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The Material Origin of Numbers

Insights from the Archaeology of the
Ancient Near East

By Karenleigh A. Overmann

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The material origin of numbers

Gorgias Studies in the Ancient Near East

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The material origin of numbers

Insights from the archaeology of the Ancient Near East

Karenleigh A. Overmann

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Cover. Mathematical tablet (HS 201) from Nippur. It dates somewhere between the Ur III (2100–2000 BCE) and Early Old Babylonian (2000–1900 BCE) periods and is part of the Frau Professor Hilprecht Collection, University of Jena, Germany. The table is a list of reciprocals, an excellent example of the relational data transforming the concept of number. In 1935, Otto Neugebauer read the last line of the right column as 32 being the reciprocal of 112.5 ('32 igi 1,52,30', *Mathematische Keilschrift-Texte*, p. 10). The photo was produced by Manfred Krebern timer and his assistant, and use of the image is courtesy of the Frau Professor Hilprecht Collection, University of Jena, Germany.

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To Bill: My first, my last, my always, and everything in between.

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CONVENTIONS

TIMELINE

Late Upper Paleolithic	30 to 12 thousand years ago
Epipaleolithic	12,000 years before present to 8300 BCE
Neolithic	8300–4500 BCE
Chalcolithic (Copper Age)	4500–3300 BCE
Pre-Uruk V	8500–3500 BCE
Uruk V	3500–3350 BCE
Uruk IV	3350–3200 BCE
Early Bronze Age	3300–2000 BCE
Uruk III	3200–3000 BCE
Jemdet Nasr (JN)	3200–3000 BCE
Early Dynastic (ED) I/II	2900–2700 BCE
Early Dynastic (ED) IIIa	2600–2500 BCE
Early Dynastic (ED) IIIb	2500–2340 BCE
Old Akkadian (OA)	2340–2200 BCE
Lagaš II	2200–2100 BCE
Ur III	2100–2000 BCE
Early Old Babylonian	2000–1900 BCE
Old Assyrian (Old Assyr.)	1950–1850 BCE
Old Babylonian (OB)	1900–1600 BCE
Middle Assyrian (MA)	1400–1000 BCE
Neo-Assyrian (NA)	911–612 BCE
Neo-Babylonian (NB)	626–539 BCE

ABBREVIATIONS FOR MUSEUM DESIGNATORS

A	Oriental Institute, University of Chicago, Illinois, USA
AO	Louvre Museum, Paris, France
Ashm	Ashmolean Museum, Oxford, England
CUNES	Cornell University, Ithaca, New York, USA
DV	State Hermitage Museum, St. Petersburg, Russian Federation
IM	Iraq Museum, Baghdad, Iraq
JA	National Museum of Syria, Raqqa, Syria
JRL	University of Manchester, England
MS, MT, MW	Private, anonymous collections in Europe

NIM	National Museum, Tehran, Iran
NMSDeZ	National Museum of Syria, Der-ez-Zor, Syria
NMSR	National Museum of Syria, Raqqa, Syria
OIM A	Oriental Institute, University of Chicago, Illinois, USA
Sb	Louvre Museum, Paris, France
T	National Museum of Syria, Damascus, Syria
UM	University of Pennsylvania Museum of Archaeology and Anthropology, Philadelphia, Pennsylvania, USA
VA, VAT	Vorderasiatisches Museum, Berlin, Germany
W	Artifact found at the ancient city of Uruk in Iraq

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CHAPTER 1.

INTRODUCTION

I had been at Oxford for only two weeks when my doctoral supervisor, cognitive archaeologist Lambros Malafouris, introduced me to an eminently distinguished and intimidatingly famous archaeologist, a scholar whose contributions to the field were so legendary they had been recognized not with mere knighthood but peerage. I speak, of course, of Colin Renfrew, Baron of Kaimsthorn, Fellow of the British Academy and the Society of Antiquaries, Emeritus Disney Professor of Archaeology at Cambridge University, Senior Fellow of the McDonald Institute for Archaeological Research, one of the leading pioneers of cognitive archaeology, and Lambros' own doctoral supervisor. With a flattering interest, Renfrew asked me the topic of my thesis. Upon learning it was the clay tokens used as numerical counters in Neolithic Mesopotamia, he snapped, 'Those have been done to death! Why study them? Do something else!' His emphatic doubt about what, if anything, the least bit interesting might be left to say about these artifacts has lingered at the back of my mind the several years since, daring me to address it.

