Symmetry and asymmetry between inferences between disjunctions and conditionals

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Abstract

How people make inferences between disjunctions and conditionals is a current important question that can test existing main psychological accounts (mental logic, the probabilistic approach, the original and revised mental model theory) for propositional reasoning. In order to test these accounts, one experiment investigated how relations (material implication, subcontrariety, contradiction, and contrariety) between two basic components (A and C) in disjunctions (e.g., A or C ; $\text{not-}A$ or C) and conditionals (e.g., *if not- A then C* ; *if A then C*) and inference directions (disjunction-to-conditional versus conditional-to-disjunction) between disjunctions and their corresponding conditionals affect human inferences between both. It was found that participants' inferences were symmetric between the two inference directions in compatible relations and incompatible relations where two basic components were on different dimensions, but not in the other relations. Which of the two inference directions was easier depended on relations between two basic components, because some relations tended to elicit particular interpretations of premises and conclusions, or belief biases. The present overall response pattern is beyond all the existing accounts for inferences between disjunctions and conditionals. Inferences between disjunctions and conditionals are complex and so there may not be a unified account for them.

Keywords: inferences between disjunctions and conditionals; relations between basic components; symmetry; asymmetry; belief bias

In the psychology of propositional reasoning, a current question is how people make inferences between disjunctions and conditionals. For example, for a card, given that the statement *the card is blue or the card is square* is true, does it follow from this premise that *if the card is not blue then it is square*? In this question, the conditional (with the formula, *if not-A then C*) in the conclusion is logically equivalent to the disjunction (with the formula, *A or C*) in the premise, and so the logical answer is Yes. For a ball, given that the statement *if the ball is blue then it is plastic* is true, does it follow from this premise that *the ball is not blue or the ball is plastic*? In this question, the disjunction (with the formula, *not-A or C*) in the conclusion is logically equivalent to the conditional (with the formula, *if A then C*) in the premise, and so the logical answer is Yes. However, previous studies show that in the absence of training the majority of people do not accept inferences between disjunctions and conditionals that are equivalent (Espino & Byrne, 2013; Oberauer et al., 2011).

In the above examples, the four formulas have two basic affirmative components: *A* and *C*. Relations between two basic components may be compatible (with the definition that *A* and *C* can both be true at the same time) or incompatible (with the definition that *A* and *C* cannot both be true at the same time). A compatible relation may be a subcontrariety (with the definition that *A* and *C* cannot both be false at the same time, but may both be true) or a material implication (that is, $A \supset C$, logically equivalent to *not-A or C* where *A* and *C* can both be true at the same time, and can both be false at the same time). The representational formula of the former may be an inclusive disjunction, *A or C* (e.g., *the card is red or it is square*), or the equivalent conditional, *if not-A then C* (e.g., *if the card is not red then it is square*). The representational formula of the latter may be an inclusive disjunction, *not-A or C* (e.g., *the card is not red or it is square*), or the equivalent material

conditional, *if A then C* (e.g., *if the card is red then it is square*). An incompatible relation may be a contrariety (with the definition that *A* and *C* cannot both be true at the same time, but may both be false), or a contradiction (with the definition that *A* and *C* cannot both be true at the same time, and cannot both be false at the same time). A contrariety can be represented as two equivalent sentences: *not-A or not-C* (e.g., *the card isn't triangular or it isn't square*) and *if A then not-C* (e.g., *if the card is triangular then it isn't square*). The disjunction *not-A or not-C* is logically inclusive in that *not-A* and *not-C* can both hold at the same time. A contradiction can be represented as an exclusive disjunction, *A or C* (e.g., *the card is triangular or it is square*), or the equivalent biconditional interpretation, *if not-A then C* and *if A then not-C*. The disjunction *A or C* may be inclusive (with the meaning *A or C or both*, e.g., *the card is red or it is square*) or exclusive (with the meaning *A or C but not both*, e.g., *the card is triangular or it is square*), depending on background knowledge and context (Fillenbaum, 1974; Quelhas & Johnson-Laird, 2017; Khemlani, et., al., 2018; Newstead, et., al., 1984; Noveck, et., al., 2002; Roberge, 1976a, 1996b). Overall, there are four relations between basic components *A* and *C*: subcontrariety, material implication, contradiction, and contrariety.

Table 1 presents the complete formulas and the logical truth conditions of disjunctions and conditionals in the four relations. We for the first time specify the logical framework that organizes various disjunctions and conditionals in the four relations under compatible and incompatible relations.

Each relation has two equivalent sentential forms with different logical connectives: a disjunction and the equivalent conditional. A relation is the essential intension of the corresponding two sentential forms. Sentential forms with logical connectives are representational forms that show explicit syntactical meanings. Sentential forms with different logical connectives can represent the

same intensional relation. When two different sentential forms represent the same intensional relation, both have the same truth conditions and so are logically equivalent (see Table 1). Thus, sentential forms are representational forms of intensional relations. For example, the subcontrariety between A and C can be represented as two equivalent sentences: A or C (e.g., *the card is red or it is square*), or the conditional, *if not- A then C* (e.g., *if the card is not red then it is square*). In both sentences, the card can be red and square, but cannot be neither red nor square. Thus, the two sentences are logically equivalent. The contrariety between A and C can be represented as two equivalent sentences: *not- A or not- C* (e.g., *the card is not round or it is not square*), or the conditional, *if A then not- C* (e.g., *if the card is round then it is not square*). In both sentences, the card can be neither round nor square, but cannot be both round and square. The two sentences are logically equivalent in that both share the same truth conditions (see Table 1). The formulas of disjunctions and conditionals in the four relations in Table 1 have the concrete disjunctions and conditionals (in Table 4) used in current research.

