COMMENT

A resolution of the Trans-Planckian problem in the $R_h = ct$ universe

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The recent measurement of a cutoff $k_{\min}$ in the fluctuation power spectrum $P(k)$ of the cosmic microwave background may vitiate the possibility that slow-roll inflation can simultaneously solve the horizon problem and account for the formation of structure via the growth of quantum fluctuations in the inflaton field. Instead, we show that $k_{\min}$ may be interpreted more successfully in the $R_h = ct$ cosmology, as the first mode exiting from the Planck scale into the semi-classical Universe shortly after the Big Bang. In so doing, we demonstrate that such a scenario completely avoids the well-known trans-Planckian problem plaguing standard inflationary cosmology.

KEYWORDS:

cosmological parameters; cosmology: observations; cosmology: early Universe; cosmology: theory; gravitation; cosmology: inflation

1 | INTRODUCTION

The Friedmann-Lemaître-Robertson-Walker (FLRW) metric, based on the cosmological principle and its assumption of isotropy and homogeneity on large scales, is the backbone of modern cosmology. All the available observational evidence appears to support its essential spacetime basis (Melia, 2020), so any conceptual or foundational hurdles arising with the expansion of the Universe are attributed to other factors—notably an incomplete understanding of the physics underlying the evolution of its contents.

Over the past four decades, several crucial amendments and additions have been introduced to the basic picture in order to address some of these difficulties, chief among them the well-known horizon problem associated with the uniformity of the cosmic microwave background (CMB) temperature, T_{cmb} . In the context of Λ CDM, CMB photons emitted near the surface of last scattering (LSS) at redshift $z \sim 1080$ from opposite sides of the sky would be causally disconnected without an anomalous accelerated expansion in the early Universe (Melia, 2013). Yet T_{cmb} has the same value in all directions, save for $\sim 10^{-5}$ variations associated with fluctuations seeded at, or shortly after, the Big Bang.

A very elegant solution to this problem was introduced in the early 1980's (Guth, 1981), based on an expected phase transition in grand unified theories (GUTs), when the strong and electroweak forces may have separated at an energy scale $\sim 10^{16}$ GeV, or $\sim 10^{-35}$ seconds after the Big Bang. As long as the scalar field, ϕ , associated with this spontaneous symmetry breaking had the 'right' potential, $V(\phi)$, one could envisage an evolution at almost constant energy density, ρ_ϕ , producing a transient near-de Sitter cosmic expansion (Linde, 1982). Such an inflationary phase would have exponentially stretched all observable features well beyond the Hubble radius, $R_h = c/H$, where $H(z)$ is the redshift-dependent Hubble parameter, causally connecting the spacetime throughout the visible Universe today.

Perhaps even more importantly, this event is believed to have also produced the large-scale structure via the seeding of quantum fluctuations in ϕ and their subsequent growth to classically relevant scales during the inflated expansion (Kodama & Sasaki, 1984; V. F. Mukhanov, Feldman, & Brandenberger, 1992). A near scale-free spectrum $P(k)$ would have been generated as modes with comoving wavenumber k successively crossed R_h and classicalized, freezing their amplitude at a mode-dependent crossing time t_k (Maldacena, 2003). Thus,

inflation appears to have simultaneously solved the T_{cmb} horizon problem and provided an explanation for the origin of $P(k)$.

In spite of this initial success, however, the inflationary paradigm is nonetheless conceptually incomplete for several reasons. For example, the recent discovery of the Higgs particle (Aad et al., 2012) has reminded us that ΛCDM is subject to several horizon problems—at several different epochs—not just one, so the GUT transition at $\sim 10^{-35}$ seconds is looking more like an overly customized solution focusing solely on T_{cmb} , rather than providing an over-arching paradigm to account for our entire causally-connected Universe. A second well-motivated phase transition should have occurred when the electric and weak forces separated at a critical temperature $T_{\text{Higgs}} \sim 159.5 \pm 1.5$ GeV, i.e., $t \sim 10^{-11}$ seconds after the Big Bang (Barr, Dolan, Englert, de Lima, & Spannowsky, 2015; Dolan, Englert, & Spannowsky, 2012; Fileviez Pérez, Patel, Ramsey-Musolf, & Wang, 2009; Noble & Perelstein, 2008; Weir, 2018), too far beyond the GUT scale to have been affected by the hypothesized first transition (Melia, 2018c). This second spontaneous symmetry breaking would have inevitably led to its own horizon problem, having to do with the ‘turning on’ of the Higgs field and its vacuum expectation value, which today appears to be universal, even on scales exceeding the regions that were causally connected at the time of the electroweak phase transition.

