A Note on Non-deterministic Turing Machine and the "P vs. NP (P versus NP problem)"

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摘要: 一个非确定型图灵机 NDTM 的计算过程, 可以相当于其对应的确定型图灵机 DTM 的幂集。如果接受 ZF 公理系统的幂集公理, “P 对 NP” 问题最可能的答案是: 对于确定型图灵机, $P \neq NP$ 。可以从另外 3 个角度对它进行一定的解释。

ABSTRACT: The calculation process of a non-deterministic Turing machine (NDTM) can be equipotent to the power set of its corresponding deterministic Turing machine (DTM). If accepting the “Axiom of power set” of the ZF axiom system (Zermelo–Fraenkel set theory), the most likely answer to the "P vs. NP" (P versus NP problem) is: For a deterministic Turing machine, $P \neq NP$ (P is not equal to NP). This answer can be explained from three other perspectives.

关键词: P 对 NP, 非确定型图灵机, 幂集公理, Zermelo – Fraenkel 集合论公理系统, 确定型图灵机, P 不等于 NP, $P \neq NP$

KEY WORDS: P versus NP problem, non-deterministic Turing machine, Axiom of power set, Zermelo–Fraenkel set theory, deterministic Turing machine, P is not equal to NP, $P \neq NP$

数学证明有演绎和归纳两大类^[1]。对于演绎证明, 任何证明都是相对的, 因为该证明是根据未被证明的前提得到的^[2]。The deduction and induction are two types of mathematical proofs^[1]。For deductive proofs, any proof is relative, because the proof is based on certain unprovable assumptions^[2]。柴庭定理是一个更具体化的说明^[3]。Chaitin's theorems are some more specific explanations for the deductive proofs^[3]。

对于 “P 对 NP”^[4-12], 演绎证明可以得到三种不同的结论^[13-22]: ① 在非确定型图灵机 NDTM 中, P 等于 NP; ② 在确定型图灵机 DTM 中, P 不等于 NP; ③ 没有关于所采用的理论计算机模型的必要说明, 则具有独立性。For "P versus NP" (P versus NP problem)^[4-12], the deductive proofs can get three different conclusions^[13-22]: ① For a NDTM (non-deterministic Turing machine), $P = NP$ (P is equal to NP); ② For a DTM (deterministic Turing machine), $P \neq NP$ (P is not equal to NP); ③ If there is no necessary explanation about the theoretical computer models used, it is independent.

1. ZF 中的幂集公理和非确定型图灵机 “Axiom of power set” of the ZF axiom system (Zermelo–Fraenkel set theory) and the non-deterministic Turing machine

The details can see “A non-canonical example to support that P is not equal to NP” (Transactions of Tianjin University, 2011)^[19] and “第二类计算机构想 Conception of the second class computer” (中国电子科学研究院学报, 2011, Journal of China Academy of Electronics and Information Technology, 2011)^[18].

Encyclopedia of Mathematics has some related entries about the ZF^[23,24].

2. “P 对 NP”另外 3 个角度的解释 Three other perspectives to explain the “P vs. NP (P versus NP problem)”

(1) “P 对 NP”的核心是“指数方式的增长”。从《图论》的角度看,问题的图论模型中包含非平面图 Kuratowski graphs ($K_{3,3}$ and K_5)是“指数方式的增长”一个充分条件^[25-27].

(2) 在无穷化版本下, NPI (NP-Intermediate)的存在性,就是“连续统假设 Continuum hypothesis”^[28].: 可数无穷基数 a 和连续统基数 c 之间是否存在一个其它的无穷基数。

(3) 概率意义下^[29]的“对于 DTM, $P \neq NP$ ”,即“对于 DTM, $P \neq NP$ 几乎处处 (almost everywhere, a. e.)成立。”例如,如果按照正态分布指定边(edge)的权重,完全图(complete graph)上的 TSP^[30] 几乎处处需要指数时间。

3. 从 “P 对 NP” 到 “第二类计算机” From "P vs. NP" to "the Second Class Computer"

如果真的“对于 DTM, $P \neq NP$ ”,则应该研究能够“将几何曲线作为整体处理”^[31-33]的计算机(第二类计算机)。

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