Table 1 Logical truth tables of conditionals and disjunctions in the four relations

Relations between A and C										
Cases	Compatible relations		Incompatible relations							
	Clauses		Material implications		Subcontrarities		Contradictions		Contrarities	
	A	C	$\neg A$ or C	If A then C	A or C	If $\neg A$ then C	A or C	If $\neg A$ then C	$\neg A$ or $\neg C$	If A then $\neg C$
								If A then $\neg C$		
$A \ \& \ C$	T	T	T	T	T	T	F	F	F	F
$A \ \& \ \neg C$	T	F	F	F	T	T	T	T	T	T
$\neg A \ \& \ C$	F	T	T	T	T	T	T	T	T	T
$\neg A \ \& \ \neg C$	F	F	T	T	F	F	F	F	T	T

Note: T for true and F for false. In each relation, the disjunction and the conditional are logically equivalent.

Intuitively, relations between two basic components may influence human inferences between disjunctions and conditionals, as is suggested by the principle of modulation. This principle states that particular background knowledge can block some possibilities of compound assertions as premises, thereby reducing number of possibilities of premises, and so it can modulate reasoning about compound sentences (Khemlani et al., 2018; Quelhas et al., 2010; Quelhas & Johnson-Laird, 2017). The principle of modulation in the mental model theory can explain whether disjunctions (A or C) are inclusive or exclusive (Khemlani et al., 2018; Noveck, et al., 2002), but not whether relations between basic components A and C in compound sentences are general compatible relations (*not- A or C*) or general incompatible relations (*not- A or not- C*). In general compatible and incompatible relations, disjunctions and conditionals both involve three possibilities (see the concrete disjunctions and conditionals in these relations in Table 4), and no particular knowledge blocks some of three possibilities and so reduces number of possibilities of premises. Thus, general compatible and incompatible relations are beyond the principle of modulation with the feature that background knowledge that can reduce number of possibilities of premises modulates reasoning. No previous studies have investigated the possible influence of these four relations on inferences between disjunctions and conditionals. Current research aims to investigate whether and how relations between basic components in disjunctions and conditionals affect human inferences between disjunctions and conditionals. Existing main psychological accounts of propositional reasoning imply different predictions for inferences between disjunctions and conditionals, and so investigating the above question can distinguish between these accounts.

Next, we specify existing theoretical accounts for inferences between disjunctions and conditionals where premises are true and so certain. There are five main accounts for inferences

between disjunctions and conditionals: classical logic, mental logic (Braine & O'Brien, 1998), the probabilistic approach (Cruz et al., 2015; Cruz et al., 2017; Elqayam & Over, 2013; Evans & Over, 2004; Oaksford & Chater, 2009; Politzer & Baratgin, 2016), the original and revised mental model theory (Khemlani et al., 2012; Khemlani et al., 2018). Except classical logic, the latter four accounts are psychological. The predictions of the four psychological accounts are shown in Table 2.

Table 2 Theoretical predictions for difficulty of inferences between disjunctions and conditionals

Relations between A and C	Premise :: Conclusion	Difficulty			
		MLT	PA	MMT1	MMT2
Material	$\neg A \text{ or } C :: \text{If } A \text{ then } C$	easy	difficult	difficult	difficult
implications	$\text{If } A \text{ then } C :: \neg A \text{ or } C$	difficult	easy	difficult	difficult
Subcontrarities	$A \text{ or } C :: \text{If } \neg A \text{ then } C$	easy	difficult	difficult	difficult
	$\text{If } \neg A \text{ then } C :: A \text{ or } C$	difficult	easy	difficult	difficult
Contradictions	$A \text{ or } C :: \text{If } \neg A \text{ then } C$	easy	difficult	easy	difficult
	$\text{If } \neg A \text{ then } C :: A \text{ or } C$	difficult	easy	easy	difficult
Contrarities	$\neg A \text{ or } \neg C :: \text{If } A \text{ then } \neg C$	easy	difficult	difficult	difficult
	$\text{If } A \text{ then } \neg C :: \neg A \text{ or } \neg C$	difficult	easy	difficult	difficult

Notes: " $\neg$ " denotes negation. MLT and PA denote the mental logic theory and the probabilistic approach, respectively. MMT1 and MMT2 denote the original and revised mental model theory, respectively. The same below.

Classical logic provides a normative account of propositional reasoning at the computational level. In classical logic, for a conclusion from a premise, the inference is valid when the conclusion is true in each condition where the premise is true (Jeffrey, 1981). Consequently, when two propositions share the same truth conditions, they are logically equivalent and so each of them

follows from the other. A disjunction, $A \text{ or } C$, is logically equivalent to the conditional, *if not- A then C* , and a conditional, *if A then C* , is logically equivalent to the disjunction, $\text{not-}A \text{ or } C$. The pair of the disjunction and its equivalent conditional in each relation is shown in Table 1. In each pair, the inference from the disjunction to the equivalent conditional and the inference from the conditional to the disjunction are both valid and so the two opposite inferences are symmetric. Hereafter, we call the former inference DCI, the latter inference CDI.