This is not so much an argument against inflation per se, though it does weaken the claim that a GUT phase transition could account for all of the major features we see today; it apparently does not. A more serious problem with the slow-roll inflationary paradigm has been uncovered by another recent study of the angular correlation function measured in the CMB by *Planck* (Planck Collaboration et al., 2018).

The inflationary model faces several other hurdles, not merely possible inconsistencies with the horizon problems. For example, recent observations of the CMB anisotropies have apparently eliminated a broad range of possible inflationary scenarios, disfavoring all the simplest potentials (Ijjas, Steinhardt, & Loeb, 2013). And the remaining range of inflaton fields are subject to serious drawbacks, making it much more difficult to understand how the universe acquired its initial conditions. Along with this outcome, many of the best-motivated inflationary scenarios appear to have been ruled out, as we shall see below. According to this recent analysis (Liu & Melia, 2020; Melia & López-Corredoira, 2018), none of the slow-roll potentials based on the ‘Quadratic,’ ‘Higgs-like’ and ‘Natural’ forms can account for the observed power spectrum while simultaneously mitigating the horizon problem. Having said this, it is not really our goal in this *Note* to argue against the

inflationary paradigm, though the difficulties faced by inflationary cosmology certainly add to our motivation for pursuing the alternative interpretation described here.

In the standard picture, the solution to the T_{cmb} horizon problem, and a generation of a near scale-free fluctuation spectrum $P(k)$, are intimately connected via the initiation and extent of the inflationary phase. But the CMB angular-correlation function now provides compelling evidence—at a confidence level *exceeding* 8σ —that $P(k)$ has a non-zero cutoff $k_{\text{min}} = (4.34 \pm 0.50)/r_{\text{cmb}}$, where r_{cmb} is the comoving distance to the LSS (Melia & López-Corredoira, 2018). Since k_{min} would have been the first mode crossing R_h during inflation, it would signal the precise time, as a function of $V(\phi)$, at which the de Sitter expansion started. Unfortunately, its measured value shows that none of the slow-roll potentials proposed thus far can simultaneously account for the uniformity of T_{cmb} across the sky and the observed $P(k)$ in the CMB (Liu & Melia, 2020). The conclusion from this is that, if slow-roll inflation is to work, it must function in a more complicated way than has been imagined thus far.

As we shall see in this *Note*, the measurement of k_{min} in the angular correlation function of the CMB not only constrains the time when inflation could have started, but apparently provides direct evidence of quantum fluctuations at the Planck scale. This topic broaches one of the most serious fundamental problems with inflation, one that has eluded satisfactory resolution for over three decades. It is generally understood that to solve the horizon problem in ΛCDM , a minimum of 60 e-folds of inflationary expansion must have occurred, even more in many variants of the basic model. Thus, cosmological scales of observational relevance today must have expanded from sub-Planckian wavelengths at the start of inflation (R. H. Brandenberger & Martin, 2001; Easter, Greene, Kinney, & Shiu, 2001; Kempf, 2001; Niemeyer, 2001). But the physics we have today cannot adequately handle such processes, a situation known as the ‘trans-Planckian problem’ (TP) (Martin & Brandenberger, 2001). (See R. H. Brandenberger & Martin (2013) for a detailed review.) This signals a potentially fatal incompleteness of inflationary theory at a fundamental physics level.

The key here is how one should interpret the evolution of quantum fluctuations seeded well below the Planck scale, where quantum mechanics and general relativity as we know them today are probably not valid (Niemeyer & Parentani, 2001). The problems that derive from attempting to follow this trans-Planckian evolution have been well documented. For example, attempts to renormalize a scalar theory with nontrivial initial conditions in flat space run into possible divergences confined to the surface where the initial conditions are imposed (Collins & Holman, 2005).

