Why win-wins are rare in complex environmental management

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Abstract

High-profile modeling studies often project that large-scale win-win solutions are widely available, but practitioners are often skeptical of win-win narratives, due to real-world complexity. Here, we bridge this divide by showing mathematically why complexity makes win-wins elusive. We provide a general proof that, under uncertainty, the probability a manager should assign to win-win outcomes (here meaning Pareto improvements) existing strictly decreases in: the number of objectives, the number of stakeholders, and the number of constraints. We also show that a measure of tradeoff severity increases in the number of objectives, and approaches a limit unaffected by tradeoff surface curvature. This is important because most empirically estimated two-dimensional tradeoff surfaces are concave—77%, we show in a meta-analysis. Concave tradeoffs are less severe, but our theory suggests this difference dissipates in higher dimensions. Our results provide precise intuition and quantitative guidance for interpreting implications of simple tradeoff studies for complex realities.

Main

High-profile studies—usually using simple models calibrated to empirical data—often project that large-scale win-win outcomes should be achievable in environmental management (e.g., refs. 1–5). For instance, Tilman *et al.*² projected that global crop yields could increase by ~50% while cutting nitrogen fertilizer use in half (see also ref. 3). Costello *et al.*⁴ projected that global fisheries could simultaneously experience increases of 16 million metric tons (MMT) of catch, \$53 billion in profit, and 619 MMT in fish biomass. Building on Costello *et al.*'s model, Burgess *et al.*⁵ projected that roughly half of studied marine mammals, sea turtles, and seabirds could begin to recover their populations under profit-maximizing fishery management. Many other such studies exist (e.g., see Dataset S1 in Supplementary Information, SI). These studies provide important insights into possibilities at large scales regarding high-priority aggregate objectives, such as ensuring food security and minimizing land clearing or greenhouse-gas emissions².

Despite this, win-win narratives are often met with skepticism by scientists, managers, and practitioners focused on specific systems, for reasons related to complexity: purported win-wins in some objectives come at costs to other objectives or stakeholders; management constraints make realizing win-wins challenging; pathways to achieving win-wins can be complex and non-linear^{6–8}. For example, rebuilding overfished fisheries increases profits and fish biomass, but might consolidate fleets, reducing equity and harming smaller operators⁹. Increasing crop yields and reducing land clearing globally through intensification can locally increase pollution and ecological

damage¹⁰. Meta-analyses of case studies have found win-wins are rarer than win-lose situations (relative to status quo)¹¹⁻¹³. Indeed, even modeling studies sometimes find unavoidable tradeoffs between large numbers of objectives or ecosystem services¹⁴.

Here, we formally and precisely reconcile these two important perspectives on win-wins. We demonstrate mathematically how increasing the number of objectives, stakeholder groups, or constraints reduces the availability of win-wins and increases the severity of tradeoffs. In so doing, we offer tradeoff modelers a means to precisely understand why managers and practitioners often have complexity-related skepticism towards simple win-win narratives. We offer managers and practitioners a precise means with which to communicate this skepticism, and to contextualize results of simple tradeoff models related to their study system that they encounter in the literature and other scientific forums.

Results

Our analysis defines a ‘win-win’ as a Pareto improvement—a change in the outcome resulting in no objective (or stakeholder group) being worse off, and at least one being better off. (With three objectives, this might be called a ‘win-win-win’ instead of a ‘win-win’, but we use ‘win-win’ for simplicity.) We then perform a meta-analysis of estimated tradeoff severity in 280 published two-dimensional tradeoff surfaces, and we combine these results with our model to illustrate how scientists and managers might extrapolate estimates of two-dimensional tradeoff severities to higher-dimensional tradeoffs. Our analysis defines a ‘tradeoff surface’ as a Pareto-efficiency frontier—the set of points from which a win-win (Pareto improvement) is impossible (Fig. 1B,C).