The mental logic theory (MLT) argues that propositional reasoning is based on logic-like abstract rules, that is, mental logic (Braine & O'Brien, 1998). It interprets a DCI as the following inference schema:

The major premise: $A \text{ or } C$;

The minor premise: $\text{not } A$

The conclusion: C

The conjunction of the minor premise and the conclusion implies *if not- A then C* . Therefore, the conditional follows from the disjunction $A \text{ or } C$, and so the DCI is valid (López-Astorga, 2013, 2015, 2016). In MLT, no inference schema is applicable to a CDI, and so the CDI is not reasonable. Thus, MLT predicts that for the pair of a disjunction and its equivalent conditional, people will be more likely to accept DCI than CDI (see the prediction in Table 2), and so the two inferences will elicit asymmetric performances.

The probabilistic approach (PA) assumes the principle of probabilistic validity that a one-premise inference is valid when the probability of its conclusion is higher than or equal to the probability of its premise; otherwise, it is invalid (Baratgin & Politzer, 2016; Cruz et al., 2015; Cruz et al., 2017; Gilio & Over, 2012). In the probabilistic approach, the probability that $A \text{ or } C$ is true equals $[P(A) +$

$P(C) - P(AC)] = [P(A) + P(\neg A) \cdot P(C/\neg A)]$, and the probability that *if not-A then C* is true equals the conditional probability, $P(C/\neg A)$, which equals $[P(A) \cdot P(C/\neg A) + P(\neg A) \cdot P(C/\neg A)]$ (Gilio & Over, 2012; Oberauer et al., 2011). The difference between the two probabilities equals the former probability minus the latter probability, that is, $P(A) \cdot [1 - P(C/\neg A)]$. This difference is larger than or equal to zero, and so the probability of the conditional is less than or equal to the probability of the disjunction, as is consistent with the prediction in Baratgin and Politzer's article (2016). Thus, generally speaking, the inference from the conditional to the disjunction should be more likely than the reverse inference. In a CDI with a certain conditional as the premise, the premise probability is $P(C/\neg A)$ (that is, 1), and the difference is zero, and so the conclusion probability is 1, that is, the conclusion is true. Thus, the CDI is valid. In a DCI (from *A or C* to *if not-A then C*) with a certain disjunction as the premise, the conclusion probability is $P(C/\neg A)$. When $P(\neg A)$ is larger than 0, $P(C/\neg A)$ equals 1 and so the conclusion is true. When $P(\neg A)$ equals 0, $P(C/\neg A)$ is undefined and so the conclusion is uncertain. Thus, the DCI is not always valid. Consequently, for the pair of a disjunction and its equivalent conditional, people will be more likely to accept the CDI than the DCI (see the prediction in Table 1), and so the two opposite inferences will elicit asymmetric performances. This is the unique prediction of the probabilistic approach.

The mental model theory (Hereafter, MMT) proposed the principle of core meanings that the core meaning of a compound assertion refers to the set of possibilities allowed by it (Johnson-Laird, 2001; Johnson-Laird & Byrne, 2002, p 650). Possibilities represent possible states of affairs that are *semantically compatible* with a compound assertion (Johnson-Laird, 2001). Possibilities envisaged by people form mental models, and each mental model represents a possibility according to the premises. Mental models represent what is true in a possibility, but by default not what is false

(Johnson-Laird, 2001). Such mental models are initial models, which can become fully explicit models in case of need. An inclusive disjunction (*A or C*) allows that *A* & *not-C* is possible, *not-A* & *C* is possible, and *A* & *C* is possible. A conditional (*if A then C*) allows that *A* & *C* is possible, *not-A* & *C* is possible, and *A* & *C* is possible. We present the initial and full models of various disjunctions and conditionals in Table 3, which is an extension of Table 2 in Johnson-Laird & Ragni (2019). These disjunctions and conditionals has the corresponding example sentences in Table 4 in the experiment. In Johnson-Laird & Byrne (2002), the core meaning of *If A then C* refers to the possibilities:

a	c
$\neg a$	c
$\neg a$	$\neg c$

The first row is the initial model that is explicit in the conditional. The last two rows are implicit models for the possibilities in which their antecedents are not satisfied. A mental footnote on the implicit model can be used to make fully explicit models (Johnson-Laird & Byrne, 2002).

Table 3 Mental models of disjunctions and conditionals in the four relations

Relations between A and C															
Material implications				Subcontrarities				Contradictions				Contrarities			
$\neg A$ or C		If A then C		A or C		If $\neg A$ then C		A or C		If $\neg A$ then C		$\neg A$ or $\neg C$		If A then $\neg C$	
$\neg A$		A	C	A		$\neg A$	C	A		$\neg A$	C	$\neg A$		A	$\neg C$
	C				C				C		...		$\neg C$		
			...				...						$\neg C$		...
$\neg A$	C			A	C			/		/		$\neg A$	$\neg C$		
$\neg A$	$\neg C$	A	C	A	$\neg C$	$\neg A$	C	A	$\neg C$	$\neg A$	C	$\neg A$	C	A	$\neg C$
A	C	$\neg A$	C	$\neg A$	C	A	C	$\neg A$	C		/	A	$\neg C$	$\neg A$	$\neg C$
$\neg A$	C	$\neg A$	$\neg C$	A	C	A	$\neg C$	/	A	$\neg C$		$\neg A$	$\neg C$	$\neg A$	C

Note: "..." indicates an implicit possibility/model, and "/" indicates nothing.