As we shall see in the next section, the development of a quantum theory of gravity could resolve such issues, allowing us to follow the growth and evolution of quantum fluctuations from birth to their exit into the semi-classical universe (beyond the Planck domain). Attempts have also been made to modify the dispersion relation for quantum modes on short scales (see, e.g., refs. Ashoorioon, Chialva, & Danielsson 2011; Zhu, Wang, Kirsten, Cleaver, & Sheng 2016), or to alter the Heisenberg uncertainty relation (Easther et al., 2001), in order to uncover possible observational signatures that one may detect in the CMB, allowing us to make some progress in identifying new physics at the Planck scale.

Our goal in this *Note* is to provide an alternative interpretation of the cutoff $k_{\min}$ measured in the primordial power spectrum $P(k)$, in order to mitigate this current need of having to follow the evolution of inflationary quantum fluctuations where our semi-classical theories are unlikely to be valid.

2 | PHENOMENOLOGICAL APPROACH TO TP PHYSICS IN Λ CDM

One can easily understand why our inability to analyze physics below the Planck scale constitutes a potentially insurmountable problem. The Planck mass is a unit of mass defined solely using fundamental and universal units, and is given by $M_P \equiv \sqrt{\hbar c/G}$. As it turns out, the Planck length associated with M_P is of the same order as the Schwarzschild radius and the Compton wavelength of the Planck mass, suggesting that it represents a transition from classical general relativity to the quantum gravity domain. Given this ‘physical’ interpretation, we shall use in this *Note* a slightly different value of the Planck mass, which we shall call $m_P = \sqrt{\pi \hbar c/G}$ (see below), derived from strictly setting its Compton wavelength equal to its Schwarzschild radius.

It is not difficult to see that, since the former length scale increases as the latter shrinks towards the Big Bang, it is simply not possible to characterize the behaviour of modes below the Planck scale using quantum mechanics and general relativity separately. The semi-classical physics we use to describe the evolution of quantum fluctuations as the Universe expands does not apply for mode scales shorter than their Compton wavelength (R. H. Brandenberger & Martin, 2013).

This problem manifests itself in several ways, particularly via the mode normalization that one must use to calculate $P(k)$ for a comparison with the CMB data. The amplitude of the modes is typically inferred by minimizing the expectation value of the Hamiltonian, but with a time-dependent spacetime curvature at the Planck scale, the frequencies themselves depend on time and non-inertial effects. Early attempts at addressing this issue extended the birth of fluctuation modes

into the very distant conformal past, well below the Planck scale, arguing that the simple harmonic oscillator is recovered there, allowing one to impose a Minkowski vacuum—called the Bunch-Davies vacuum in this context (Bunch & Davies, 1978)—as the background for the fluctuations. But given that the physics below the Planck scale is unknown, we have a conceptual problem understanding whether or not the Bunch-Davies vacuum is even the correct choice for sub-Planckian modes.

We should point out, however, that this trans-Planckian problem arises primarily from the use of quantum mechanics with (the classical theory of) general relativity. It is believed that this problem may disappear with a viable theory of quantum gravity, such as Wheeler-DeWitt theory, loop quantum gravity or string theory (Kiefer, 2004). Significant effort is being expended in studying such models, which are established by using quantization techniques on symmetry-reduced general relativity. For a large class of wave functions, the homogeneous and isotropic FLRW metric even loses its singularity (Ashtekar, Corichi, & Singh, 2008).

But such an approach is not entirely problem free either. These models suffer from the long-standing measurement problem in quantum mechanics, having to do with the ambiguity of when exactly collapses happen and how a unique, deterministic outcome is selected. The measurement problem is even more severe in cosmology, because the universe lacks outside observers or measurement devices that could have collapsed the wave function. There is also the problem of time, given that the wave function is static in both the Wheeler-DeWitt theory and loop quantum gravity (Kiefer, 2004; Kuchař, 2011), making it difficult to envisage temporal evolution, e.g., to ascertain whether the universe is expanding or contracting. Finally, such quantum gravity theories are based solely on a wave function, without an actual metric, so it is difficult to determine the behavior of quantum fluctuations in the remote conformal past, where singularities might have emerged anyway (Dewitt, 1967).

