to be (Friberg, 2007).

Textual evidence⁸ of finger-counting has several important implications for ANE numerical cognition. First, it shows that the Mesopotamian peoples used their fingers for counting, just like extant peoples do (the latter is attested by the prevalence of decimal, quinary, and vigesimal bases and cycles in known number systems, and the persistence of “five- and ten-ness” and finger-counting despite significant levels of numerical elaboration). Second, the presence of finger-counting in modern emerging number systems implies that finger-counting may have emerged early in ANE counting (i.e., earlier than the later metrologically based systems of numerical tokens, impressions, and notations and perhaps earlier than the possible tallies found in Levantine contexts). An early emergence would be consistent with the neurological integration between the parts of the brain controlling the fingers and appreciating quantity, the use of the fingers in manipulating enumerated objects, and the ready availability of the fingers as a material device for representing quantity⁹ (Overmann, 2016c). Finally, the region contained at least three indigenous numerical traditions in contact with one another.¹⁰ Sumerian, Elamite, and Akkadian were distinct languages,¹¹ and differed in the details of their numerical representations (e.g., methods of compounding larger numbers from smaller ones). The interaction between these separate numerical traditions (demonstrated by notational adoption and shared linguistic forms for higher numbers) implies opportunities for numerical elaboration as a response to increased needs for numbers emerging from demographic factors, social purposes like resource management and trade, and the illuminating contrast of numerical differences.

The idea that ANE peoples realized their numbers in similar fashion to extant peoples is further supported by linguistic evidence. In contemporary languages, both the earliest lexical numbers (Menninger, 1992) and grammatical number (Corbett, 2000) take the general form “one, two, three” and (big and small) “many” across

⁸ Though textual evidence of numerical language occurs much later than archaeological evidence of numerical impressions, tallies, and tokens, this is likely an artifact of lacking a writing system capable of preserving both semantic and phonetic components of language (“glottographic” writing), rather than evidence of a complex, two-dimensional number system springing into existence with unprecedented and unique rapidity.

⁹ Because Mesopotamian people were characterized by anatomical and behavioral modernity, there is no reason to exclude them from sharing these traits.

¹⁰ There is also evidence of other number systems in the region, a ternary counting system and an emesal or Sumerian dialect spoken by women that contained numbers (Lambert, 1969; Whittaker, 2002). Ternary numbers occur in scribal and priestly (i.e., male) contexts (Lambert, 1969). The emesal was possibly a literary convention rather than a woman’s dialect (Whittaker, 2002), but this seems implausible because an invented counting system would be a curiously specific detail. Alternatively, either may have represented emergent number systems from a minority cultures in contact with Sumerian, Akkadian, and Elamite populations.

¹¹ Sumerian and Elamite are linguistic isolates, unrelated to any known language including each other, and Akkadian is an East Semitic language.

languages and cultures. This patterning conforms to and thus likely reflects the perceptual experience of quantity—subitizing and magnitude appreciation. While there is no linguistic evidence of the earliest form of their lexical numbers, “one-two-three-many” patterning does characterize grammatical number in Sumerian, Akkadian, and Elamite. Grammatical number, which distinguishes singulars from plurals, became expressible in written Sumerian, Akkadian, and Elamite during the third millennium (Dahl, 2015). As grammatical number emerges subsequent to lexical numbers (Corbett, 2000; Overmann, 2015), the presence of grammatical number in the three ancient languages implies an earlier emergence for lexical number words, though this is a relative (not absolute) chronology. Further, slight but significant differences in grammatical number suggest that the three languages developed the feature independently; this in turn suggests that the numerical traditions they reflect developed independently as well.

In total, the linguistic and archaeological evidence suggests that the Mesopotamian peoples developed their numbers in similar fashion to the way extant peoples do—through the perceptual experience of quantity, the comparison of sets for shared (or dissimilar) cardinality, and the use of fingers for numerical comparisons and representation—and that their numerical development may have started much earlier than the fourth millennium BC—possibly as early as the Late UP (in conjunction with or earlier than the Levantine tallies).