In MMT, reasoning depends on imagining the possibilities compatible with the premises, and a conclusion is valid if it holds in all the models of the premises (Johnson-Laird, 2001). And difficulty of propositional reasoning increases with number of models involved due to the constraint of working memory capacity, and so the fewer models an inference calls for, the easier the inference is (Johnson-Laird, 2001; Khemlani et al., 2018; Quelhas & Johnson-Laird, 2017). In MMT, any deduction that calls for three or more models should be very difficult and so yield poor performance (Espino & Byrne, 2013; Johnson-Laird & Byrne, 1992). The principle of difficulty is applicable to inferences between disjunctions and conditionals. Thus, inferences between an exclusive disjunction with two models and its equivalent conditional should be easier than inferences between an inclusive disjunction with three models and its equivalent conditional. No previous studies have investigated this prediction in the absence of training.

There are two versions of MMT as follow. “The previous version of the model theory postulated that people understand compound assertions by constructing models representing the disjunctive alternatives to which they refer. The new theory postulates instead that compounds refer to conjunctions of possibilities that each hold in default of information to the contrary” (Khemlani et al., 2018, p. 4; Johnson-Laird et al., 2015). Here, we call the previous version MMT1, and the new theory MMT2.

In MMT1, an inclusive disjunction ($A \text{ or } C$) is regarded as true when at least one of its three cases ($A \ \& \ \text{not-}C$, $\text{not-}A \ \& \ C$, and $A \ \& \ C$) is actual; otherwise, it is false (Johnson-Laird & Byrne, 2002, p. 651). Goodwin and Johnson-Laird (2018) claimed “any case which an assertion refers to as possible

should also be one in which the assertion is true.” They argued that *if A then C* is "true" if and only of one of the three cases allowed by the conditional holds. Overall, in MMT1, a disjunction (or a conditional) is true if and only of one of the three cases allowed by it holds. A disjunction and its equivalent conditional share three possible cases (see Table 3), and so have the same truth conditions (see Table 2). In particular, any case which renders a disjunction true also renders its equivalent conditional true; and any case which renders a conditional true also renders its equivalent disjunction true. Thus, in MMT1, the disjunction and its equivalent conditional should follow from each other. For a DCI, the conditional as the conclusion holds in all the models of the disjunction as the premise, and so is valid. For a CDI, the disjunction as the conclusion holds in all the models of the conditional as the premise, and so is valid. For inferences between a disjunction and its equivalent conditional, people need to construct all the models of the premise and then evaluate whether the conclusion holds in each of the models of the premise. According to the principle of difficulty, this inferential procedure is difficult for premises with three models, but relatively easy for premises with two models in contradictions (see Table 1). Thus, people will be more likely to accept inferences with premises involving two models than inferences with premises involving three models. In the same relation, a DCI and the CDI involves equal models, and have the same difficulty. Thus, both inferences should be symmetric in performance.

In MMT2, the principle of inference is that “necessary inferences are those in which the models of the premises support all and only the models of the conclusion” (Khemlani et al., 2018, p. 16). In other words, a valid inference is one in which the conclusion has the same set of epistemic possibilities as the premises. “...a basic conditional, ‘*if A then C*’, is true only if all three situations in its fully explicit models are possible: possibly(*A & C*) & possibly(not-*A & not-C*) &

possibly(not-A & C) and A & not-C is impossible”(Johnson-Laird et al., 2015, p. 206). Khemlani et al. (2018, p. 4) claimed “An inclusive disjunction, A or B or both, refers by default to the exhaustive conjunction of these default possibilities that each holds if no information is to the contrary: possible(A & not B) & possible(Not A & B) & possible(A & B).” The disjunction *not-A or C* involves the same possibilities as the conditional *if A then C* involves, and so *not-A or C* is equivalent to *if A then C*. Likewise, *A or C* involves the same possibilities as its equivalent conditional *if not-A then C* involves. It follows that *A or C* is true only if all three situations (A & not-C, not-A & C, and A & C) are possible and *not-A & not-C* is impossible. Thus, in MMT2, a disjunction and its equivalent conditional share the same conjunction of three possible cases, and so have the same truth condition. Thus, given the full explicit models of the disjunction and its equivalent conditional, both will follow from each other. In MMT2, inferences between a disjunction and its equivalent conditional require comparing the set of all the possibilities of the disjunction with the set of all the possibilities of its equivalent conditional. When the disjunction and its equivalent conditional share the same set of possibilities, both follow from each other. Otherwise, both don’t follow from each other. The comparison of sets of possibilities for the disjunction and its equivalent conditional involves four or more than four models (see Table 3), and so is very difficult. In MMT, any deduction that calls for three or more models should be very difficult (Espino & Byrne, 2013; Johnson-Laird & Byrne, 1992). Thus, it is very difficult to match the three (or two) models of the disjunction with the three (or two) models of the equivalent conditional (see Table 3), and so people will equally reject the CDI and the DCI. Thus, people will equally reject the CDI and the DCI, and so both inferences will elicit symmetric performances at a lower level (see the prediction in Table 2).

Overall, for inferences between disjunctions and their equivalent conditionals, classical logic, MMT1, and MMT2 all predicts symmetry between DCIs and CDIs, whereas MLT and PA both predicts asymmetry between DCIs and CDIs (see the prediction in Table 1). Moreover, only MMT1 predicts that inferences between disjunctions and their equivalent conditionals are easier in contradictions with two models than in the other relations with three models. No previous studies have investigated these predictions. More importantly, no previous studies have investigated how relations between basic components in disjunctions and conditionals affect human inferences between disjunctions and conditionals. Relations between basic components may affect whether human inferences between disjunctions and their equivalent conditionals are symmetric or not in performance. Investigating this possible influence can distinguish between the existing main accounts for inferences between disjunctions and conditionals and so test these basic accounts of propositional reasoning in a more general sense. Current study aims to investigate this possible influence.

