Bond Future Option Model

Bond future option is an option on bond futures. The underlier of Bond futures is basket of deliverable bonds. For most bond futures, the selling party has the option to deliver any of the instruments in the basket.

The basket is composed of government bonds from a unique issuer (country) with rules on remaining maturity, initial maturity and issue size to be eligible.

Bond future options offer significant advantages for reducing costs, enhancing returns and managing risk.

Investors use bond future options to hedge an existing bond portfolio against adverse interest rate movements or enhance the long-term performance of a portfolio of assets.

Assuming that the bond futures price at the maturity of the option is lognormal, the present value of a call bond future option is represented as:

$$PV(t) = N[F_T\Phi(d_1) - K\Phi(d_2)]D_T$$

where

$$d_{1,2} = {[ln(F_T/K) \pm \sigma^2 T/2]}\Big/{\sigma\sqrt{T}}$$

K the strike

N the notional

$F_T = [(P - C_\Sigma)\exp(r_T T) - A\,]/CF$ the forward clean price of the delivered bond (CTD) at t

T the option maturity date

D_T the discount factor

CF the conversion factor for a bond to deliver in a bond futures contract

$C_\Sigma = \sum_{t_i \le T} Cexp(-r_i t_i)$ the present value sum of all coupons of the underlying bond between t and T

A the accrual interest before T.

P the bond dirty price at t

r_T the continuously compounded interest rate between t and T

$\sigma = \alpha D y \sigma_y / CF$ the volatility of forward bond price.

σ_y forward yield volatility of the CTD bond of the underlying futures. We use the swaption volatility

α implied volatility scaling factor

y the forward yield that can be solved by $P - C_\Sigma = \sum_{T \le t_i \le T_B} Ce^{-yt_i}$

T_B the maturity of the underlying CTD bond

$D = \frac{\sum_{T \le t_i \le T_B} t_i C e^{-yt_i}}{\sum_{T \le t_i \le T_B} C e^{-yt_i}}$ the forward modified duration of the CTD bond of the underlying futures

The present value of a put bond future option is represented as:

$$PV(0) = N[K\Phi(-d_2) - F_T\Phi(-d_1)]D_T$$

Delta is the derivative of the option price with respect to the underlying futures price at valuation time. In particular, let F denote the futures price at valuation time; then

$$\text{Delta} = \frac{\partial P}{\partial F}.$$

Gamma is the derivative of Delta with respect to the underlying futures price at valuation time. That is,

$$\text{Gamma} = \frac{\partial^2 P}{\partial F^2}.$$

Vega is the derivative of the option price with respect to the underlying futures volatility. That is,

$$\text{Vega} = \frac{\partial P}{\partial \sigma}$$

where σ, the constant volatility level of the underlying futures price process, is expressed as a percentage.

Theta is the derivative of the option price with respect to the maturity, but where the maturity is measured in days. That is,

$$\text{Theta} = \frac{\partial P}{\partial T}$$

where T denotes the maturity expressed in days.

You can find more details at
https://finpricing.com/lib/EqBarrier.html