Why study the Neolithic tokens? The question is highly pertinent, since indeed, the tokens have been investigated and published upon, extensively if not exhaustively, since archaeologists like Vivian Broman,¹ A. Leo Oppenheim,² and Pierre Amiet³ first noticed their shapes, sizes, and quantities corresponded to those of the clay impressions that preceded handwritten forms for numbers in the mid-to-late 4th millennium BCE. The tokens' role in the invention of writing has long been proclaimed, they themselves painstakingly catalogued and analyzed, at least as much as they were known up until the early 1990s, by archaeologist Denise Schmandt-Besserat.⁴ Lambros himself⁵ had already explored and written about their significance as components of the cognitive system for numbers, the very inquiry I was proposing to take up yet again. What could possibly be added to such work that was worthy of the undertaking? And, if such

¹ Broman, *Jarmo Figurines*.

² Oppenheim, 'On an Operational Device in Mesopotamian Bureaucracy'.

³ Amiet, *Mémoires de la Délégation Archéologique en Iran, Tome XLIII, Mission de Susiane*.

⁴ Schmandt-Besserat, *Before Writing: From Counting to Cuneiform*.

⁵ Malafouris, 'Grasping the Concept of Number: How Did the Sapient Mind Move beyond Approximation?'.

illustrious scholars had already discovered and said everything it was possible to know on the topic of tokens, what could a mere student bring to the inquiry?

Here it's worth noting I had a somewhat different perspective on numbers than that of these antecessors. For the several years prior to starting at Oxford, I had worked at the University of Colorado at Colorado Springs with psychologist Frederick Coolidge and archaeologist Thomas Wynn. They had introduced me to the idea that change in the form of material artifacts over time could indicate cognitive change in the species making the artifacts. Wynn, another leading pioneer of cognitive archaeology, had discerned change in the spatial cognition of species ancestral to humans through their increasing imposition of shape and symmetry on stone tools.⁶ Coolidge had brought to their partnership the idea of investigating the expansion of working memory through artifacts like traps that implied such executive functions.⁷ In my own work with Coolidge and Wynn, much of my research had focused on understanding how number systems emerge from the perceptual experience of quantity humans share with other species, and how this differs from language.⁸ I had considered the questions of what numbers are and how we get them through the methods and data of psychology, language, and ethnography: what the brain does in numbers, how language puts number words together, and how peoples in different cultures and societies count. I had also investigated the links between language for numbers, cultural complexity, and numerical complexity,⁹ though by the time I arrived at Oxford, I was becoming increasingly dissatisfied with these findings, since they had only concluded, as other disciplines had,¹⁰ that there was some kind of a link, the details of which could not be determined. Further, I had not yet given much thought to what material forms for representing and manipulating numbers had to do with numerical origins and elaboration,¹¹ something Lambros encouraged me to do.

Once I started reading the Assyriological literature, I had enough background in anthropological theory to be disquieted by the claim that numbers were somehow *concrete* before they became *abstract*. This assumption pervaded the work of both Schmandt-Besserat¹² and Malafouris,¹³ as well as that of other scholars writing about numerical origins and developmental acquisition—psychologists Peter Damerow and

⁶ Wynn, *The Evolution of Spatial Competence*.

⁷ Coolidge and Wynn, 'Executive Functions of the Frontal Lobes and the Evolutionary Ascendancy of *Homo sapiens*'.

⁸ Coolidge and Overmann, 'Numerosity, Abstraction, and the Emergence of Symbolic Thinking'.

⁹ Overmann, 'Material Scaffolds in Numbers and Time'; *Numbers and Time: A Cross-Cultural Investigation of the Origin and Use of Numbers as a Cognitive Technology*; 'Numerosity Structures the Expression of Quantity in Lexical Numbers and Grammatical Number'.

¹⁰ Epps et al., 'On Numeral Complexity in Hunter-Gatherer Languages'.

¹¹ Overmann, 'The Role of Materiality in Numerical Cognition'.

¹² Schmandt-Besserat, 'An Archaic Recording System and the Origin of Writing'.

¹³ Malafouris, 'Grasping the Concept of Number'.

Jean Piaget.¹⁴ Since the 1970s, cultural anthropology had more than frowned upon categorizing societies as either primitive or advanced, and as evolving from one to the other. Such proscriptions reacted to the discipline's own origins in the work of 19th-century scholars like Edward Burnett Tylor, who announced his opinion of non-Western societies in the title of his most famous book: *Primitive Culture*.¹⁵ I had also encountered the idea of philosopher of mathematics Bertrand Russell that numbers are the recognition of cardinality shared between sets of objects.¹⁶ This seemed to me to be capable of bridging the gap between the perceptual experience of quantity and the conceptualization of number. These insights suggested the development of numerical concepts from the perceptual experience of quantity not only could but had to be explained without appealing to constructs that invoked inherently flawed and fortunately outdated assumptions about societal modes of thinking.