MESOPOTAMIAN ARTIFACTS AND THEIR EFFECTS ON NUMERICAL COGNITION

Analysis of the agency and semiotic function (Malafouris, 2013) of the material technologies for counting offers the possibility of gaining additional insight into their role in numerical cognition. One aspect of material agency is the idea of *affordance*, potential utility that organisms can recognize and exploit (Gibson, 1977; Greeno, 1994; Jenkins, 2008). Affordances can be encoded in material artifacts, making them available for other individuals and generations and distributing cognitive effort over space and time (Hutchins, 1995; Smith, 2007). Different material artifacts offer different affordances for counting, and newer material forms can represent extensions of and reactions to older material affordances. Essential to such an analysis is an extended sequence of material technologies used for counting and calculating, like that of the ANE (Table 20.1), which would have spanned the realization of initial number concepts (perhaps as early as the Late UP) and their elaboration into a complex mathematical system (by the Old Babylonian period). Each material form would have provided capabilities and structure, as well as limitations that would eventually motivate the incorporation of new material forms, which in turn would have been selected on the basis of affordances they shared and capabilities that differed from those of previous forms. New material forms would have solved one or more limitations of a previous form while injecting new limitations of their own; they would also have facilitated numerical elaboration by introducing new affordances, contrasts with older affordances, and the potential for generating novel patterns.

Table 20.1. Affordances and Limitations in Artifacts Used as Material Counting Technologies in the Ancient Near East

<i>Artifact</i>	<i>Affordances</i>	<i>Limitations</i>
Fingers	<ul style="list-style-type: none">• Neurologically integrated with quantity perception• Psychological–behavioral–material bridge• Available; manipulate the objects being counted• Linearity; stable order	<ul style="list-style-type: none">• Limited capacity; ephemeral• Commodity unspecified
Tallies	<ul style="list-style-type: none">• Linearity; stable order• Accumulation• More capacity; persistent	<ul style="list-style-type: none">• Commodity unspecified• More than three or four notches are difficult to discriminate visually; not manipulable; one-dimensional
Tokens	<ul style="list-style-type: none">• Linearity; stable order (imposed: not implicit in the form)• Accumulation; greater capacity; persistent• Commodity encoded• Grouped (more discriminable); manipulable (operations; relations); two-dimensional	<ul style="list-style-type: none">• Loose (imposing the need for containment); not concise; multivalent
Notations	<ul style="list-style-type: none">• Linearity; stable order (imposed)• Many operations; greater capacity; persistent• Commodity specified apart from number• Grouped; two-dimensional• Integrity of form; concise; monovalent• Handwritten (fusiform gyrus becomes trained; improved hand–eye coordination; improved character recognition and recall); conceptualization as objects• Whole–part relations (with tokens)• Tables (e.g., multiplication; reciprocals)• Greater complexity of calculation	<ul style="list-style-type: none">• Fixed (must calculate with tokens or manipulate relations between numbers with new algorithms)

Note: The technologies used for counting in the Ancient Near East represented four key transitions. Fingers influenced basic structure in numerical concepts (e.g., linearity; stable order). Tallies represented the transition to material culture (i.e., as opposed to using the body for counting). Tokens represented the emergence of knowledge-based numeration. Finally, written notations enabled numbers to become conceptualized as entities. Affordances and limitations show continuity and discontinuity across technologies, representing factors that motivated and influenced the selection of new material forms. Previously published as Table 1 (p. 10) in Overmann (2017), *Concepts and how they get that way, Phenomenology and the Cognitive Sciences*, available online October 31, 2017 through Springer Science + Business Media B.V. (<https://doi.org/10.1007/s11097-017-9545-8>).

Fingers

The digits of the hands are often an early material form used to represent number concepts, for several reasons. First, the neural interaction between the sensorimotor control of the fingers and the perception of quantity implies that finger-counting would be cognitively prepotent and likely explains its ubiquity and persistence across significant differences of numerical elaboration. Second, fingers manipulate enumerated objects, thereby bridging the psychological, behavioral, and material components of cognition (Gallagher, 2013; Mattens, 2013). Third, fingers are simply ready to hand to aid memory and mitigate attentional limits as needed. Across cultures, finger-counting becomes habitually ordered, often from an outside finger across the hand (Overmann, 2014), imparting linearity and stable order, or *ordinality*, sequential ordering by increasing magnitude (Gelman & Gallistel, 1978). This culturally inflected patterning makes numerical information more accessible (relative to random ordering or starting with the inner fingers) by decreasing demands on attention and memory and improving visual distinguishability (the latter is particularly important in *finger-montring*, the display of finger-counting patterns to others). Ordinality emerges from the interaction of psychological, behavioral, and material components: memory and ordinal sequencing, motor efficiency and habituation, and the hand's pentadactylism. However, fingers do not specify what they count, information that must be maintained separately, either in context or in memory, where it is subject to processes like interruption and forgetting. Fingers also have a relatively limited capacity for enumeration (most typically, five per hand), and they are perishable in the sense that sooner or later, the hands are needed for something other than recording quantity. These limitations may motivate the incorporation of new material forms with greater persistence and capacity for higher quantities.

