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# Gas distribution network topology problem*

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Problem of gas flow in looped network is nonlinear problem and these kind of problems have to be solved using some kind of iterative procedure. For the pipeline network, two topology matrices can be written; i.e. loop matrix and node matrix. The node matrix is related to the first Kirchhoff's law while the loop matrix is related to second Kirchhoff's law. Here will be shown efficient method in which both matrices, i.e. the node and the loop matrix are united in one coherent procedure for solution of looped gas pipeline problem.

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## 1. Introduction

Problem of gas flow in looped network is nonlinear problem (relation between flow and pressure drop is not linear while relation between electric current and voltage is). Water flow in looped network is also nonlinear problem, and hence these problems have to be solved using some kind of iterative procedure. For the pipeline network, two topology matrices can be written; i.e. loop matrix and node matrix. The node matrix is related to the first Kirchhoff's law while the loop matrix is related to second Kirchhoff's law. First Kirchhoff's law must be satisfied in each of the iterations, while the second one must be fulfilled in the end of calculation, both in the case of loop oriented methods like this presented in this paper. Otherwise, situation is opposite. Here will be shown efficient method in which both matrices, i.e. the node and the loop matrix are united in one coherent procedure for solution of looped gas pipeline problem. Today, most used methods are based on the loop matrix solution based on idea of Hardy

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Cross in which paper matrix calculation is not used. Here shown method is great step forward.

Common relation for gas flow, i.e. for determination of lost of pressure due a gas flow in plastic (polyethylene) pipes typical for gas distribution pipelines in towns is Renouard equation (2.1) [1].

$$\Delta c = p_1^2 - p_2^2 = \frac{4810 \cdot Q_{st}^{1.82} \cdot L \cdot \rho_r}{D_{in}^{4.82}} \tag{2.1}$$

Details of Renouard relation (2.1) can be found in the paper of Coelho and Pinho [2]. In the present practice of calculation of gas networks, especially for those made by polyethylene, Renouard equation (2.1) is not only most used by the petroleum engineers from Serbia but also from other countries.

First derivative of Renouard relation where the flow is treated as variable will also be used in our calculation (2.2):

$$c' = \frac{\partial(\Delta c)}{\partial Q} = \frac{1.82 \cdot 4810 \cdot Q_{st}^{0.82} \cdot L \cdot \rho_r}{D_{in}^{4.82}} \tag{2.2}$$

## 2. Methods for solution of pipe network problems

Two groups of methods for calculation of flows through pipes of looped gas pipeline networks can be used. These general groups are, first based on solution of loop equations, and the second one is based on solution of node equations. Here will be shown *the node-loop method*. *The node-loop method* belongs to so called loop oriented methods with auxiliary use of nodes in calculation procedures. Strictly node oriented methods, as well as node oriented methods with auxiliary use of loops in calculation procedures generally have limited applicability in practise.

The Hardy Cross method introduced in 1936. is the first useful procedure for the flow calculation in looped pipelines [3]. Improvement of Hardy Cross method made by Epp and Fowler reduce significantly number of iteration necessary to reach demanded accuracy in calculation [4].

The main strength of *the node-loop method* introduced in 1972 by Wood and Charles [5] does not reflect in noticeably reduced number of iteration compared to the modified Hardy Cross method [4]. Main advantage of this method is in the capability to solve directly the pipe flow rate rather than flow correction. Unfortunately, these corrections of flow calculated after original or improved (modified) Hardy Cross methods should be added to or subtracted from flow calculated in previous iterations according to complicated algebraic rules [6]. Wood and Rayes in 1981 introduced improvement in *the node-loop method* [7].

## 3. Topology matrices for gas distribution network

In Figure 1 is given an example of one pipeline network with three loops. Similar network as shown in this figure was used in the paper of Brkić [8] as example of similarities and differences between water and gas distribution networks. In this paper convergence of five different methods are examined including *the node-loop method* [8].