Fig. 1. Hypothetical fishery example. **A** shows a typical concave relationship between fishing effort and (equilibrium) fishery economic benefits—small changes in effort, starting from the maximum (point 1), have little effect on economic benefits [the slope of the tangent (dashed) at point 1 is zero]. **(B)** With a second objective (ecological benefits), small changes in fishing strategies, away from the economic maximum (point 1) to improve ecological benefits have little negative effect on economic benefits [the slope of the tangent (dashed) at point 1 is again zero], but a large positive effect on ecological benefits. The efficiency frontier (i.e. the tradeoff surface) is concave. **C** illustrates how increasing the number of objectives reduces the scope for win-wins. When only looking at economic and ecological benefits, it appears win-wins from Point 2 are possible (grey region). However, once equity is considered, point 2 is no longer inefficient (i.e. win-win-wins are not possible), because any simultaneous improvement in ecological and economic benefits would necessarily come at an equity cost. (Panels **B** and **C** are inspired by figure 1 in ref. 15).

The intuition of our main is that adding a new objective, stakeholder group, or constraint to a tradeoff analysis never creates potential for a win-win that did not previously exist, and sometimes eliminates the potential for a win-win that previously existed. Fig. 1 illustrates this intuition in the context of economic, ecological, and equity objectives in a hypothetical fishery. If it is possible to simultaneously improve in all three objectives, then it is also possible to simultaneously improve in any two; but knowing it is possible to improve in profits and ecological benefits does not necessarily imply it is possible to improve in all three objectives (Fig. 1B,C). Thus, under uncertainty, managers assign a progressively smaller probability to the existence of a win-win outcome as the number of objectives (or stakeholder groups) increases. This principle is somewhat analogous to the ‘vetocracy’ concept in political science (i.e. large numbers of stakeholders having a veto make collective decision-making impossible)¹⁶; and to evolutionary multi-objective optimization theories finding greater numbers of non-inferior (i.e. Pareto-efficient) solutions with more objectives¹⁷.

To show this mathematically, suppose there are n objectives, which can be thought of as representing either aggregate objectives such as profits, or as the interests of different stakeholder groups. Suppose the manager considers the objectives in a specific order: 1, 2, ..., n (we relax this assumption below). Given uncertainty, the manager assigns a probability, p_1 , to there being an achievable improvement on the status quo in objective 1. Due to uncertainty, we assume $p_1 < 1$ (though it may be arbitrarily close to 1). Next, we define p_2 as the probability the manager assigns to there being an achievable win-win in objectives 1 and 2, conditional on there being an achievable improvement in objective 1. We again assume that $p_2 < 1$ due to uncertainty. We define probabilities $p_3, \dots, p_n$ in a similar manner, and assume each is less than 1 due to uncertainty. Following this ordering of the objectives, which we denote o , the unconditional probability of a win-win existing in n objectives, which we denote $P_o(n)$, is given by:

$$(1) \quad P_o(n) = \prod_{i=1}^n p_i.$$

Because all $p_i < 1$ for all i , $P_o(n)$ is strictly decreasing in n .

Next, consider the probability of a win-win existing over all possible orderings of objectives. Specifically, suppose that the manager draws n objectives in a random order, from a universe of N possible objectives, and asks—before drawing the objectives—what the probability is that they will draw an ordering in which a win-win exists—we denote this probability $P(n)$. Assuming all orderings are equally likely to be drawn, $P(n)$ is equal to the average of the $P_o(n)$ ’s:

$$(2) \quad P(n) = \frac{(N-n)!}{N!} \sum_o P_o(n).$$

To see how $P(n)$, too, strictly decreases in n , compare $P(n)$ with $P(n+1)$. Notice that each ordering of $n + 1$ objectives is an ordering of n objectives [i.e. one of the orderings, o , in equation (2)] with an additional objective added on. Let $\bar{p}_{o,n+1}$ denote the average of all possible values of p_{n+1} for objectives drawn after the first n objectives in order o . Thus, $\bar{p}_{o,n+1}$ is the probability of there being a win-win in $n + 1$ objectives, conditional on there being a win-win in n objectives drawn in order o . Notice that $\bar{p}_{o,n+1} < 1$ (because all $p_{n+1} < 1$ due to uncertainty). With these assumptions:

$$(3) \quad P(n+1) = \frac{(N-n)!}{N!} \sum_o P_o(n) \bar{p}_{o,n+1} < P(n).$$

The probability the manager assigns to a win-win existing also decreases in the number of constraints by identical logic: each time a constraint is added in a particular order, it either leaves the scope for win-wins unchanged or decreases it. Thus, under uncertainty, the probability the manager assigns to a win-win existing must strictly decrease with the addition of a constraint [analogous to equation (1)]. The probability of a win-win therefore also decreases on average, across all possible orders, as the number of constraints increases [analogous to equation (3)].