Tallies

Material forms incorporated to address the fingers' limitations are likely to be structured similarly (e.g., notched tallies; knotted strings). Attributes like linearity and stable order are reinforced by the interactivity of psychological and behavioral capabilities with the material form, thereby enhancing the accessibility of numerical information. Like fingers, tallies do not specify what they count, necessitating keeping the information in context or memory. Unlike fingers, tallies solve the problems of persistence and capacity. Further, the motor activity of making notches may help enable the realization of numerical concepts like accumulation and relations like "one more." Tallies are not manipulable: It is difficult to move or subtract a notch once it has been made. Because of the subitization constraint, notches become increasingly difficult to distinguish in quantities of more than three or four; this motivates strategies like grouping or separating notches into subitizable or recognizable amounts during the manufacture process (e.g., groups of five). Because grouping and separating relate notches to each other in addition to whatever they are counting, they add a second dimension of exponential representation (Figure 20.1). Finally, tallies represent the incorporation of materiality external to the body into the cognitive system for numbers, as well as the transition to the use of materiality as a collaborative medium.

Tokens

Artifacts containing both impressions and tokens show that numerical impressions were ordered by increasing magnitude (Nissen et al., 1993). This implies that tokens were ordered in similar fashion, much as fingers and tallies had been and as would be consistent with their efficient and effective use. However, the source of that linearity and stable order is no longer apparent in the material form itself (i.e., loose tokens have a flexibility for rearrangement that notches on tallies and fingers lack). Tokens mitigated the effects of the subitization constraint through bundling, relations between token values that were encoded through conventions of shape and size (and, as discussed earlier, non-subitizable bundling amounts like 6 and 10 imply the ability to count). The shape/size conventions additionally specified what was being enumerated, resolving another limitation inherent in both fingers and tallies. Because they were loose, tokens were manipulable to a degree not possible with tallies or fingers. Manipulability enabled the performance of bundling and debundling operations, affording opportunities to conventionalize operations like subtraction and reciprocation (a type of division), form explicit relations between token values, and develop algorithms for calculating (operations, relations, and algorithms are important because they form the basis of mathematics). However, tokens were loose, and in the mid-fourth millennium, they were placed inside containers made of clay (bullae and envelopes); within centuries, the containers' external surfaces were marked with shapes resembling tokens, obviating the need for tokens inside and today widely credited as the origin of writing and the ability to represent quantity and commodity separately (e.g., Schmandt-Besserat, 1992b, 2010).

Impressions and Notations

Early numerical impressions and proto-cuneiform notations resembled tokens in their shapes, sizes, and bundling relations, conventions used to encode both quantity and commodity. In comparison, cuneiform notations separated quantity and commodity and would ultimately develop a system of place value. Impressions and notations shared the linearity and stable order of the earlier technologies. Unlike tokens, impressions and notations were fixed, making them suitable only for recording and communicating numerical information; until suitable operations and algorithms for manipulating notations were developed, tokens were still used for calculating (Friberg, 1981; Høyrup, 2002). Use of tokens for calculating and impressions/notations for recording implies comparisons between the two material forms that would have facilitated the appreciation of whole-part relations (e.g., those contained in the notion of reciprocals, as 60 relates to 2 and 30) and thus the explication and elaboration of relations between numbers. Unlike any previous technology, cuneiform notations were concise, enabling significant volumes of numerical information to be recorded (e.g., multiplication tables, tables of reciprocals) in a non-perishable, transportable, and easily recreatable medium. Scribes practiced writing tables as part of their training and in the process, learned numerical relations. This gave scribes more options for calculating than using tokens: They could additionally use information from tables or memory, a factor in developing new, complex algorithms for manipulating relations between numbers. The effects of handwriting cuneiform notations (e.g., trained

object-recognition processing) would have intensified the conception of numbers as discrete objects, while the ability to write about manipulating numbers would have facilitated the development of operational concepts (Overmann, 2016a).

Conceptual Change and Structural Persistence

As concepts of discrete quantity, numbers are initially equivalences realized through behaviors like one-to-one correspondence that match fingers and objects. As forms like tallies are incorporated, numbers become collections that share quantity with enumerated objects, and the source of productive grouping once provided by fingers is no longer apparent. As manipulable forms like tokens are incorporated, numbers begin to share quantity with each other, and the source of their linearity and stable order is no longer apparent. Finally, once numbers become entities related numerically to similar entities, the source of the manipulability needed to develop relations and algorithms is no longer apparent. Simply put, numbers acquire attributes like linearity, stable order, productive grouping, and manipulable relations from the material forms used for counting; these attributes remain implicit in both the number concepts themselves and the behaviors that make and use material technologies for counting, but may no longer be implicit in newer material forms. These phenomena will be called, respectively, *conceptual change* and *structural persistence*.

