Open Journal of Mathematics and Physics | Volume 3, Article 224, ISSN: 2674-5747
https://doi.org/10.31219/osf.io/cjdwg | published: 22 Mar 2021 | https://ojmp.org

IU

[knowledge base]
Diamond Open Access [waiting peer review]

Linear Transformations

Open Mathematics Collaboration*†

March 22, 2021

Abstract

LINEAR TRANSFORMATION and its underlying definitions are presented in this white paper [knowledge base (http://omkb.org)].

keywords: linear transformation, vector space, abstract algebra, knowledge base

The most updated version of this white paper is available at
https://osf.io/cjdwg/download

*All *authors* with their *affiliations* appear at the end of this white paper.
†Corresponding author: mplobo@uft.edu.br | **Open Mathematics Collaboration**

Definition

1. **Linear Transformation (Vector Space Homomorphism)**

$$T : V \to W$$

   (a) $\forall u, v \in V : T(u+v) = T(u) + T(v), \quad (T \text{ is additive}),$

   (b) $\forall k \in F \ \forall v \in V : T(kv) = kT(v), \quad (T \text{ is homogeneous}).$

   $T :=$ function
   $V, W :=$ vector spaces over F
   $F :=$ field

   [1]

Example

2. **Dilation**

$$S : V \to V \text{ such that } \forall v \in V, \ \forall m \in \mathbb{R}, \ S(v) = mv$$

   $S :=$ function
   $V :=$ vector space over $\mathbb{R}$

   [1]

Prerequisites

3. **Vector Space over a Field (F-vector space)**

$$(V, \mathcal{F}, \oplus, \cdot)$$

   (a) $(V, \oplus) :=$ commutative group

   (b) Closure under (left) scalar multiplication:
   $\forall k \in F,\ \ \forall v \in V,\ \ kv \in V$

   (c) Scalar (left) multiplication identity: $(1 \in F,\ v \in V) \to (1v = v)$

   (d) Associativity of (left) scalar multiplication:
   $\forall j, k \in F,\ \ v \in V,\ \ (jk)v = j(kv)$

   (e) (left) Distributivity of 1 scalar over 2 vectors:
   $\forall k \in F,\ \ v, w \in V,\ \ k(v + w) = kv + kw$

   (f) (left) Distributivity of 2 scalars over 1 vector:
   $\forall j, k \in F,\ \ v \in V,\ \ (j + k)v = jv + kv$

$V, F :=$ sets
$\mathcal{F} :=$ field
$\oplus :=$ binary operation on V
$\cdot :=$ scalar multiplication (between elements of F and V)
$\cdot : F \times V \to V$
$\mathcal{F} = (F, +, \cdot)$
$+ :$ binary operation on F
$\cdot :=$ binary operation on F

[1,2]

4. **Field**

$$(F, +, \cdot)$$

(a) $(F, +)$:= commutative group
(b) $(F^*, \cdot)$:= commutative group
(c) Multiplication is distributive over addition in F
(d) $0 \neq 1$

F := set, $\quad F^* = F \backslash \{0\}$
$+, \cdot$:= binary operations on F (addition and multiplication)
0 := additive identity, $\quad 1$:= multiplicative identity

[1,2]

5. **Group**

$$(G, \star)$$

(a) Associativity: $\forall x, y, z \in G,\ (x \star y) \star z = x \star (y \star z)$
(b) Identity: $\exists e \in G : \forall x \in G,\ e \star x = x \star e = x$
(c) Inverse: $\forall x \in G\ \exists y \in G :\ x \star y = y \star x = e$

G := set
$\star$:= binary operation

[1]

6. **Commutative group (Abelian)**

$$G_b$$

$$\forall g_1, g_2 \in G_b,\ g_1 g_2 = g_2 g_1$$

G_b := group

[2]