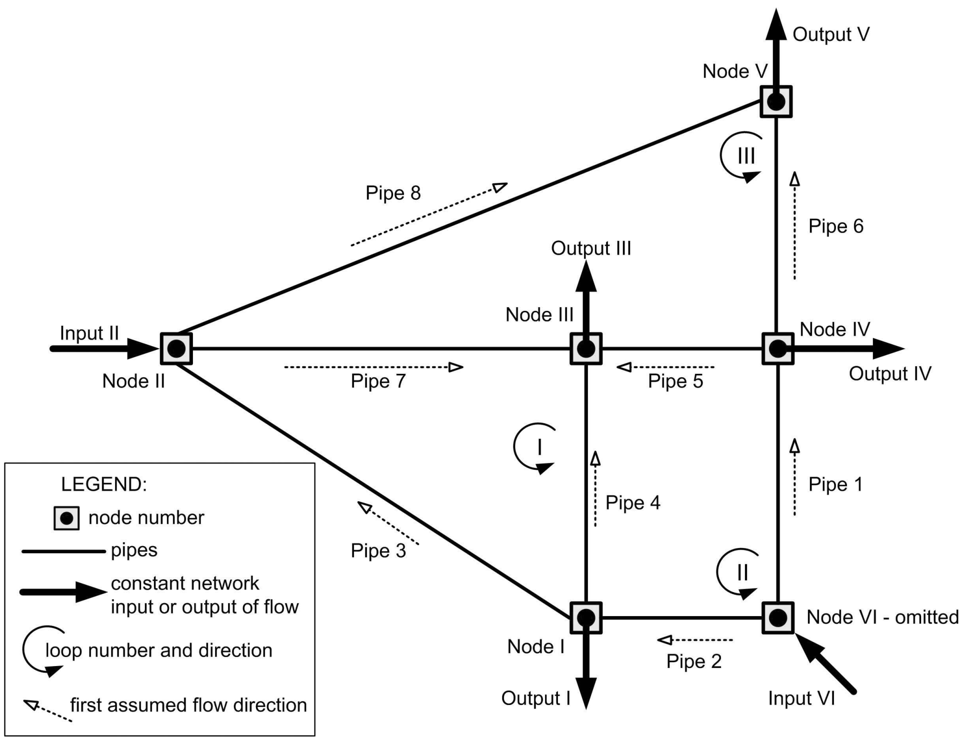


Figure 1: Example of pipeline network for gas distribution

First task is to determinate constant consumption assigned to each node while the second one is to assigned flow to each pipes to satisfy first Kirchhoff's law for all nodes. Almost always for these initial flows per pipes, the second Kirchhoff's law is not fulfilled. Using one of the iterative methods, these initial flows have to be changed with calculated ones obtained in iterative procedure but always keeping satisfied condition after first Kirchhoff's law. At the end of the calculation second Kirchhoff's law has to be satisfied after demanded accuracy (tolerance). This is applicable for the loop oriented methods such as *the node-loop method* while the situation in calculations performed using node oriented methods is opposite as mentioned before in previous text.

Graph has X branches (pipes) and Y nodes where in our example, X=8 and Y=6. Graph with Y nodes (in our case 6) has Y-1 independent nodes (in our case 5) and X-Y+1 independent loops (in our case 3). Tree is a set of connected branches chosen to connect all nodes, but not to make any closed path (not to form a loop). Branches, which do not belong to a tree, are links (number of links are X-Y+1). Loops in the network are formed using pipes from tree and one more chosen among the link pipes. Number of the loops is determined by number of links.

### 3.1. Node matrix

Network from our example has six nodes and five independent nodes. One node can be omitted from calculation while no information on the topology in that way will be lost. Rows in the node matrix with all node included are not linearly independent. To obtain linear independence any row of the node matrix has to be omitted. For example, pipe 6 is between node IV and V, and reasonable assumption is that if node IV is output node for flow through pipe 6, then node V must be input node for flow through this pipe. In our example, node VI will be omitted.

First Kirchhoff's law for the initial flow pattern shown in the figure 1 can be written using set of equations (3.2 - 3.6):

$$node_I \sim -Q_I + Q_2 - Q_3 - Q_4 = 0 \tag{3.1}$$

$$node_{II} \sim Q_{II} + Q_3 - Q_7 - Q_8 = 0 \tag{3.2}$$

$$node_{III} \sim -Q_{III} + Q_4 + Q_5 + Q_7 = 0 \tag{3.3}$$

$$node_{IV} \sim -Q_{IV} + Q_1 - Q_5 - Q_6 = 0 \tag{3.4}$$

$$node_V \sim -Q_V + Q_6 + Q_8 = 0 \tag{3.5}$$

$$node_{VI} \sim Q_{VI} - Q_1 - Q_2 = 0 \tag{3.6}$$

Node VI will be omitted from the node matrix to assure linear independency of the rows (3.7):

$$[N] = \begin{bmatrix} 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \tag{3.7}$$

First Kirchhoff's law must be fulfilled in all iterations for all nodes.

### 3.2. Loop matrix

Second Kirchhoff's law for the initial flow pattern shown in the figure 1 can be written using set of equations (3.8 - 3.10):

$$loop_I \sim -c_3 + c_4 - c_7 = C_I \tag{3.8}$$

$$loop_{II} \sim c_1 - c_2 - c_4 + c_5 = C_{II} \tag{3.9}$$

$$loop_{III} \sim -c_5 + c_6 + c_7 - c_8 = C_{III} \tag{3.10}$$

Second Kirchhoff's law for the initial flow pattern shown in the figure 1 also can be noted in matrix form using loop matrix (3.11):

$$[L] = \begin{bmatrix} 0 & 0 & -1 & 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 & -1 \end{bmatrix} \tag{3.11}$$

Second Kirchhoff's law must be fulfilled for all loops at the end of calculation with demanded accuracy.