Other matters that immediately confronted my understanding of what numbers were like were the Assyriological claims that tokens were not used with number words, and that they were the first material form used for counting. From the vantage of my admittedly meager knowledge and experience, I intuited these simply had to be incorrect. Firstly, when numbers initially emerge in language, they are limited to, and quite consistent with, the perceptual experience of quantity: *one* and *two*, possibly *three*, and occasionally *four*, with numbers above that range being *many*, a characteristic noted for number words around the world, across significant spans of time, and by observers from vastly different fields.¹⁷ No known number system uses material forms to represent numbers in the hundreds and thousands, as the Mesopotamian tokens did at the point where impressions provide insight into how the tokens were used, while concomitantly lacking a comparable lexicon of number words; similarly, no known numerical lexicon lags behind the values represented materially to the extent being claimed for Mesopotamian numbers. These things suggested there would have been a numerical lexicon, even if the evidence for it was both limited and much later.

Secondly, although tokens may look rudimentary when viewed from the perspective of the complexity that mathematics reach by the Old Babylonian period (1900–1600 BCE), they do not from the perspective of how numbers first emerge. I had been studying the vocabularies, behaviors, and devices of traditional peoples taking their first, early steps into numbers; the tokens in comparison were overwhelmingly complex, as was the material culture of urbanized Mesopotamian agriculturalists when contrasted with that of nomadic hunter–gatherers. For the tokens, this complexity lay

¹⁴ Damerow, 'Number as a Second-Order Concept'; 'Prehistory and Cognitive Development'; Piaget, *The Child's Conception of Number*; 'Logique Génétique et Sociologie'.

¹⁵ Tylor, *Primitive Culture: Researches into the Development of Mythology, Philosophy, Religion, Art, and Custom*.

¹⁶ Russell, *Introduction to Mathematical Philosophy*; 'The Theory of Logical Types'.

¹⁷ Conant, *The Number Concept: Its Origin and Development*; Greenberg, 'Generalizations about Numeral Systems'; Ifrah, *The Universal History of Numbers: From Prehistory to the Invention of the Computer*; Menninger, *Number Words and Number Symbols: A Cultural History of Numbers*; Ore, *Number Theory and Its History*; Tylor, *Primitive Culture*.

in their being related to each other, with some quantity of lower-value tokens being represented by and exchanged with a single token of higher value. This characteristic was something other than one-to-one correspondence, and it implied that tokens would have been preceded by other, simpler, technologies, with forms and structures that could explain how the first numbers consistent with the perceptual experience of quantity might eventually yield the relatively complex numerical concepts represented by tokens and later notations. Part of any investigation, then, needed to address the question of precursor technologies, in terms of what might constitute evidence for them and how they might influence numerical content, structure, and organization.

Ultimately, my attempts to reconcile the Assyriological assumptions with my prior anthropological, psychological, and linguistic understandings of numbers provided me an opportunity to say something new, not just about the Neolithic tokens in Mesopotamian numbers but also about the role of material structures in human cognition more generally. This work, an updated version of my doctoral thesis, presents my view of how the material structures used to represent and manipulate numbers inform their content, structure, and organization: whether numbers are conceptualized as equivalences, collections, or entities; whether and to what extent they possess properties like linearity and magnitude ordering; and how closely tied they are as concepts—or not—to particular material structures. Essentially, I view numbers as abstract from their inception and materially bound at their most elaborated. This view collapses the historical distinction between the so-called concrete and abstract modes of thinking. It also provides an elaborational mechanism, namely, the incorporation of additional material forms, that explains why and how number concepts change over time, reveals their inherent similarities and differences across cultures at a time and within societies over time, and explains how societies comprised of average individuals can be capable of realizing complex cultural systems like numeracy and mathematics in the first place.