Conceptual change occurs in conjunction with change in the materiality used to manipulate and represent number concepts. Change in material form is a type of abstraction (i.e., decontextualization, the extraction of content from its original circumstances to remove their influence on its meaning). Such abstraction can effectively mask immediate affordances, allowing for the emergence of novel, less constrained behavioral possibilities (Clowes & Mendonça, 2016). An example of such change is the separation of the representation of quantity and commodity (conjoined in tokens) as numerical impressions and pictographic labels, which enabled their further elaboration as mathematical and literate traditions (Schmandt-Besserat, 1992a, 1992b). Another is the development of cuneiform notations, whose concision enabled the creation of numerical tables for representing large volumes of information about the numerical relations between numbers, which in turn helped numbers become conceptualized as discrete entities related numerically to like entities. Similarly, the incorporation of tokens into a tally-based system would have fostered a concept of number that was more manipulable than an ordinal counting sequence and which contained more relations between numbers than were realizable with tallies or fingers.

Concepts may change when material forms change, but structure can persist (e.g., as in the persistence of linearity from fingers to tallies to tokens), for several reasons. One is that the material component, considered across space and time, is really a patchwork in which older and newer forms often coexist (especially when multiple, separately developed traditions interact). Another reason that structure persists is neurological, like the interaction between the intraparietal sulcus and angular gyrus that underlies, potentializes, and perpetuates finger-counting. Third, learning and practice, which represent structured interactions with materiality, are also social behaviors that transmit such knowledge between individuals and generations. Language, used for teaching, also likely reinforces qualities like linearity, as the temporal ordering of speech demands that numbers be expressed sequentially. Additionally, when numbers

are entities with linear order, productive grouping, and manipulable relations, material structure is implicit in the concepts themselves, and this implicit structure may facilitate the retention and use of older material forms. Finally, materiality has agency: What a material form is capable of enables, limits, and structures what people can do with it. The structure of older material forms also influences the structuring potential of newer forms because materiality conditions people to things working in certain ways, and this narrows the range of possible behavioral outcomes. The structuring of potential solutions is also a limitation, since it narrows the range of possibilities to those that bear affinity to previous solutions. This may be why the presentation of unrelated material forms has become an effective technique for sparking novel solutions (Kirsh, 2014).

Several points need highlighting here. First, the development of ANE number concepts is not envisioned as a universal or deterministic process, but rather one whose affinities to other number systems reflects the involvement of shared psychological, physiological, and behavioral capacities, and whose variability reflects unique material choices and combinatorial decisions. Second, behaviors with material forms must be repeated at some threshold of sustained engagement to cause training effects in the brain (e.g., neuronal repurposing or regional coordination). The required threshold is currently undefined, though a similar repurposing, developing literacy from writing behaviors, appears to emerge under the behavioral demand associated with a state-level bureaucracy (Overmann, 2016a). Behavioral sustainment in turn depends on a level of social demand (also currently undefined, through likely more than what would be associated with traditional lifestyles; see e.g., Epps et al., 2012).

UP/MSA ARTIFACTS

Working forward from the evolutionary precursors and then backward from the first unambiguous numbers and their associated material forms allowed the consideration of their likely precursors and affordance interactions. This yielded new insights into the role of materiality in numerical cognition: finger-counting emerges as a function of neurological interactivity, hand availability, and object manipulation, with linearity and stable order emerging from the interactivity between the psychological, behavioral, and material components; follow-on technologies share structure and affordances with previous technologies, while injecting new capabilities and limitations into the cognitive system for numbers; and second-dimensional relations reflect those of both previous technologies and enumerated objects. These insights have implications for interpreting artifacts possibly used for counting in even earlier prehistoric periods in terms of what material forms and their change over time might suggest about matters like agency and affordances, and the availability and extent of a numerical lexicon.