Moving beyond existence or absence of win-wins, we now consider the relationship between the number of objectives (or, equivalently, stakeholder groups) and the severity of tradeoffs. As a measure of tradeoff severity, we ask: with n objectives, what is the largest fraction—which we denote $X(n)$ —of the maximum possible value of each objective, simultaneously achievable for all objectives [$X(n) \leq 1$]? For instance, if at most 80% of all n objectives' maxima could be simultaneously achieved, then $X(n) = 0.8$.

$X(n)$ is determined by (Fig. 2): (i) the number of objectives (n) (Fig. 2A), (ii) the curvature of the tradeoff surface [$X(n)$ will be larger with a more concave surface] (Fig. 2B,C), and (iii) how suboptimal maximizing each objective individually is for each other objective (Fig. 2B). We measure factor (iii) by the projection of the tradeoff surface at the maximum value of objective i onto objective j 's axis—expressed as a fraction of the maximum value of objective j , and denoted a_{ij} ($a_{ij} < 1$).

Fig. 2. Tradeoff severity and number of objectives. Panel A shows how the downward-sloping nature of tradeoff surfaces causes $X(n)$ to decrease in n . Panel B illustrates the assumptions of our symmetric tradeoff model, with key parameters a (defining the surface edge) and θ (measuring surface curvature) in two dimensions. Panel C illustrates the difference between convex, concave, and linear tradeoff surface shapes. Panel D shows the gradual convergence of $X(n)$ over a range of tradeoff surface curvatures towards a ($a = 0.5$ in the examples shown).

The fact that (by definition) tradeoff surfaces slope downwards in all directions implies that $X(n)$ is non-increasing—and usually decreasing—in n (Fig. 2A). $X(n)$ decreases when a new objective is added, unless the normalized outcome for the n th objective—at the point on the tradeoff surface corresponding to $X(n-1)$ —was already greater than or equal to $X(n-1)$ [e.g. this could occur if $a_{in} \geq X(n-1)$, for all i ; see Supplementary Methods and Fig. S1A].

Moreover, while $X(n)$ increases with increasing tradeoff surface concavity (i.e. concave tradeoffs are less severe) with few objectives (n), we find, surprisingly, that the limit of $X(n)$ as n approaches infinity is unaffected by surface curvature. We prove this below in a symmetric tradeoff model, and we prove it in the general case in the Supplementary Methods.

In a symmetric tradeoff surface, we assume $a_{ij} = a$ for all pairs of objectives (i, j) . To measure surface curvature, we use the angle, θ (see Fig. 2B), between: (i) the line segment connecting a corner of the tradeoff surface (e.g., the point $\{1, a, \dots, a\}$) to the point corresponding to $X(n)$ (i.e., the point $\{X(n), X(n), \dots, X(n)\}$), and (ii) the outer edge of the n -dimensional hypercube extending from the same corner of the tradeoff surface (e.g., the point $\{1, X(n), \dots, X(n)\}$) (see Methods). (Although we assume symmetry in these line segments, our definition of θ does not preclude non-uniform nor non-symmetric curvatures of the tradeoff surface itself.) Because tradeoff surfaces are downward-sloping, $\theta > 0$. The closer θ is to zero, the more concave the surface is ($\theta = 0$ means no tradeoff). When the surface is linear, $\theta = \pi/4$ (45°); and $\theta = \pi/2$ (90°) is the limit of convexity. Regardless of the curvature, $0 < \theta \leq \pi/2$, and $X(n) \geq a$, for any n (see Fig. 2B), because $X(n)$ must lie on the tradeoff surface and—by definition of a —all points on the surface are greater than or equal to a in all objectives.