Mesopotamia proved an ideal case study. It has an unusually long and detailed sequence of material devices used for counting—if one admits evidence from a variety of sources and then infers from it freely. Admittedly, such inferences have the potential to irk those of a historical bent, but they are not uncommon in archaeology. Textual evidence, which provides insight into the *five-plus* formations of the Sumerian numbers *six*, *seven*, and *nine*, suggests finger-counting, an assessment based on the presence and similar interpretation of identically compounded number words in other languages. This evidence occurs more than a thousand years after the invention of writing, and there is no known way of determining exactly when such number words originated. Positioning finger-counting as the earliest form of materially structured counting in Mesopotamia was inferred from the way finger-counting appears to work in other societies globally. Further, the fingers and hand have not traditionally been classified as *material* structures for counting, though an extensive literature attests to their being a somatic basis for it. Archaeological evidence suggests there were early tallies, which take the

form of worked bones from the Epipaleolithic Levant.¹⁸ Not only is this time and place temporally, geographically, and culturally distinct from the later Mesopotamian societies who used tokens, examinations that can categorize notches as possibly quantificational in nature¹⁹ have not been performed on these artifacts, to the best of my knowledge. Tallies are the weakest link in my chain of inferences, though they fill what seems an otherwise inexplicable gap in the sequence of material devices, the transition off the body to one-dimensional material devices. The Neolithic clay tokens are fairly well known at this point, mainly through the extensive publications of Schmandt-Besserat, as are their correspondences with later, unambiguous, numerical notations.²⁰ However, much like fingers and hands, written signs have not been previously investigated as material structures in the elaboration of number. But given status as a material structure, they too fill an explanatory gap, the conceptual change historically described as the transition from concrete to abstract numbers.

Whatever inferential holes mar the argument that fingers, tallies, tokens, and numerical notations form a material sequence elaborating initial number concepts into counting sequences and mathematics, however non-generalizable the sequence may be to societies other than Mesopotamia, the resultant framework nonetheless holds the potential to explain numerical prehistory cross-culturally. That is, material devices have long been recognized as representing and manipulating numbers.²¹ What is new about the present work is that it explains what those material structures do in number concepts—not just anchoring and stabilizing them,²² not only acting as proxies for their properties,²³ but providing the very mechanism of elaboration. New devices for representing and manipulating numbers extend some of the capabilities provided by older devices, resolve some of their limitations, and inject new limitations that at some point may motivate the incorporation of even newer devices. This idea draws upon the work of ecological psychologist James Gibson,²⁴ whose theory of material structures having exploitable properties (or *affordances*) was most illuminating and useful in this regard. Importantly, analysis of the properties, capabilities, and limitations of the devices used in Mesopotamian counting revealed an internal consistency to the sequence.

¹⁸ Coinman, ‘Worked Bone in the Levantine Upper Paleolithic: Rare Examples from the Wadi Al-Hasa, West-Central Jordan’; Reese, ‘On the Incised Cattle Scapulae from the East Mediterranean and Near East’.

¹⁹ d’Errico, ‘Memories out of Mind: The Archaeology of the Oldest Memory Systems’; ‘Microscopic and Statistical Criteria for the Identification of Prehistoric Systems of Notation’.

²⁰ Friberg, ‘Preliterate Counting and Accounting in the Middle East: A Constructively Critical Review of Schmandt-Besserat’s *Before Writing*’; Nissen, Damerow, and Englund, *Archaic Bookkeeping: Early Writing and Techniques of Economic Administration in the Ancient Near East*.

²¹ Ifrah, *From One to Zero: A Universal History of Numbers*; *The Universal History of Numbers: From Prehistory to the Invention of the Computer*; *The Universal History of Computing: From the Abacus to the Quantum Computer*.

²² Hutchins, ‘Material Anchors for Conceptual Blends’.

²³ Frege, *The Foundations of Arithmetic: A Logical-Mathematical Investigation into the Concept of Number*.

²⁴ Gibson, *The Ecological Approach to Visual Perception*; ‘The Theory of Affordances’.

For this analysis, Malafouris' Material Engagement Theory (MET)²⁵ proved an apt framework. MET is able to compare cognitive states without implicitly favoring later states over earlier ones, addressing my concern with avoiding the potential pitfalls of cultural categorization and societal modes of thinking. Because MET envisions cognition as the interaction of brain, body, and world, it let me consider the respective contributions of, and interactions among, the psychological, behavioral, and material dimensions of numbers. The framework has aspects that are debatable, if not controversial. For example, MET views cognition as both extended (cognition includes materiality as a constitutive element) and enactive (cognition *is* the interaction of brain, body, and materiality). It's a good question whether investigating the role of materiality in numerical cognition requires that materiality be constitutive of cognition, rather than causally linked to it. I was skeptical on this exact point when I first encountered Malafouris' work because, as one of my professors once expressed it, causal linkage is easy to demonstrate, constitutivity much harder.