The consensus today is that Planck-scale physics probably should have created an imprint on the CMB, but with no established theory of quantum gravity, no one knows how to predict such features with any confidence. Instead, the approaches taken over the past two decades have been based on phenomenological treatments, including (1) modifications to the dispersion relation for quantum modes on short scales (Ashoorioon et al., 2011; R. H. Brandenberger & Martin, 2001; Jorás & Marozzi, 2009; Martin & Brandenberger, 2001; Niemeyer, 2001; Zhu et al., 2016); (2) the use of string-inspired changes to the Heisenberg uncertainty relation (Easther et al., 2001; Hassan & Sloth, 2003; Kempf, 2001); and (3) non-commutative geometry (R. Brandenberger & Ho, 2002; Chu,

Greene, & Shiu, 2001; Lizzi, Mangano, Miele, & Peloso, 2002).

All of these are really probes of the CMB to suggest how basic theory ought to be modified rather than robust attempts at using a well-justified model of physics at short distances to predict a trans-Planckian signature. To understand the scale we are considering here, we define the Planck length λ_P to be the Compton wavelength $\lambda_C \equiv 2\pi/m_P$ of a (Planck) mass m_P for which λ_C equals its Schwarzschild radius $R_h \equiv 2Gm_P$. The Planck energy is therefore $m_P \approx 1.22 \times 10^{19}$ GeV. Estimates of how big trans-Planckian corrections might be, based on the above phenomenological approaches, range from $(\lambda_P/R_h[t_{\text{inf}}])^2$ (see, e.g., Kaloper, Kleban, Lawrence, & Shenker 2002; Kempf & Niemeyer 2001) to as large as $\lambda_P/R_h(t_{\text{inf}})$ (R. H. Brandenberger & Martin, 2002; Danielsson, 2002; Easther et al., 2001; Easther, Greene, Kinney, & Shiu, 2002). In these expressions, $R_h(t_{\text{inf}})$ is the Hubble radius during inflation (which is more or less constant in the slow-roll approximation).

Thus, if inflation is associated with a GUT phase transition at $\sim 10^{16}$ GeV, these phenomenologically motivated corrections fall in the range 10^{-6} to 10^{-3} . Additional support for such a claim—especially towards the high-end of this range—is provided by arguments (R. H. Brandenberger & Martin, 2002; Danielsson, 2002) that curvature effects at the Planck scale probably produce deviations of the ϕ quantum state from the local vacuum state on the order of $\lambda_P/R_h(t_{\text{inf}})$, but no one really knows for sure. If reasonable, this range includes effects potentially large enough to affect the primordial power spectrum $P(k)$ in measurable ways (see, e.g., Easther et al. 2002). Of course, a final resolution of whether or not trans-Planckian effects manifest themselves observationally must await the formulation of a well-motivated quantum gravity theory.

Certainly, the measurement of k_{min} already seems to argue against the premise that inflation might have started early enough to solve the temperature horizon problem, while simultaneously explaining the origin of $P(k)$. All three major satellite missions designed to study the CMB—COBE (Hinshaw et al., 1996); WMAP (Bennett et al., 2003); and *Planck* (Planck Collaboration et al., 2018)—have uncovered several large-angle anomalies with the temperature fluctuations, including missing correlations at angles exceeding $\sim 60^\circ$ and an unexpectedly low power in multipoles $2 \lesssim \ell \lesssim 5$. These unexpected features stand in sharp contrast to the general level of success interpreting the CMB anisotropies with Λ CDM at angles $\lesssim 2^\circ$, i.e. at $\ell \gtrsim 10$.