With these assumptions, we can prove that:

$$(5) \quad \lim_{n \rightarrow \infty} X(n) = a.$$

We prove this by contradiction—showing that, if $\lim_{n \rightarrow \infty} X(n) > a$ (recall that $X(n) \geq a$), it would imply there was no tradeoff (i.e. $\theta = 0$). This follows from the fact that the cosine of θ , given n , a , and $X(n)$, is (see Methods and Fig. 2B):

$$(6) \quad \cos \theta = \frac{\sqrt{Y}}{\sqrt{[1-X(n)]^2 + Y}}, \text{ where } Y = (n-1)[X(n) - a]^2.$$

From (6), it follows that if $\lim_{n \rightarrow \infty} X(n) > a$, then $\lim_{n \rightarrow \infty} Y = \infty$, and thus $\lim_{n \rightarrow \infty} \cos \theta = 1$. This proves that:

$$(7) \quad \lim_{n \rightarrow \infty} \theta = 0, \text{ if } \lim_{n \rightarrow \infty} X(n) > a.$$

Equation (7) is not possible because $\theta = 0$ implies no tradeoff. Thus, $\lim_{n \rightarrow \infty} X(n) \leq a$. Combined with $X(n) \geq a$, it must be that $\lim_{n \rightarrow \infty} X(n) = a$, completing the proof.

This implies that the limit of $X(n)$ with very large numbers of objectives (or stakeholder groups) (n) depends only on the tradeoff surface's corners, and not at all on its curvature (see Supplementary Methods for more general proof). In stark contrast, with fewer objectives, concave tradeoff surfaces have substantially less severe tradeoffs than convex surfaces, as measured by $X(n)$. Fig. 2D illustrates these properties.

We performed a meta-analysis of published two-dimensional environmental tradeoff surfaces (there are very few published surfaces in higher dimensions), and found 280 surfaces from 116 studies (see Methods and Dataset S1). We only included surfaces that were empirically calculated (usually with a model calibrated to data). For each surface, we assessed visually whether it was approximately concave, convex, linear, or none of these (e.g., S-shaped¹⁸) (see Fig. 3A), and we calculated $X(2)$ —the maximum jointly achievable fraction of both single-objective maxima—using graph digitization software (see Methods, Datasets S1 and S2, and Supplementary Video S1). Figs. 3B and 3C show the results of this meta-analysis.

Fig. 3. Measuring and extrapolating empirical tradeoff severities. Panel A illustrates the different types of empirical tradeoff surfaces we encountered, with different combinations of objectives that were maximized ('Good'; e.g., profits) and objectives that were minimized ('Bad'; e.g., pollution). Tradeoff surfaces with 'Bad' variables on one or both axes required defining concavity and convexity imagining the 'Bad' axis flipped 180 degrees. Panel B shows the results of our meta-analysis of the shapes of empirical tradeoff surfaces (concave, convex, linear, other). Panel C shows the distributions of $X(2)$ and a_{ij} observed in the meta-analysis (144 observations), using a kernel smoother. Dashed lines show median values of a_{ij} (red), $X(2)$ (blue), and the midpoint between median a_{ij} and 1 (gray), which represents what the median $X(2)$ approximately would have been if all tradeoffs had been linear. Panel D shows extrapolations of our observed $X(2)$'s (left box) to higher numbers of objectives (5, 10, 15, 20) using our symmetric model (see text and SI Notebook S1). Boxes represent 25th-75th percentile range; white lines represent medians; and whiskers show min-max range.

We found 77% (216/280) of these tradeoff surfaces were concave (Fig. 3B). We found the median a_{ij} was 0.28 (0.0011-0.87 10th-90th percentile range), and the median $X(2)$ was 0.81 (0.5-0.96 10th-90th percentile range) (Fig. 3C). [We could only calculate a_{ij} and $X(2)$ for ‘good’ objectives, which can be scaled by their non-zero maximum value, leaving 144 tradeoffs in our sample; see Methods and Supplementary Video S1.]

It is not surprising that most tradeoff surfaces are concave, since relationships between objectives and control variables are often concave (see, e.g., Fig. 1A). Indeed, if a single objective is maximized without binding constraints on control variables (i.e. an interior solution), then the tangent of the tradeoff surface at the single-objective maximum must be perpendicular to the axis (illustrated in Fig. 1A,B), which implies the surface is at least locally concave at this point. This property also implies that—when moving from maximizing a single objective to balancing two—there is initially very little cost in the original objective, and potentially a large gain for the other objective, as the control variables move towards more advantageous values for the second objective. However, this property does not hold necessarily when moving from two to three objectives (or more) (Fig. 1B,C).