Previous empirical studies

Some previous studies found that some background knowledge blocks some possibilities of some conditionals and disjunctions, and so modulates conditional reasoning and disjunctive syllogism (Brisson, et al., 2018; Johnson-Laird & Byrne, 1992; Quelhas et al., 2010; Quelhas & Johnson-Laird, 2017). These studies found that participants showed better performances on inferences within compound sentences with less models. However, these studies did not concern inferences between conditionals and disjunctions. Some studies investigated paraphrases between conditionals and disjunctions with the help of using connectives as cues of paraphrases (Ormerod & Richardson, 2003; Richardson & Ormerod, 1997). They found that paraphrasing tasks yielded better

performances than reasoning tasks that asked participants to make inferences between conditionals and disjunctions.

Oberauer et al. (2011) investigated whether participants accepted inferences between *A or C* and *If $\neg A$ then C*, and inferences between *$\neg A$ or C* and *If A then C*, following truth-table tasks of these sentences. They found that the former pair yielded better performances than the latter pair, indicating whether there were negations in disjunctions affected performances. Espino and Byrne (2013) investigated inferences between conditionals and inclusive disjunctions with two connectives (e.g., *A or C or both*). They found overall poor valid inferences between conditionals and disjunctions, and the compatibility heuristic that participants tended to accept inferences between a conditional and a disjunction when they can envisage a compatible initial or alternative model between both, but not when they are unable to envisage it. However, they didn't investigate the influence of relations between basic components, and failed to find systematical asymmetry between DCIs and CDIs.

Overall, no previous studies have investigated whether relations between basic components in disjunctions and conditionals affect human inferences between disjunctions and conditionals, and how relations between basic components in disjunctions and conditionals affect whether DCIs and CDIs are symmetric or not. One experiment investigated these important questions.

Experiment

Method

Participants

The participants were 1200 undergraduates from Shaanxi Normal University (435 males and 765 females). They have not studied any logic courses before. The experiment received ethical approval

from the Shaanxi Normal University Human Research Ethics Committee. All the participants provided written informed consent.

Experimental Design and Materials

We used a paper-pen questionnaire study that investigated how people make inferences between disjunctions and conditionals. The design was a two factor between-subjects design. One factor was inference direction that was varied with two directions: DCI and CDI. Both were opposite in direction, but used the same disjunction and its equivalent conditional. The other was relation between basic components *A* and *C* in disjunctions and conditionals that was varied with four relations: material implications, subcontrarities, contradictions, contrarities. Moreover, there were two types of incompatible relations (see Table 4): contrariety (1) where *A* and *C* were on different dimensions, and contrariety (2) where *A* and *C* were on the same dimension. Thus, the two factors combined into ten between-subjects conditions. In each condition, there were two between-subjects problems with different materials (cards and balls). On the whole, we used the between-subjects design to avoid possible interactions between different conditions in a within-subjects design.

We present three example problems in subcontrariety as follow. The other problems had the same instruction and the same problem format as these three example problems, except that they used the respective disjunctions and conditionals in Table 4.

The DCI problem about cards in subcontrariety:

Instruction

Please read and complete the following problem independently and carefully, and tick the option you agree with. Thank you for your sincere cooperation.

There is a pack of cards. And a card was randomly drawn from it. For this card, it is true that the card is blue or the card is square. Does the conclusion below follow from this premise?

Conclusion: If the card is not blue, then it is square. (Yes No)

The CDI problem about cards in subcontrariety had the same instruction and the problem form as the above problem, except the disjunction and the conditional swapped places.

The CDI problem about balls in subcontrariety:

[The instruction is identical to that in the above problem]

There is a box of balls. And a ball was randomly drawn from it. For this ball, it is true that if the ball is not blue, then it is metal. Does the conclusion below follow from this premise?

Conclusion: The ball is blue or the ball is plastic. (Yes No)

Table 4 Pairs of disjunctions and conditionals used in the experiment

Relations between A and C	Disjunctions	Conditionals
	$\neg A$ or C	If A then C
1 Material implications	The card is not red or the card is square	If the card is red then it is square
	The ball is not blue or the ball is plastic	If the ball is blue then it is plastic
	A or C	If $\neg A$ then C
2 Subcontrarities	The card is red or the card is square	If the card is not red then it is square
	The ball is blue or the ball is plastic	If the ball is not blue then it is plastic
	A or C	If $\neg A$ then C
3 Contradictions	The card is triangular or the card is square	If the card is not triangular then it is square
	The ball is metal or the ball is plastic	If the ball is not metal then it is plastic
	$\neg A$ or $\neg C$	If A then $\neg C$
4 Contrariety (1)	The card is not red or the card is not square	If the card is red then it is not square
	The ball is not blue or the ball is not plastic	If the ball is blue then it is not plastic
	$\neg A$ or $\neg C$	If A then $\neg C$
5 Contrariety (2)	The card is not triangular or the card is not square	If the card is triangular then it is not square
	The ball is not metal or the ball is not plastic	If the ball is metal then it is not plastic

The theoretical predictions for reasoning difficulty in the ten conditions are shown in Table 2.