7. **Distributive**

$$\forall x, y, z \in R : x \cdot (y + z) = x \cdot y + x \cdot z \qquad \text{left distributive}$$

$$\forall x, y, z \in R : (y + z) \cdot x = y \cdot x + z \cdot x \qquad \text{right distributive}$$

[1–3]

8. **Homomorphism**

$$f^h$$

   (a) $f^h : G \to H$
   $\forall x, y \in G : f^h(x * y) = f^h(x) \circ f^h(y)$

   f^h := function
   G, H := sets
   $*, \circ$:= binary operations
   $(G, *), (H, \circ)$:= groups

   [1, 2, 4]

9. **Function** from A to B

$$f : A \to B$$

$$\forall a \in A \ \exists! b \in B \ ((a, b) \in f)$$

   f, A, B := sets
   $\exists!$:= exists exactly one
   (a, b) := ordered pair

   [5]

10. **Cartesian product**

$$A \times B = \{(a, b) \mid a \in A,\ b \in B\}$$

$A, B :=$ sets
$A \times B :=$ Cartesian product
$(a, b) :=$ ordered pair

[5]

11. **Ordered pair**

$$(a, b) = \{\{a\}, \{a, b\}\}$$

$a :=$ first coordinate
$b :=$ second coordinate

[1, 5]

12. **Binary operation**

$$\star : S \times S \to S$$

$S :=$ set
$S \times S :=$ Cartesian product

[1]

Open Invitation

Review, *add* content, and **co-author** this white *white paper* [6, 7].
Join the **Open Mathematics Collaboration**.
Send your contribution to `mplobo@uft.edu.br`.

Open Science

The **latex file** for this *white paper* together with other *supplementary files* are available in [8].

Ethical conduct of research

This original work was pre-registered under the OSF Preprints [9], please cite it accordingly [10]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

Acknowledgements

+ **Center for Open Science**
  `https://cos.io`

+ **Open Science Framework**
  `https://osf.io`

References

[1] Warner, Steve. *Abstract Algebra for Beginners*. GET 800, 2018. `https://books.google.com/books?id=UFleyAEACAAJ`

[2] Dummit, David Steven, and Richard M. Foote. *Abstract Algebra*. Vol. 3. Hoboken: Wiley, 2004. `https://books.google.com/books?id=znzJygAACAAJ`

[3] Cain, Alan J. *Nine Chapters on the Semigroup Art*. AJC Porto & Lisbon, 2020. `http://www-groups.mcs.st-and.ac.uk/alanc`

[4] Rotman, Joseph J. *A first course in abstract algebra*. Pearson College Division, 2000.
https://books.google.com/books?id=ctEZAQAAIAAJ

[5] Velleman, Daniel J. *How to prove it: A structured approach*. Cambridge University Press, 2019.
https://books.google.com/books?vid=ISBN0521861241

[6] Lobo, Matheus P. "Microarticles." *OSF Preprints*, 28 Oct. 2019.
https://doi.org/10.31219/osf.io/ejrct

[7] Lobo, Matheus P. "Simple Guidelines for Authors: Open Journal of Mathematics and Physics." *OSF Preprints*, 15 Nov. 2019.
https://doi.org/10.31219/osf.io/fk836

[8] Lobo, Matheus P. "Open Journal of Mathematics and Physics (OJMP)." *OSF*, 21 Apr. 2020.
https://doi.org/10.17605/osf.io/6hzyp
https://osf.io/6hzyp/files

[9] COS. *Open Science Framework*. https://osf.io

[10] Lobo, Matheus P. "Linear Transformations." OSF Preprints, 22 Mar. 2021. https://doi.org/10.31219/osf.io/cjdwg

The Open Mathematics Collaboration

Matheus Pereira Lobo (lead author, mplobo@uft.edu.br)[1,2]
https://orcid.org/0000-0003-4554-1372

[1]Federal University of Tocantins (Brazil)
[2]Universidade Aberta (UAb, Portugal)