Our meta-analysis of two-dimensional tradeoffs suggests abundant opportunities to achieve ~70% or more of the maximum values of both objectives [25th percentile $X(2)$ = 0.70; Fig. 3C]. However, our theory suggests this lower-dimensional picture is optimistically misleading when moving to higher dimensions.

In Fig. 3D, we use our symmetric model from Fig. 2D to approximately extrapolate the observed $X(2)$ value—in each of the 144 empirical tradeoffs in our meta-analysis—to higher dimensions [i.e. to generate an approximate $X(n)$ with n objectives]. For each tradeoff, we calculate an average a value (mean of the two observed a_{ij} ’s), and an approximate θ value (see Methods and Notebook S1) specified by this average a and the observed $X(2)$. We then extrapolate $X(n)$ to higher dimensions (n) assuming the average a and θ remain constant (Fig. 3D). In other words, we are extrapolating what $X(n)$ one would expect with larger numbers of objectives, assuming that the observed two-dimensional tradeoff properties (as captured by the observed average a and θ) remain consistent as more objectives are added. A manager or practitioner—if presented with a tradeoff curve from a study or model of their system—could use our code in Notebook S1 to make this extrapolation themselves. By observing how the extrapolated $X(n)$ varies over n , they can form an approximate expectation of how adding objectives in their system may affect the tradeoff severity [as measured by $X(n)$]. We apply this approach across our 144 published tradeoffs in Fig. 3D, and we find that these extrapolations result in median $X(n)$ = 0.71, 0.64, 0.57, with 5, 10, and 20 objectives respectively [compared to median $X(2)$ = 0.81 in the observed tradeoffs].

Discussion

Our results provide a precise and general theoretical link between two important strands of research on environmental tradeoffs: (i) modeling studies focusing on small—but important—subsets of objectives and projecting win-win solutions are widely available (e.g., refs. 1-5); and

(ii) case-studies finding win-wins are imperfect and rarer than expected due to real-world complexity (e.g., refs 6-8,11-13). We prove mathematically why complexity—measured as increasing numbers of objectives, stakeholder groups, or management constraints—decreases the expected availability of win-win outcomes. Our proof is simple, general, and makes no assumptions about the nature of interactions between objectives. For these reasons, it provides modelers a precise means of understanding the limitations of simple win-win results in more complex realities; and it provides practitioners, managers, and on on-the-ground researchers with a precise framework and toolbox (e.g., Notebook S1) for understanding and contextualizing results of simple modeling studies, and their limitations, in their systems.

Most tradeoff studies focus on two dimensions at a time, for clear reasons related to visualization and tractability (see Dataset S1). We show that most (77%) estimated two-dimensional tradeoff surfaces are concave (Fig. 3B). We show why this is conceptually unsurprising (Fig. 1A,B), and we show how the importance of tradeoff-surface concavity to tradeoff severity decreases in higher dimensions (Fig. 2D). Lastly, we show how to approximately extrapolate results of two-dimensional tradeoff studies to higher dimensions—absent specific knowledge of higher-dimensional tradeoffs—in order to account for complexity increasing tradeoff severity (Fig. 3C,D). This provides practitioners with a precise tool (Notebook S1) for contextualizing results from low-dimensional tradeoff models of their system.

Crucially, our results do not diminish the importance of global modeling studies. Such studies typically focus—appropriately—on optimizing some of the most important objectives: food production, climate mitigation, economic output, and biodiversity, to name a few. It is critical to environmental agenda setting to know the large-scale aggregate possibilities in these objectives. Additionally, our results do not imply that win-wins are never possible—indeed many case-study examples exist^{8,11-13}. Moreover, win-win and ‘bright spot’ narratives can be important to motivating stakeholders to undertake key behavior changes¹⁹. Instead, our results offer an important bridge between modeling and management, adding precision to case-study-based intuitions about win-win infeasibility, and adding realism to model-based projections of widespread win-wins.

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Methods:

Formal definition of θ and additional details related to proof of equation (5)

As stated in the main text (and illustrated in Fig. 2B), we define the curvature of our hypothetical symmetric tradeoff surface by the angle, θ , between a line connecting any of the surface's corners (it does not matter which corner, due to the symmetry) and the outer edge of the n -dimensional hypercube extending from the same corner (and having lengths 1 in all directions; se Fig. 2B).