Difficult inferences implied that endorsement rates would significantly be lower than 50% (the level of chance). Easy inferences implied that endorsement rates would significantly be higher than 50%.

The theoretical predictions for symmetry/asymmetry between DCIs and CDIs are presented in the introduction.

Procedure

The different problems were randomly arranged to participants. Each participant received one problem and a little gift. Participants took about three minutes to complete the task in classrooms.

Results

In each condition, the card problem and the ball problem did not differ in performance and so the results were collapsed across the two problems. The overall results (the endorsement rates) are shown in Table 5. In Table 5, the five relations are numbered 1, 2, 3, 4, and 5, for the convenience of comparing results of different relations.

Table 5 The results (percentages of affirmative answers) in the experiment

Relations between A and C	Inference directions			
	DCIs	Yes	CDIs	Yes
1 Material implications	$\neg A$ or C :: If A then C	30.0	If A then C :: $\neg A$ or C	33.3
2 Subcontraries	A or C :: If $\neg A$ then C	40.0	If $\neg A$ then C :: A or C	65.0
3 Contradictions	A or C :: If $\neg A$ then C	66.7	If $\neg A$ then C :: A or C	82.5
4 Contrariety (1)	$\neg A$ or $\neg C$:: If A then $\neg C$	35.0	If A then $\neg C$:: $\neg A$ or $\neg C$	40.0
5 Contrariety (2)	$\neg A$ or $\neg C$:: If A then $\neg C$	67.5	If A then $\neg C$:: $\neg A$ or $\neg C$	38.3

Note: $N = 120$ in each condition.

We compared the different conditions by using chi-square independence tests. we first tested the influence of relation between A and C. DCIs received more endorsements in contradiction than in each of relations 1, 2, and 4: for relations 3 versus 1, $\chi^2(1) = 32.30, p = .000 < .001, W = .37$; for relations 3 versus 2, $\chi^2(1) = 17.14, p = .000 < .001, W = .27$; for relations 3 versus 4, $\chi^2(1) = 24.07, p = .000 < .001, W = .32$. DCIs showed no significant differences between contradiction with two possibilities and contrariety (2) with three possibilities. DCIs showed no significant differences between the three relations: material implication, subcontrariety, and contrariety (1). DCIs received more endorsements in contrariety (2) than in contrariety (1), $\chi^2(1) = 25.37, p = .000 < .001, W = .33$. Overall, DCIs received higher endorsement rates in contradiction and contrariety (2), but lower endorsement rates in the other three relations. Thus, endorsement rates of DCIs varied with relations.

CDIs received more endorsements in contradiction than in each of the other relations, for relations 3 versus 1, $\chi^2(1) = 59.51, p = .000 < .001, W = .50$; for relations 3 versus 2, $\chi^2(1) = 9.49, p = .002 < .01, W = .20$; for relations 3 versus 4, $\chi^2(1) = 45.66, p = .000 < .001, W = .44$; for relations 3 versus 5, $\chi^2(1) = 48.94, p = .000 < .001, W = .45$. CDIs received more endorsements in subcontrariety than in material implication and two contrarieties, for relations 2 versus 1, $\chi^2(1) = 24.07, p = .000 < .001, W = .32$; for relations 2 versus 4, $\chi^2(1) = 15.04, p = .000 < .001, W = .25$; for relations 2 versus 5, $\chi^2(1) = 17.09, p = .000 < .001, W = .27$. CDIs showed no significant differences between the three relations: material implication, contrarieties (1) and (2). Overall, CDIs received higher endorsement rates in subcontrariety and contradiction, but lower endorsement rates in the other three relations. Thus, endorsement rates of CDIs varied with relations.

Next, we tested the influence of inference direction. In subcontrariety and contradiction, CDIs received more endorsements than DCIs, for subcontrariety, $\chi^2(1) = 15.04$, $p = .000 < .001$, $W = .28$; for contradiction, $\chi^2(1) = 7.95$, $p = .005 < .01$, $W = .18$, and so both inferences showed asymmetric performances. In contrariety (2) where A and C were on the same dimension, DCIs received more endorsements than CDIs, $\chi^2(1) = 20.49$, $p = .000 < .001$, $W = .29$, and so both inferences showed asymmetric performances. In both material implication and contrariety (1), DCIs and CDIs showed no significant differences in performance and so are symmetric. Overall, whether DCIs and CDIs were symmetric or asymmetric depended on relations between basic components A and C , and so there was an interaction of inference direction and relation between basic components. This confirms our conjecture in the introduction.

Discussion

The finding that subcontrariety elicited less endorsements of DCIs than contradiction showed that participants interpreted the premises “the card is blue or the card is square” and “the ball is blue or the ball is plastic” as inclusive. If these two disjunctions were interpreted as exclusive, they would have the same number of possibilities as the premises in exclusive disjunctions and so yield the equal endorsements as the latter. Yet, this was not the case. Thus, the interpretations of the two disjunctions should be inclusive.

For CDIs, contradiction with two possibilities elicited more endorsements than the other relations with three possibilities. For DCIs, contradiction with two possibilities elicited more endorsements than the other relations with three possibilities except contrariety (2), as favors the difficulty prediction of MMT1 over MMT2. For DCIs, contradiction and contrariety (2) both had the endorsement rate of about $2/3$ and showed no significant differences. The difference in number of

models between these two relations did not result in a difference in performance. This disfavors the difficulty prediction of MMT1. Overall, whether difficulty of inferences between disjunctions and conditionals increased with number of possibilities involved depended on relations. Thus, the principle of difficulty is not always applicable.