Conservatively, prehistoric artifacts like beads, bones, and handprints need not represent numerical concepts, just as modern rosaries do not; one element (bead, notch) is simply matched with one of whatever is being counted (with a rosary, prayers). However, activities like making notches and stringing beads might plausibly generate numerical concepts (e.g., “accumulation,” “one more”), since they represent psychological–behavioral–material interactivity. As mentioned previously, the earliest

counting may leave no archaeological trace.¹² Artifacts of non-perishable materials may therefore represent a subsequent elaboration and are likely to be one-dimensional technologies (i.e., bridging fingers and two-dimensional technologies) associated with number concepts and terms occurring in the restricted range (i.e., up to 20). One-dimensional accumulation enumerates objects or events, which may or may not be recognizable from the numbers themselves once removed from cultural context (e.g., recognizable: days in a lunar cycle; not recognizable: bride-price). Given increased social needs for numbers, material forms like tallies are predicted to change over time in two ways: first, the quantities they record will increase (i.e., numbers will go up), and second, the quantities will reflect a strategy (e.g., grouping) that facilitates visual differentiation of the marks. Such changes would predictably develop under social pressures for greater efficiency in numbers, typically associated with demographic factors like larger group size and inter-group contact.

A material form like a tally entails interactivity between the psychological, behavioral, and material dimensions of cognition and thus is capable of scaffolding numerical concepts. However, a series of notches is unlikely to generate the same concept of number as that associated with, for example, a numerical notation. That is, the concept is related to the material form used to scaffold and represent it. A tally is a collection that cumulatively represents a discrete quantity. The concept of number instantiated by a tally is accessed by counting the notches or comparing them to a standard, and it represents relatively few relations between numbers (e.g., “more than,” “less than,” and “as many as”). In contrast, a written numeral (e.g., Old Babylonian sexagesimal numbers) is an entity that represents a discrete quantity by itself. The concept of number instantiated by a numeral is apparent from but not reducible to the form and exists within a relational system of numerical entities.

Further, notches on a tally may be collected over time, while numerals are generally handwritten as complete units. The movements and behaviors associated with each productive process engage different psychological processes and imply different opportunities for neuronal reorganization. For example, while numerosity is thought to be repurposed for recognizing symbolic numbers (Dehaene & Cohen, 2007), the extent to which similar reorganization occurs in tally-based numerical recognition is unclear. Similarly, fostering inter-regional coordination, as in recruiting the angular gyrus for arithmetic facts (Grabner, Ansari, et al., 2009; Grabner, Ischebeck, et al., 2009), depends on the availability of arithmetic facts (e.g., relations between numbers) that may not obtain in tally-based numerical representation. Neuronal engagement and training effects also require recurrence of the instantiating behavior, something plausibly associated with social demand. Whether the creation and use of, for example, an MSA tally would fall above the behavioral recurrence threshold required to repurpose neuronal functionality (in one or more individuals) is unknown. This is further complicated by the circumstance that demand thresholds for behavioral recurrence have not been established.

¹² Early counting may leave linguistic traces (e.g., “one-two-three-many” patterning, “five-plus” compounds) that provide insight into past psychological, behavioral, and material characteristics. However, language reconstruction is limited to roughly the past 10,000 years, less or more depending on whether the focus is lexical similarity or structural features (Dunn, Terrill, Reesink, Foley, & Levinson, 2005).

UP/MSA artifacts like the beads from Blombos Cave (d'Errico et al., 2005), the notched bone from Border Cave (d'Errico et al., 2012), and the stenciled handprints in Cosquer Cave (Rouillon, 2006) are either ungrouped (notches and handprints) or ambiguous regarding grouping (beads). This implies number concepts falling within the restricted range, as well as a lack of social pressure for higher numbers and grouping (and this is consistent with established demographic factors). The lack of social pressure, in turn, suggests a limited participation in behaviors associated with counting and thus a limited amount of neural repurposing. Absent social purposes and associated behaviors for numbers related to demographic factors, the elaboration of number concepts to any significant extent may have occurred as late as the Neolithic.

Some 30,000 years separate the Blombos beads from the Lebombo bone, another 15,000 the Lebombo bone and Cosquer handprints. Though all of them are the kind of material forms predictable for early counting, none settles the question of numerical origins or can be reasonably associated with more than restricted numbers. If at least some of them represented counting, as may be the case for the tallies in particular, the temporal span suggests a relative stasis between realizing and elaborating numbers that may in turn reflect the lack of significant demographic pressure. Beyond demographic factors are cultural characteristics like resourcing and technology (e.g., subsistence, sedentism, and specialization) with implications for the elaboration of material culture that emerged in MSA Africa and UP Europe. A rich material culture may be critical to realizing and elaborating numbers: Material culture is an integral part of the cognitive system generally but also provides objects whose value motivates counting, objects whose form enables and structures counting, and objects unrelated to either that may help spark the innovation and creativity that transcend habit and entrenched material structure. And once available, numbers are a cognitive technology that enables the management of complexity, allowing for even greater complexity to emerge.

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