On Fast Spectral Solving Triharmonic Equation with Simply Supported Type Condition

Agah D. Garnadi

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Department of Mathematics,
Faculty of Mathematics and Natural Sciences,
Bogor Agricultural University
Jl. Meranti, Kampus IPB Darmaga, Bogor, 16680 Indonesia

Abstract

This note is addressed to fast solving simply supported boundary value problem of triharmonic equation in the unit rectangle using spectral method.

Key words: Triharmonic equation, simply supported boundary value problem, spectral method.

1 Introduction

This note is an extension of a work by Fortunato and Townsend [6] on fast solver for Poisson equation to simply supported boundary value problem of Triharmonic equation. Henrici's work [2] on fast solver for Poisson's equation.

The need to solve discretization of Triharmonic equation is motivated by its occurrence from computer graphics [7], outside of classical applications such as elasticity and fluid flows [1, 3].

2 Triharmonic equations with simply supported boundary conditions.

Suppose that S is a unit square,

$$S = \{(\xi, \eta); 0 \leq \xi, \eta \leq 1\}.$$

We are going to solve biharmonic equation:

$$\Delta^3 u(x, y) = f(x, y),$$

in the rectangle S . Furthermore, we deal with homogeneous simply supported boundary value problem,

$$\left. \begin{array}{l} u = 0, \\ \Delta u = 0, \\ \Delta^2 u = 0, \end{array} \right\} \text{ on } \partial\Omega .$$

Rewrite as coupled system:

$$\Delta u(x, y) = v; \quad \Delta v(x, y) = w(x, y), \quad \Delta w(x, y) = f(x, y), \quad (1)$$

with the boundary condition:

$$\left. \begin{array}{l} u = 0, \\ v = 0, \\ w = 0, \end{array} \right\} \text{ on } \partial\Omega .$$

To solve the problem using spectral methods on S , is easily executed since the availability of fast spectral method for Poisson Equation as given by [6].

3 Epilogue

We show an algorithmic development of solving Triharmonic equations in unit rectangle. The implementation of spectral fast solver of Poisson equations in MATLAB in [6], made the fast spectral Triharmonic problem with simply supported conditions is realizable.

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