After a careful re-analysis of the latest release of the *Planck* data, it now appears that both of these discrepancies have common origin: a sharp cutoff of the primordial power spectrum $P(k)$ at the aforementioned minimum wavenumber k_{min} . In the first of these studies (Melia & López-Corredoira, 2018), such

a cutoff was shown, not only to suppress the expected correlation at large angles but, to actually significantly improve the fit of the angular correlation function at all angles. This analysis demonstrated that a zero k_{min} was ruled out at a confidence level exceeding 8σ . The second study focused on the impact of k_{min} on the CMB angular power spectrum itself, and concluded that the missing power at low multipoles was equally well explained by this cutoff, while none of the other 11 or 12 Λ CDM parameters optimized by *Planck* were materially affected by the introduction of k_{min} .

The combined result of these two analyses is that a single value of k_{min} completely mitigates both large-angle anomalies. But as shown in Liu & Melia (2020), the interpretation of this cutoff as the first mode to cross the Hubble radius R_h once slow-roll inflation began is then inconsistent with the accelerated expansion required to provide us with a causally-connected Universe today. The principal reason is that the time associated with this crossing was too late to have permitted the quasi-de Sitter phase to have inflated the universe sufficiently to solve the CMB temperature horizon problem. In the next section, we present an alternative interpretation of k_{min} that avoids these conceptual problems and, at the same time, completely eliminates the trans-Planckian inconsistency.

3 | A RESOLUTION OF THE TP PROBLEM IN $R_H = cT$

The FLRW cosmology known as the $R_h = ct$ universe (Melia, 2007, 2016, 2017b; Melia, 2020; Melia & Abdelqader, 2009; Melia & Shevchuk, 2012) is essentially Λ CDM, though with an additional constraint motivated by both the observational evidence and a careful application of the Local Flatness Theorem in general relativity. It too has an energy density ρ dominated by various combinations of matter, ρ_m , radiation, ρ_r and dark energy, ρ_{de} , depending on the cosmic epoch, but the equation-of-state of this ‘cosmic fluid’ always satisfies the so-called zero active mass condition, $\rho + 3p = 0$, where $\rho = \rho_m + \rho_r + \rho_{\text{de}}$ and p is the corresponding total pressure $p = p_m + p_r + p_{\text{de}}$. An introductory review of this model may be found in ref. Melia (2019b), and a more thorough presentation of its foundational support, both observational and theoretical, will be presented in the upcoming monograph Melia (2020), to be released in Fall, 2020.

By now, this model has been subjected to comparative tests with basic Λ CDM using over 27 different kinds of cosmological data, and has accounted for the observations at least as well as the standard model, actually better than latter in the majority of cases. A recent summary of these comparative tests may be found in Table 2 of Melia (2018b).

The $R_h = ct$ cosmology was first postulated on the basis of such empirical evidence showing that the *apparent* horizon $R_h \equiv c/H$, averaged over a Hubble time, equals the comoving distance, ct , light could have traveled since the Big Bang (Melia, 2018b). In general relativity, this horizon separates null geodesics receding from the observer from those approaching him/her, but is generally not the same as the event horizon, which signals the causal limit in the asymptotic future (Melia, 2018a). As such, there is no reason why R_h should always equal ct , unless there exists some specific, fundamental reason forcing this condition.

Recently, the theoretical basis for $R_h = ct$ was strengthened considerably with a thorough re-examination of the lapse function (i.e., the coefficient g_{tt} in the FLRW metric; Melia 2019a). The issue of whether or not a lapse function $g_{tt} = 1$ is consistent with a non-inertial Hubble flow has been paid scant attention in the past. In this recent work, the Local Flatness Theorem (Weinberg, 1972) in general relativity was used to prove that $g_{tt} = 1$ is in fact valid for only two specific equations of state: an empty Universe with $\rho = p = 0$ (i.e., Minkowski space) and the aforementioned zero active mass condition, $\rho + 3p = 0$. As we now know, a pressure $p = -\rho/3$ produces a constant expansion rate and, more importantly, forces the equality $R_h = ct$. The empirical evidence pointing to this constraint was therefore a pre-confirmation of the subsequent theoretical analysis based on the Local Flatness Theorem, showing that the use of FLRW is valid only when the cosmic fluid satisfies this equation of state.