We can formally define θ using two vectors, which constitute the hypotenuse and adjacent (to θ) sides of a right-angle triangle. The first vector—which we'll call v_1 —connects the edge of the tradeoff surface (e.g., the point $\{1, a, \dots, a\}$) to the point corresponding to $X(n)$ (i.e., the point $\{X(n), X(n), \dots, X(n)\}$). Thus, the magnitude of v_1 is given by:

$$(8) \quad \|v_1\| = \sqrt{[1 - X(n)]^2 + Y}, \text{ where } Y = (n - 1)[X(n) - a]^2.$$

The second vector—which we'll call v_2 —connects the edge of the tradeoff surface (e.g., the point $\{1, a, \dots, a\}$) to the point on the outer edge of the length-1 hypercube directly 'above' the point corresponding to $X(n)$; i.e., its coordinates (relative to edge point $\{1, a, \dots, a\}$) are: $\{1, X(n), \dots, X(n)\}$. Thus, the magnitude of v_2 is given by:

$$(9) \quad \|v_2\| = \sqrt{Y}.$$

The cosine of θ is equal to $\|v_2\|/\|v_1\|$ (adjacent/hypotenuse), which is equation (6).

The linear tradeoff case is also useful for illustrating the intuition of equation (5), because we can analytically define the surface, and thus the value of $X(n)$ at any n . Specifically, let x_i denote the objective value (expressed as a fraction of maximum achievable) in objective i . The linear surface [where $\theta = \pi/4$ (45°)] is defined by the equation:

$$(10) \quad 1 + (n - 1)a = \sum_{i=1}^n x_i,$$

$X(n)$ corresponds to the point on this surface—i.e. satisfying (10)—where all x_i 's are equal; i.e., $x_i = X(n)$ for all i and $nX(n) = \sum_{i=1}^n x_i$. Inserting this into (10) and solving for $X(n)$, we get:

$$(11) \quad X(n) = \frac{1}{n} + \left(\frac{n-1}{n}\right)a.$$

From (11), we can see that $\lim_{n \rightarrow \infty} X(n) = a$. [i.e. equation (5) is satisfied].

Meta-analysis methods

We began our meta-analysis with an opportunistic search of the literature, beginning with highly-cited papers in the field and those we were previously aware of (e.g., refs. 2, 3, 5, 18, 20-23). We conducted a supplementary search on November 23, 2020 using Google Scholar to review and characterize the additional literature on tradeoff services. To conduct this supplementary review, we used the search term “tradeoffs” paired with the following additional search terms: “marine spatial planning”, “land use”, “fisheries”, “energy”, “environmental”, “ecosystem service”, and “agriculture”. We included all variants of the term tradeoff (i.e. tradeoff, trade-off, and trade off) in our search. Though these search terms were unlikely comprehensive, they encompass a broad range of environmental disciplines and research areas in which tradeoff analyses are used to characterize win-wins. We collected and recorded the information of all papers within the first 100 results from each search and removed any duplicate entries from papers that appear in multiple searches.

Using these methods, our search returned 1052 unique papers related to tradeoffs. We then reviewed each paper to identify those which included tradeoff surfaces. Papers that did not include an empirical or empirically-inspired tradeoff curve were excluded from our analysis. Although there is a growing body of literature on tradeoffs including greater than two objectives, we restricted our search to papers that only included bivariate tradeoff surfaces (still the vast majority of tradeoff studies). We also excluded any papers earlier than the year 2000. However, we note that 5 papers returned in our search from before the year 2000 included tradeoff surfaces. Ambiguous papers were reviewed by at least two coauthors.

From our review, we identified 116 papers that quantified 280 unique tradeoff surfaces. From each paper, we recorded the number of unique tradeoff surfaces developed, their shape [e.g. convex, concave, linear (also called ‘direct’ in the literature, e.g. ref. 18), or other (also called ‘multiple scenarios’¹⁶)], their objectives, and the corresponding figure containing the studied tradeoff surface. We note that not all tradeoff surfaces were developed by maximizing both objectives—some objectives are minimized, which affects the interpretation of curvature (Fig. 3A). As such, we characterize each objective as either ‘good’ or ‘bad’ depending on if the objective was maximized or minimized, respectively.