The comparisons of DCIs and CDIs showed that both inferences were symmetric in material implication and contrariety (1), but asymmetric in the other three relations. In the former two relations, inference directions made no differences to inferences. In the latter three relations, inference directions made differences to inferences. Thus, there was a significant interaction between relations and inference directions. The influence of inference directions depended on relations. The overall response pattern shows that the influence of inference directions (whether DCIs and CDIs are symmetric or asymmetric) depends on relations between basic components A and C. Which of DCIs and CDIs are easier depends on relations between basic components A and C, and so is relative. It seems be no certain answers to which of DCIs and CDIs are easier. Some certain answers to the question suggested by the previous studies (Espino & Byrne, 2013; Oberauer et al., 2011; Ormerod & Richardson, 2003; Richardson & Ormerod, 1997) may be untenable.

The finding of symmetry between DCIs and CDIs favors the predictions of MMT1 and MMT2 over the other accounts. The finding of asymmetry between DCIs and CDIs disfavors the predictions of MMT1 and MMT2. In both subcontrariety and contradiction, CDIs received more endorsements than DCIs. This finding favors the prediction of PA over the other accounts. In contrariety (2), the asymmetry that DCIs received more endorsements than CDIs favors the prediction of MLT over the other accounts. MMT1 predicted the relatively high endorsement rates (more than 60%) in contradiction. Overall, each account predicted some partial results, but no

accounts can explain all the findings. The overall response pattern is beyond all these existing accounts.

Next, we explain why the influence of inference directions depended on relations. The interaction between inference directions and relations may be due to that some relations tended to elicit particular interpretations of premises and conclusions, or belief biases. We specify this explanation as follows.

For CDIs, the endorsement rate (65.0%) in subcontrariety was higher than that (33.3%) in two contrarieties, but lower than that (82.5%) in contradiction. The former difference is inconsistent with the prediction of MMT1 and MMT2 that the two relations would not differ in performance because both had the same number (three) of possibilities. MMT1 and MMT2 were unable to explain this difference. The difference pattern between the three relations may be due to that in subcontrariety, some participants interpreted *If $\neg A$ then C* as *If A then $\neg C$* , showing the biconditional interpretation with two possibilities; whereas some other participants interpreted *If $\neg A$ then C* as an ordinary conditional involving three possibilities. In subcontrariety, the biconditional interpretation made CDIs easier relative to material implication with three possibilities, whereas the ordinary conditional interpretation made CDIs more difficult relative to contradiction with two possibilities.

In subcontrariety, the biconditional interpretation of *If $\neg A$ then C* may contribute to asymmetry between CDIs and DCIs. For *If $\neg A$ then C :: A or C* , the biconditional interpretation of *If $\neg A$ then C* implies that A and C are exclusive and so *A or C* as an exclusive disjunction is acceptable, and so can improve CDIs. For the reverse inference, *A or C :: If $\neg A$ then C* , the default interpretation of the premise is an inclusive disjunction with three possibilities, but the biconditional interpretation of *If*

$\neg A$ then C implies that A and C are exclusive, being in conflict with the premise as inclusive, and so the conditional is unacceptable. Thus, the biconditional interpretation of *If $\neg A$ then C* can inhibit the reverse inference. Overall, the biconditional interpretation of *If $\neg A$ then C* can improve CDIs but inhibit DCIs, resulting in asymmetry between CDIs and DCIs in subcontrariety.

In contradiction (e.g., the pair of *The card is triangular or the card is square* and *If the card is not triangular then it is square*), the relatively low endorsement rate of DCIs in the asymmetry between CDIs and DCIs may be due to the belief bias that some participants rejected the conclusions according to that the conclusions were inconsistent with their general knowledge. For example, for the DCI problem with the premise “The card is triangular or the card is square”, the conclusion “If the card is not triangular then it is square” is inconsistent with general knowledge that a card not being triangular may not be square, and so is more likely to be rejected.

In contrariety with three possibilities (see Table 3), contrariety (2) where A and C were on the same dimension yielded more endorsements of DCIs than contrariety (1) where A and C were on different dimensions. This difference between the two kinds of valid DCIs in contrarieties may be due to the belief bias effect that according to background knowledge, the conclusions are more believable in contrariety (2) than in contrariety (1). For example, consider the following two inferences with similar materials:

(1) “the card is not red or the card is not square.” :: “if the card is red then it is not square.”

(2) “the card is not triangular or the card is not square.” :: “if the card is triangular then it is not square.”

According to general empirical knowledge, the conditional in (2) is more believable than the conditional in (1), because a card that is triangular cannot be square, whereas a card that is red may

be square. The difference in believability of conclusion resulted in that (2) received more endorsements than (1). Thus, whether a valid conclusion is endorsed or not depends on whether it is more believable or not according to general empirical knowledge. In the above two inferences, each conclusion is both logically and empirically possible for the premise, and empirical knowledge does not block the conclusion possibility allowed by the premise. The difference in performance between (2) and (1) cannot be explained by MMT1/MMT2. According to both MMT, the two premises share the same number (three) of mental models and the two conclusions also share the same number (three) of mental models. Thus, the two inferences involve the same number of mental models, and so both should not differ in performance. However, this was not the case.

