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Metrizable Topological Space

Open Mathematics Collaboration*†

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Abstract

METRIZABLE TOPOLOGICAL SPACE and its underlying definitions are presented in this white paper (knowledge base).

keywords: metrizable topological space, metric, topology, knowledge base

The most updated version of this white paper is available at

https://osf.io/f8vez/download

Definition

1. Metrizable Topological Space

$$(S, \mathcal{T})$$

$$\exists d : \mathcal{T} \text{ is generated from the open balls in } (S, d)$$

$$(d \text{ induces } \mathcal{T})$$

$S :=$ set
$\mathcal{T} :=$ metric topology on S induced by d
$d :=$ metric
$d : S \times S \to \mathbb{R}$

[1, 2]

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Prerequisites

2. **Metric (distance function)**

$$d : S \times S \to \mathbb{R}$$

$S :=$ set

(a) $\forall x, y \in S, \quad (d(x,y) = 0) \leftrightarrow (x = y)$

(b) $\forall x, y \in S, \quad d(x,y) = d(y,x)$

(c) $\forall x, y \in S, \quad d(x,z) \le d(x,y) + d(y,z)$

[1,2]

3. **Metric Space:** (S, d)

$S :=$ set
$d :=$ metric
$d : S \times S \to \mathbb{R}$

[1,2]

4. **Arbitrary Union**

$$\bigcup X$$

$X :=$ collection of sets
$\bigcup X := \{y \mid \exists Y \in X,\ y \in Y\}$
$\bigcup X$ is the union of the elements of X

[1,2]

5. **Arbitrary Intersection**

$$\bigcap X$$

$X :=$ collection of sets
$\bigcap X := \{y \mid \forall Y \in X,\ y \in Y\}$
$\bigcap X$ is the intersection of the elements of X

[1, 2]

6. **Topology** on S

$$\mathcal{T}$$

$S :=$ set
$\mathcal{T} :=$ collection of open subsets of S
$X, Y :=$ collection of sets
$\bigcup X :=$ arbitrary union
$\bigcap Y :=$ arbitrary intersection

   (a) $\varnothing, S \in \mathcal{T}$
   (b) $(X \subseteq \mathcal{T}) \to (\bigcup X \in \mathcal{T})$
       [$\mathcal{T}$ is closed under arbitrary unions]
   (c) $(Y \subseteq \mathcal{T},\ Y \text{ finite}) \to (\bigcap Y \in \mathcal{T})$
       [$\mathcal{T}$ is closed under finite intersections]

   [1, 2]

7. **Topological space**

$$(S, \mathcal{T})$$

$S :=$ set
$\mathcal{T} :=$ topology on S

[1, 2]

8. **Open Ball** (center a, radius r)

$$B_r(a) = \{x \in S \mid d(a,x) < r\}$$

$S :=$ set
$d :=$ metric
$(S,d) :=$ metric space
$a \in S; \quad r \in \mathbb{R}^+$
$B_r(a) \equiv B_r(a;d)$

[1, 2]

Open Invitation

Review, *add* content, and **co-author** this *white paper* [3, 4].
Join the **Open Mathematics Collaboration**.
Send your contribution to `mplobo@uft.edu.br`.

Open Science

The **latex file** for this *white paper* together with other *supplementary files* are available in [5].

Ethical conduct of research

This original work was pre-registered under the OSF Preprints [6], please cite it accordingly [7]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

Acknowledgement

+ **Center for Open Science**
  `https://cos.io`

+ **Open Science Framework**
  `https://osf.io`

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