After characterizing each curve, we used a graph digitizing tool (WebPlotDigitizer: <https://automeris.io/WebPlotDigitizer/>) to estimate the unique $X(2)$ and a_{ij} value for each tradeoff surface (with a separate a_{ij} values for each objective). However, we note that it was not possible to objectively calculate $X(2)$ and a_{ij} values for tradeoffs with “bad” axes, as $X(2)$ and a_{ij} are defined relative to a single-objective maximum and zero. There is no objective analog to zero for “bad” axes, and thus any subjective definition would have specified $X(2)$ and a_{ij} , such that any inference drawn from these values would be tautological. Thus, we were able to calculate $X(2)$ and a_{ij} for 144 tradeoffs (out of 280). We recorded these values and save them with the characterized tradeoff curves as seen in Datasets S1, S2. We include a step-by-step video walk through of how we digitize tradeoff surfaces in the Supplementary Information (Supplementary Video S1).

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Supplementary Information: The Supplementary Information includes Supplementary Methods, Datasets S1 and S2, Notebook S1 (Mathematica code for Figs. 2D and 3C,D), and Supplementary Video S1. These data and code files are also publicly available at www.github.com/mattgburgess/tradeoffs.

Supplementary Information for:
Why win-wins are rare in complex environmental management

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Supplementary Information Contents:

Supplementary Methods (this pdf)

Datasets S1, S2,

Notebook S1 (Mathematica code for Figs. 2D and 3C,D)

Supplementary Video S1

In the main text, we analyze a measure of tradeoff severity, $X(n)$ —the maximum fraction of all single-objective maxima that is simultaneously achievable across all objectives. When the tradeoff is symmetric—meaning that each objective’s single-objective maximum corresponds to a fraction, a , of all other objectives’ single-objective maxima—we show that the limit of this measure is unaffected by the tradeoff surface curvature: $\lim_{n \rightarrow \infty} X(n) = a$ [equation (5)].

Figure 1 is a 3D plot illustrating the Efficiency Frontier (EF) in a three-objective optimization problem. The axes are labeled Objective 1 (x_1), Objective 2 (x_2), and Objective 3 (x_3). The EF is shown as a shaded orange surface. Three points are marked on the EF: $X(2)$, $X(3) = X(2)$ (same x_1 values), and $X(3)$ (different x_1 values). Dotted lines project these points onto the axes. A label "EF, projected onto x_1 - x_2 plane" points to the projection of the EF onto the x_1 - x_2 plane.

Fig. S1. Non-symmetric tradeoffs. Panel **A** illustrates a case of a non-symmetric tradeoff in three dimensions, where the point $\{X(n), \dots, X(n)\}$ does not lie on the tradeoff surface [whereas the point $\mathbf{X}(n)$ does]. Panel **B** illustrates how angles, θ_i , differ across corners of the tradeoff surface when the tradeoff is non-symmetric.

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2 in Fig. S1A) and the largest possible values achievable in other objectives, subject to all objectives having at least a value of $X(n)$. Unlike the point $\{X(n), \dots, X(n)\}$, the vector $\mathbf{X}(n)$ defines a point that must be on the tradeoff surface.

Second, our measure of tradeoff surface curvature, θ —the angle between a corner of the tradeoff surface, the point $\{X(n), \dots, X(n)\}$, and the unit hypercube (illustrated in Fig. 2B in the symmetric case)—no longer has the same value at all corners of the tradeoff surface (as illustrated in Fig. S1B). Instead, this angle has a potentially unique value at each corner of the tradeoff surface, which we denote θ_i at the corner where $x_i = 1$ (Fig. S1B). (Here and in Fig. S1, x_i refers to the fraction achieved of Objective i 's maximum at any particular point.)

Below, we prove 2 propositions that together generalize the idea that tradeoff-surface curvature does not affect the limits of $X(n)$ and $\mathbf{X}(n)$ as n approaches infinity.

Proposition 1: In the limit as n approaches infinity, the point $\{X(n), \dots, X(n)\}$ is only found on the tradeoff surface if the tradeoff surface is symmetric [i.e. all $a_{ij} = a$, implying equation (5) holds].