Now after several years of learning to see cognition from a MET perspective, I have become a thorough convert. Readers of the present work will have to decide their level of commitment for themselves, of course. Here it will suffice to note that redrawing the boundaries of cognition to include materiality, as Malafouris invites us to do,²⁶ raises the possibility of gaining new insights. I doubt the present work would have been possible without such redistricting. For example, as I investigated numerical notations as material structures, I also looked at how they differed from signs for non-numerical language, and how both changed over time. After this analysis and much thought,²⁷ I came to see reading as an unambiguous example of cognition that is both extended and enactive, because it is a cognitive state that does not—indeed, cannot—exist without engaging the material form that is writing. It is difficult to imagine what sort of thing reading could be, without engaging its material form. Once reading is admitted as example of extended and enacted cognition, what follows is the recognition that the material form has become increasingly effective at eliciting specific behaviors and psychological responses in its users. For other material forms, including those used for numerical representation and manipulation, what follows is not *whether* they are constitutive of cognition, but *how*. Mathematician Brian Rotman sees mathematical calculation by means of notations as an amalgam of thinking and scribbling,²⁸ something Malafouris would describe as brain, body, and material forms being constitutively intertwined. And, as I explain in the book, the reason numerical notations differ from non-numerical signs becomes explicable through Malafouris' distinction between material and linguistic signs, and as a function of their different material prehistories.

²⁵ Malafouris, 'At the Potter's Wheel: An Argument for Material Agency'; 'The Cognitive Basis of Material Engagement: Where Brain, Body and Culture Conflate'; Malafouris and Renfrew, *The Cognitive Life of Things: Recasting the Boundaries of the Mind*.

²⁶ Malafouris, *How Things Shape the Mind: A Theory of Material Engagement*.

²⁷ Overmann, 'Beyond Writing: The Development of Literacy in the Ancient Near East'; 'Thinking Materially: Cognition as Extended and Enacted'.

²⁸ Rotman, *Mathematics as Sign: Writing, Imagining, Counting*.

This work is only an initial step in understanding how peoples use material forms to realize concepts of numbers, and how those concepts change over time through the incorporation of new material forms. Accordingly, this work needs to be, and hopefully will be, subject to correction, revision, and expansion by other scholars interested in answering questions of numerical origins and change. One loose end that might be unraveled further is whether admitting materiality into numerical concepts sheds any new light on historical perspectives of what numbers are. That is, as concepts, numbers have properties that are unique and peculiar, like their intersubjective verifiability and our confidence in apprehending them.²⁹ Certainly, peoples widely separated in space and time have come up with the same sequence of counting numbers, and given that sequence and a notion of divisibility, will converge on the same prime numbers. These properties have led thinkers since Plato to postulate that numbers are discovered, not invented, and as things that are discovered, must in some sense exist. If such properties truly originate in things like material semiosis and conceptual anchoring by the material forms used to represent and manipulate numbers, there may be some implications in this for mathematical realism. Along similar lines, the involvement of materiality in numerical concepts seems to at least partly refute the rather introspectionist idea that numbers are completely mental phenomena, the intuitionism of mathematician Luitzen Egbeurtus Jan Brouwer.³⁰ Such extensions, however, must be worked out in future efforts, as they fall outside the scope of the present work.

I hope this work will further two ambitions. First, I want to change the language we use to characterize the differences between and within number systems. Beyond their being the rather distasteful fruit of an outdated view on societal difference, the terms *abstract* and *concrete* are just insufficiently descriptive. Characterizing numbers instead by their content, organization, and structure will help us, I believe, gain traction on how peoples become numerate by using material structures, something with implications for research into how numbers originate, how they vary between cultures, and how they change over time, along with the much larger issue of how material forms inform the ongoing cognitive change of our species. Second, I want this work to build upon the past research that is its basis, not start a wholly new way of conceiving and researching Ancient Near Eastern numbers. I don't merely wish to avoid the problem of there being two competing approaches, I also want to recognize that previous scholars have pointed out real phenomena, matters the current state of research into numerical cognition now enables us to understand from a different perspective.

With these potential contributions to understanding numerical origins and elaboration, as well as the role of materiality in numerical cognition specifically and in human cognition and the development of complex cultural systems generally, I hope this work at last answers Professor Renfrew's useful caveat and spur in a satisfactory and meaningful manner.

²⁹ Frege, 'The Thought: A Logical Inquiry'; Hersh, *What Is Mathematics, Really?*

³⁰ Brouwer, *Brouwer's Cambridge Lectures on Intuitionism*.