Yield Curve Study

- Yield curve is defined as the relationship between the yield-to-maturity and the maturity. It is derived from observed market instruments.
- Yield curve is a representation for a two dimensional array of maturity date and corresponding continuously compounded zero-coupon interest rate pairs.
- Given yield curve, one can compute either a discount factor to a given maturity date or a forward discount factor between two given maturity dates.
- Based on yield curve, one can also compute either a zero-coupon rate to a given maturity date or a forward zero-coupon rate between two given maturity dates
- Yield curve assumes an ACT/365 day-counting convention. Assume that the "USD" curve consists of a two-dimensional array of continuously compounded zero-coupon rate and maturity date pairs, $\left(R_{T_i}, T_i\right)$, for $i = 1, \ldots, n$, where $t_0 < T_1 < \ldots < T_n$.
- If the starting date, t_1, coincides with the valuation date, t_0, then computes a discount factor to the maturity date, T. In particular, if $T = T_i$, for some $i \in \{1, \ldots, n\}$, then

$$Dfactor(t_0, t_0, T) = \exp\left(-R_{T_i} \times \frac{T_i - t_0}{365}\right).$$

- Moreover, if the date, T, does not match any point in the array of input *"USD"* curve maturity dates, but falls in between two consecutive input maturity points, T_{i_1} and T_{i_2} (i.e., $T_{i_1} < T < T_{i_2}$), then the discount factor calculation is based on the log-linear interpolation of the discount factors at the bracketing points, T_{i_1} and T_{i_2}; for example,

$$Dfactor(t_0, t_0, T) = Dfactor(t_0, t_0, T_{i_1}) \times \left[\frac{Dfactor(t_0, t_0, T_{i_2})}{Dfactor(t_0, t_0, T_{i_1})}\right]^{\frac{T - T_{i_1}}{T_{i_2} - T_{i_1}}}.$$

- The term structure of zero rates is constructed from a set of market quotes of some liquid market instruments such as short term cash instruments, middle term futures or forward rate agreement (FRA), long term swaps and spreads.
- If the starting date, t_1, is different from the valuation date, t_0, then *Dfactor* computes a discount factor between the starting date, t_0, and the maturity date, T, as follows,

  $$Dfactor(t_0, t_1, T) = \frac{Dfactor(t_0, t_0, T)}{Dfactor(t_0, t_0, t_1)}.$$

- Note that *ZeroRate* assumes an ACT/365 day-counting convention. If the starting date, t_1, coincides with the valuation date, t_0, then *ZeroRate* computes a zero-coupon rate, R_T, to the maturity date, T, according to the specified quotation convention.
- Moreover, if the maturity date, T, does not match any point in the array of input *"USD"* curve maturity points, but falls in between two consecutive input maturity points, T_{i_1} and T_{i_2} (i.e., $T_{i_1} < T < T_{i_2}$), then the zero-coupon rate is calculated according to the quotation convention as shown below:
- The yield term structure is increasingly used as the foundation for deriving relative term structures and as a benchmark for pricing and hedging.
- continuous convention,

  $$\exp\left(-R_T \times \frac{T - t_0}{365}\right) = d_T,$$

- linear convention,

  $$\frac{1}{1 + R_T \times \frac{T - t_0}{365}} = d_T,$$

- annual convention,

  $$\frac{1}{(1 + R_T)^{\frac{T - t_0}{365}}} = d_T,$$

where

$$d_T = Dfactor(t_0, t_0, T_{i_1}) \times \left[\frac{Dfactor(t_0, t_0, T_{i_2})}{Dfactor(t_0, t_0, T_{i_1})} \right]^{\frac{T - T_{i_1}}{T_{i_2} - T_{i_1}}}.$$

- When the starting date, t_1, is different from the valuation date, t_0, then *ZeroRate* computes a forward zero-coupon rate, $R_{t_1,T}$, as follows:

- continuous convention,

$$\exp\left(-R_{t_1,T} \times \frac{T - t_1}{365} \right) = d_T,$$

- linear convention,

$$\frac{1}{1 + R_{t_1,T} \times \frac{T - t_1}{365}} = d_T,$$

- annual convention,

$$\frac{1}{\left(1 + R_{t_1,T}\right)^{\frac{T - t_1}{365}}} = d_T,$$

where

$$d_T = \frac{Dfactor(t_0, t_0, T)}{Dfactor(t_0, t_0, t_1)}.$$

Reference:

https://finpricing.com/lib/EqBarrier.html