A notable feature of the expansion implied by this scenario is that it lacks any horizon problem, eliminating the need for an inflated expansion of the early Universe. Thus, if the zero active mass condition was evident at the earliest times, t , it is straightforward to show (Melia, 2017a) that an incipient (though non-inflationary) scalar field ϕ would have had the well-defined potential

$$V(\phi) = V_0 \exp \left\{ -\frac{2\sqrt{4\pi}}{m_P} \phi \right\}. \quad (1)$$

ϕ is therefore a special member of the class of minimally coupled fields explored in the 1980's, that produced power-law inflation (Abbott & Wise, 1984; Barrow, 1987; Liddle, 1989; Lucchin & Matarrese, 1985) except that, with the zero active mass equation-of-state, this ϕ produced a constant expansion rate $a(t) = t/t_0$ and did not inflate.

In $R_h = ct$, quantum fluctuations in ϕ with a wavelength $\lambda_k < 2\pi R_h$, where k is the comoving wavenumber and R_h is the Hubble radius, oscillate, while those with $\lambda_k > 2\pi R_h$ do not (Melia, 2017a). Thus, mode k oscillated in the semi-classical Universe once it emerged across the Planck scale. But the critical question is “When did it emerge?” From the expression $k = 2\pi a(t)/\lambda_k(t)$, it is clear that the observed value

of $k_{\min}$ indicates the time $t_{\min}$ when the first mode appeared. Therefore,

$$t_{\min} = \frac{4.34 t_P}{\ln(1 + z_{\text{cmb}})}, \quad (2)$$

in terms of the redshift, z_{cmb} , at the surface of last scattering.

In the concordance Λ CDM model, $z_{\text{cmb}} \sim 1080$, for which $t_{\min} \sim 0.6 t_P$. With the expansion scenario implied by $R_h = ct$, this redshift could be quite different, but the dependence of $t_{\min}$ on the location of the last scattering surface is so weak, that even a redshift $z_{\text{cmb}} \sim 50$ would result in an initial emergence time of $t_{\min} \sim 1.1 t_P$. Therefore, it appears that $k_{\min}$ in $R_h = ct$ represents the first mode exiting the Planck region at about the Planck time, a compelling indication that the cutoff $k_{\min}$ corresponds to the first mode that could have physically emerged into the semi-classical Universe after the Big Bang.

Unlike the situation with an inflaton field, in which these modes were seeded in the Bunch-Davies vacuum and oscillated across the trans-Planckian region, the quantum fluctuations associated with a non-inflationary scalar field in the $R_h = ct$ cosmology could well have been formed at the Planck scale and then evolved according to standard physical principles in the semi-classical Universe. Such an idea—that modes could have been created at a particular (perhaps even fixed) spatial scale—is not new. It has been proposed and discussed in the literature before, notably by Hollands & Wald (2002), but also by R. Brandenberger & Ho (2002) and Hassan & Sloth (2003), among others.

Whereas inflaton quantum fluctuations would have been born randomly at various times in the distant conformal past, our new proposal is that all of the non-inflaton fluctuations emerged into the semi-classical universe at the *same spatial scale*, i.e., the Planck length. They could very well have had a past history prior to reaching λ_P , but the difference is that we don't need to know how to dynamically evolve them below this scale using our current version of quantum mechanics and general relativity in order to self-consistently evolve them at $t \geq t_k$. In contrast, the conventional picture requires that we use these semi-classical theories in a domain where they are probably not valid. Without invoking a Bunch-Davies vacuum and following their evolution prior to t_P , we cannot ensure that the inflaton field simultaneously fixed the horizon problem and produced the required primordial power spectrum $P(k)$.

Another key difference between the quantum fluctuations associated with an inflaton field and those generated with the field potential in Equation (1), is that the solution to the Mukhanov-Sasaki equation (Kodama & Sasaki, 1984; V. Mukhanov, 2005), using an expansion factor $a(t) = t/t_0$ consistent with this potential, can be easily shown to have a *constant* frequency—dependent only on the time t_k at which the quantum fluctuation emerged out of the Planck domain. It was therefore a true harmonic oscillator and its amplitude

may be determined via canonical quantization in flat space-time or, equivalently, by minimizing the Hamiltonian. This is due entirely to the zero active mass condition, which produced zero acceleration, even at the Planck scale. Compare this with the conventional case, in which the extreme spacetime curvature at λ_p obviates such a straightforward approach and one must instead evolve the quantum fluctuations starting with the Bunch-Davies vacuum in the very remote past.