## 4. *The node-loop method* for looped gas pipeline networks

The nodes and the loops equations shown in previous text here will be united in one coherent system by coupling these two set of equations. To introduce matrix calculation, the node-loop matrix [NL], matrix of calculated flow in observed iteration [Q], and [V] matrix will be defined (4.1).

$$[NL] \times [Q] = [V] \tag{4.1}$$

Further, vector [Q] of the unknown flows can be calculated in the first iteration (4.2):

$$[Q] = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = inv[NL] \times [V] \tag{4.2}$$

Possible sign minus in a front of flow Q in the matrix [Q] means that calculated flow direction is opposite compared to shown one in the previous iteration (or in the figure 1 in our case for the first calculated values of flows compared with initials flow pattern).

The node-loop matrix [NL] can be defined using node matrix, loop matrix and first derivative of Renouard function (4.3):

$$\begin{bmatrix} 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mid c_3^{'} \mid & \mid c_4^{'} \mid & 0 & 0 & -\mid c_7^{'} \mid & 0 \\ \mid c_1^{'} \mid & -\mid c_2^{'} \mid & 0 & -\mid c_4^{'} \mid & \mid c_5^{'} \mid & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\mid c_5^{'} \mid & \mid c_6^{'} \mid & \mid c_7^{'} \mid & -\mid c_8^{'} \mid \end{bmatrix} \tag{4.3}$$

Matrix [V] has to be defined also (4.3):

$$\begin{bmatrix} |Q_I| \\ -|Q_{II}| \\ |Q_{III}| \\ |Q_{IV}| \\ |Q_V| \\ -C_I + (-\mid c_3 \mid \cdot \mid c_3^{'} \mid + \mid c_4 \mid \cdot \mid c_4^{'} \mid - \mid c_7 \mid \cdot \mid c_7^{'} \mid) \\ -C_{II} + (\mid c_1 \mid \cdot \mid c_1^{'} \mid - \mid c_2 \mid \cdot \mid c_2^{'} \mid - \mid c_4 \mid \cdot \mid c_4^{'} \mid + \mid c_5 \mid \cdot \mid c_5^{'} \mid) \\ -C_{III} + (-\mid c_5 \mid \cdot \mid c_5^{'} \mid + \mid c_6 \mid \cdot \mid c_6^{'} \mid + \mid c_7 \mid \cdot \mid c_7^{'} \mid - \mid c_8 \mid \cdot \mid c_8^{'} \mid) \end{bmatrix} \tag{4.4}$$

If all values of C calculated after (3.8 - 3.10) are not approximative zero with reasonable accuracy, calculation has to be repeated using values calculated in previous iteration.

## 5. Modification of *the node-loop method* for water distribution systems

*The node-loop method* can be used for calculation of waterworks, ventilation systems in mines or buildings.

For the calculation of water distribution system, Renouard relation have to be replaced with Darcy-Weisbach relation (5.1):

$$\Delta c = p_1 - p_2 = \lambda \cdot \frac{L}{D^5} \cdot \frac{8 \cdot Q^2}{\pi^2} \cdot \rho \tag{5.1}$$

Darcy friction factor can be calculated using Colebrook equation [9, 10] (5.2):

$$\frac{1}{\sqrt{\lambda}} = -2 \cdot \lg\left(\frac{2.51}{Re \cdot \sqrt{\lambda}} + \frac{\epsilon}{3.71 \cdot D}\right) \tag{5.2}$$

Excellent book for waterworks calculation by Boulos et al. [11] can be recommended for further reading. In this book, unfortunately, the Hazen-Williams relation is used to correlate water flow, pressure drops in pipes and hydraulics frictions. Introduced in the early 1900s, the Hazen-Williams equation determines pipe friction head loss for water, requiring a single roughness coefficient. Unfortunately even for water it may produce errors when applied outside a limited and somewhat controversial range of Reynolds numbers, pipe diameters and coefficients [12]. Not only inaccurate the Hazen-Williams equation is conceptually incorrect [13]. Also in the book by Boulos et al. [11], authors instead to omit one node in the node matrix to preserve linear independency of rows in this matrix, they introduce one pseudo-loop in loop matrix. These procedure is not practical because at least two node with equal pressure must be found in the network. This is not always possible. Further in that way the node-loop matrix has two additional rows which could be avoided.

For some details on natural ventilation airflow networks consult paper of Aynsley [14].

## 6. Conclusion

Here presented *the node-loop method* is powerful numerical procedure for calculation of flows in looped fluid distribution networks. Main advantage is that flow in each pipe can be calculated directly, which is not possible after the most other available methods. In other methods, results of calculation are flow corrections which have to be added to flows calculated in previous iteration using complex algebraic rules.

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