



Last two digits of 2^{googol}

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Abstract

We compute the last two digits of $2^{10^{100}}$.

keywords: number theory, googol, modular arithmetic

Last two digits with modular arithmetic

1. **Problem:** Compute the last two digits of 2^{googol} .
2. $2^{10} = 1024 \equiv 24 \pmod{100}$.
3. $2^{20} = (2^{10})^2 \equiv 24^2 = 576 \equiv 76 \pmod{100}$.
4. Then $2^{40} = (2^{20})^2 \equiv 76^2 = 5776 \equiv 76 \pmod{100}$.
5. So $2^{80} \equiv 2^{160} \equiv 2^{320} \equiv 2^{640} \equiv 76 \pmod{100}$.
6. Consequently, $2^{1000} = 2^{640} 2^{320} 2^{40} \equiv 76 \cdot 76 \cdot 76 = 438976 \equiv 76 \pmod{100}$ [1].
7. From (5), and the principle of mathematical induction, the general statement

$$2^{10 \cdot 2^n} \equiv 76 \pmod{100} \text{ is true for all natural numbers } n.$$

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8. Let $g = \text{googol} = 10^{100}$.
9. $x = 2^g = 2^{10^{100}}$.
10. $x = 2^{1000} \cdot 2^{1000} \cdot \dots \cdot 2^{1000}$.
11. x has 10^{97} terms of 2^{1000} .
12. From (6), $2^{1000} \equiv 76 \pmod{100}$.
13. From (4) and (6), $76^3 \equiv 76^2 \equiv 76 \pmod{100}$.
14. In general, $76^k \equiv 76^{k-1} \pmod{100}$, for all natural $k \geq 2$.
15. Since $2^{10^{100}} > 2$, then from (10), (12), and (14), $x = 2^{10^{100}} \equiv 76 \pmod{100}$.

Note

16. What are the last two digits of the number $3^{2^{100}}$?
17. Note that $3^4 = 81$, $3^8 \equiv 61 \pmod{100}$, $3^{16} \equiv 21 \pmod{100}$, $3^{32} \equiv 41 \pmod{100}$ e $3^{64} \equiv 81 \pmod{100}$.
18. Arguments in (17) ensure that $3^{2^6} \equiv 81 \pmod{100}$ and $81^{2^4} \equiv 81 \pmod{100}$.
19. You are welcome to send the solution of the problem (16) to the Open Journal of Mathematics and Physics [2].

Final Remarks

20. The last two digits of 2^{googol} are 76.

Open Invitation

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Ethical conduct of research

This original work was pre-registered under the OSF Preprints [3], please cite it accordingly [4]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

References

- [1] Dummit, David Steven, and Richard M. Foote. *Abstract algebra*. Vol. 3. Hoboken: Wiley, 2004.
- [2] Open Journal of Mathematics and Physics. <https://ojmp.wordpress.com/>
- [3] COS. *Open Science Framework*. <https://osf.io>
- [4] Lobo, Matheus P. “Last Two Digits of 2^{googol} .” *OSF Preprints*, 15 Aug. 2019. <https://doi.org/10.31219/osf.io/csv2y>

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