Assuming that all subsequent modes continued to emerge across the Planck scale with a wavelength $\lambda_k = 2\pi\lambda_p$, though at progressively later times $t_k \equiv k\lambda_p t_0$, it is trivial to show (Melia, 2017a) that the resultant power spectrum is almost scale free, with an index n_s slightly less than one, consistent with the value measured by *Planck* (Planck Collaboration et al., 2018). Thus, a non-inflationary scalar field in the $R_h = ct$ universe can account for both the measured cutoff $k_{\min}$ and for the observed distribution of fluctuations in the CMB. Most importantly for the main theme of this paper, the first reliable measurement of a minimum cutoff in the power spectrum $P(k)$ signals a direct link between the CMB anisotropies—and the subsequent formation of structure in the Universe—and quantum fluctuations at the Planck scale. In so doing, this interpretation eliminates one of the principal inconsistencies with the basic slow-roll inflationary model, i.e., the well-known trans-Planckian problem.

In short, this principal distinction between the standard inflationary scenario and the mechanism described here is that quantum fluctuations in the former had to be seeded on scales well below the Planck length, at a time well before the Planck time in order to produce the observed anisotropies in the CMB, while in the latter they could have formed at the Planck scale, with the first emerging at the Planck time. All subsequent modes would also have formed at the Planck scale, though after the Planck time, thereby producing a near scale free spectrum, and always evolving dynamically according to standard quantum mechanics and general relativity in the semi-classical Universe. The mechanism we are describing here therefore completely avoids the trans-Planckian problem, because our treatment of the quantum fluctuations relies solely on the physics we know, based on initial conditions at the Planck scale—not below it.

4 | CONCLUSION

In this paper, we have discussed the implications of the fact that, in addition to the well-studied power spectral index n_s and amplitude of the CMB fluctuations, we now have a robust measurement of a third parameter characterizing the primordial perturbation spectrum, i.e., the wavenumber cutoff $k_{\min}$, which differs from zero at a confidence level exceeding 8σ (Melia

& López-Corredoira, 2018). This cutoff appears to invalidate basic slow-roll inflationary models attempting to simultaneously account for the 60 e-folds of exponential expansion at the GUT scale and the generation of anisotropies in the CMB from quantum fluctuations in the inflaton field (Liu & Melia, 2020). An additional well-known inconsistency with this scenario is the trans-Planckian problem, referring to the required transition of modes from below the Planck scale into the semi-classical Universe, a process that cannot adequately be described by quantum mechanics and general relativity separately.

Contrasting with this deficiency in the standard model, we have also demonstrated that the interpretation of $k_{\min}$ in the $R_h = ct$ cosmology suggests it corresponds to the first quantum fluctuation that could have physically emerged from the Planck scale shortly after the Big Bang. This scenario thus avoids the trans-Planckian problem if one invokes the idea that all fluctuations in the incipient scalar (though non-inflationary) field were seeded at a fixed spatial scale—in this case, the Planck scale—though at progressively later times depending on the wavenumber k of the mode. This interpretation is fully consistent with the quantum mechanical meaning of the Planck length, representing the shortest physical size of any causally connected region in the early Universe.

Looking to the future, this interpretation of $k_{\min}$ may offer clues concerning how to extend our current semi-classical description of the early Universe to scales below the Planck length, thereby heralding the initiation of an observationally-motivated quantum gravity theory. In concert with such ideas, we point out that, if the $R_h = ct$ cosmology turns out to be correct, the potential of the (non-inflationary) scalar field present just after the Big Bang is precisely known (Eq. 1), allowing us eventually to also focus more directly on possible extensions to the standard model of particle physics.

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Author contributions

Fulvio Melia is the sole author of this paper.

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