We for the first time found the effect of belief biases on inferences between disjunctions and conditionals in relations 2 (subcontrariety) and 5 (contrariety). The effect of belief biases shows that particular background knowledge can enhance or reduce the believability of a logically valid conclusion, depending on whether empirical knowledge or prior belief is consistent with the conclusion or not. This is similar to the effect of belief biases in syllogistic reasoning (Evans, Barston, & Pollard, 1983; Klauer, Musch, & Naumer, 2000). In the present effect of belief biases, a conclusion is one possibility allowed by the premise and empirical knowledge does not reduce number of logical possibilities of the premise. Thus, the effect of belief biases is different from the principle of modulation in MMT that particular knowledge can block or remove some logical possibilities of disjunctions/conditionals, thereby reducing number of possibilities of premises, and so affect reasoning about these compound sentences. The principle of modulation can explain the relatively high endorsement rates of inferences between conditionals and exclusive disjunctions (e.g., *the card is triangular or it is square*). For example, the disjunction is interpreted as exclusive

due to the modulation of knowledge that the card cannot be triangular and square at the same time, and so it allows only two possibilities: the card is possibly triangular and it is possibly square. The relatively less possibilities improve inferences between conditionals and exclusive disjunctions. However, the principle of modulation is unable to explain the effect of belief biases in which there is no reduction of number of possibilities of premises.

The present finding that the majority of participants accepted inferences between exclusive disjunctions and their corresponding conditionals, disfavors the indirect prediction of the compatibility heuristic proposed by Espino and Byrne (2013, p 106) that for inferences with no compatible possibilities, participants should most often reject inferences between conditionals and disjunctions when they cannot think of a compatible initial possibility or compatible alternative. As is shown in Table 3, for a pair of an exclusive disjunction (*A or C*) with two initial models (*A, C*) and its corresponding conditional (*If $\neg A$ then C*) with the initial model possibly ($\neg A \ \& \ C$) and the alternative model possibly ($A \ \& \ \neg C$), the two models of the conditional does not match the two initial models of the exclusive disjunction. Thus, according to the indirect prediction of the compatibility heuristic, there is no compatible model between the two propositions, and so participants should most often reject inferences between both. This prediction is contrary to the case. Likewise, the indirect prediction is also contrary to the other two high endorsement rates in Table 5. Thus, the present results indirectly disfavor the compatibility heuristic by disconfirming the indirect prediction of it. At least the compatibility heuristic is inapplicable to the present tasks using logically typical disjunctions with one “or”.

Participants’ response patterns were different between three main studies on inferences between disjunctions and conditionals: the present study, Espino & Byrne (2013), and Oberauer et al, (2011).

Systematic asymmetries occurred in the present study, but not in the other two studies. The compatibility heuristic occurred only in Espino & Byrne (2013). The effect of negation that disjunctions with negative clauses inhibited valid inferences was general only in Oberauer et al, (2011) with abstract letter tasks. In the present study, DCIs in Table 5 showed no significant differences between relations 1, 2, and 4, each of which involved three possibilities. In these three relations, the premises of DCIs were respectively $\neg A \text{ or } C$, $A \text{ or } C$, and $\neg A \text{ or } \neg C$. Here, negations in the premises did not affect inferences. These differences in response pattern may be due to three differences in research method between the three studies. The first difference was that the present study used a between-subjects design where there were no interactions between different problems, whereas the other two studies used within-subjects design where there may be some interactions between different problems. The second difference was that we and Oberauer et al (2011) used logical disjunctions with only one “or”, whereas Espino & Byrne (2013) used disjunctions with two “or”. The third difference was that Oberauer et al (2011) arranged truth-table training tasks before inference tasks, whereas we and Espino & Byrne (2013) used only inference tasks. These differences in research method should be responsible for the differences in response pattern between the three studies. However, current research method is ecologically more valid because we used both the between-subjects design and disjunctions with only one “or”. Disjunctions with one “or” are logically typical, and more frequent in natural language than disjunctions with two “or” (Oaksford, Over, & Cruz, 2019). Thus, disjunctions with one “or” are ecologically more valid than disjunctions with two “or” in Espino & Byrne (2013), and so the present findings are also ecologically more valid.

In summary, when disjunctions as premises were interpreted as exclusive, participants showed

more endorsements for DCIs; and when conditionals as premises were interpreted as biconditionals, participants showed more endorsements for CDIs. In the other interpretations, the majority of participants generally rejected inferences between disjunctions and their corresponding conditionals. Essentially, when basic components *A* and *C* are exclusive (contradictory), transformation between a disjunction and its corresponding conditionals is easy; otherwise, it is difficult. Thus, interpretations of relations of basic components *A* and *C* affect whether people accept inferences between disjunctions and their corresponding conditionals. This favors the principle of difficulty in MMT. However, both MMT1 and MMT2 are unable to explain the three asymmetries between DCIs and CDIs in the present results. The other factors such as belief biases also affect inferences between disjunctions and their corresponding conditionals. Overall, inferences between disjunctions and their corresponding conditionals are affected by the two main factors: interpretations of relations between basic components *A* and *C*, and belief biases. The influence of inference directions (whether DCIs and CDIs are symmetric or asymmetric; which of DCIs and CDIs are easier) depends on relations between basic components *A* and *C*, because some relations tend to elicit particular interpretations of premises and conclusions, or belief biases. The present overall response pattern is beyond all the existing accounts for inferences between disjunctions and conditionals. Inferences between disjunctions and conditionals are complex and so there may not be a unified account for them.

Declarations

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