Effectively, assuming $\{X(n), \dots, X(n)\}$ lies on the surface assumes that we are only considering objectives that are constraining to $X(n)$, and disregarding all others. So, if we are excluding all objectives not limiting to $X(n)$, with infinite objectives, this implies that we must exclude all objectives less limiting than the most limiting objectives—i.e., we exclude any objective, k , where $a_{ik} > a$ for any i . So, if $\{X(n), \dots, X(n)\}$ is on the tradeoff surface with infinite n , then the tradeoff surface is symmetric and the curvature does not affect the limit of $X(n)$ [by equation (5)]. If the point $\{X(n), \dots, X(n)\}$ does not lie on the surface, then the 45° line (where $x_i = x_j$, for all i, j) must not intersect the surface (as shown in Fig. S1A), which implies that the curvature of the surface must be irrelevant to the value of $X(n)$. Thus, Proposition 1 implies that tradeoff-surface curvature does not affect the limit of $X(n)$ as n approaches infinity.

Proof of Proposition 1:

Consider the limit of $X(n)$ as n approaches infinity, in a case where we assume that $\{X(n), \dots, X(n)\}$ lies on the tradeoff surface. The proposition claims that this assumption implies that the tradeoff surface is symmetric (i.e. all $a_{ij} = a$).

The case of a linear tradeoff surface demonstrates the intuition for why this is true. A linear tradeoff surface would have the form:

$$(S1) \quad \sum_{i=1}^n x_i = c,$$

where c is a constant. At corner of the surface corresponding to objective i , $x_i = 1$ and $x_j = a_{ij}$. Each corner must be on the surface, which means that:

$$(S2) \quad 1 + \sum_{j \neq i} a_{ij} = c, \text{ for all } i.$$

Because equation (S2) applies to all corners of the surface, it must be the case that:

$$(S3) \quad \sum_{j \neq i} a_{ij} = \sum_{j \neq k} a_{kj}, \text{ for all pairs, } i \text{ and } k.$$

Thus, we can rewrite $\sum_{j \neq i} a_{ij}$ as $(n - 1)$ multiplied by the mean of all a_{ij} 's, denoted $\bar{a}$:

$$(S4) \quad \sum_{j \neq i} a_{ij} = (n - 1) \frac{\sum_{j \neq i} a_{ij}}{n-1} = (n - 1) \bar{a}.$$

Combining (S1), (S2), and (S4), we can define the surface as:

$$(S5) \quad 1 + (n - 1) \bar{a} = \sum_{i=1}^n x_i.$$

If we assume that $\{X(n), \dots, X(n)\}$ is on the surface, equation (S5) at this point is:

$$(S6) \quad 1 + (n - 1) \bar{a} = nX(n), \text{ which implies:}$$

$$(S7) \quad X(n) = \frac{1}{n} + \left(\frac{n-1}{n}\right) \bar{a}.$$

From (S7), we can see that:

$$(S8) \quad \lim_{n \rightarrow \infty} X(n) = \bar{a}.$$

However, recall that—because we assume $X(n)$ is on the tradeoff surface—it must be the case that $X(n) \geq a_{ij}$, for all i and j (see Fig. 2B). Thus, if $\lim_{n \rightarrow \infty} X(n)$ is equal to the mean of a_{ij} 's, but also cannot be smaller than any a_{ij} , it must be the case that all a_{ij} 's are equal; i.e., $a_{ij} = a$, for all i, j , and therefore $\lim_{n \rightarrow \infty} X(n) = a$ [equation (5)].

Moving to the non-linear case, we now consider—as we do in the main text—the cosine of the angle θ_i . The vector that defines the hypotenuse (for the purposes of the cosine) relative to θ_i , v_1 , connects a point, with a_{ij} in the j th position and 1 at the i th position, to the point $\{X(n), X(n), \dots, X(n)\}$. The magnitude of v_1 is given by:

$$(S9) \quad \|v_1\| = \sqrt{[1 - X(n)]^2 + Z}, \text{ where } Z = \sum_{j \neq i} [X(n) - a_{ij}]^2 = (n - 1) \frac{\sum_{j \neq i} [X(n) - a_{ij}]^2}{n-1}.$$

The vector adjacent to θ_i , v_2 , connects the point, with a_{ij} in the j th position and 1 at the i th position, to the point on the outer edge of unit hypercube directly 'above' the point $\{X(n), X(n), \dots, X(n)\}$, which has coordinates $X(n)$ in all positions except the i th, where it has the coordinate 1. The magnitude of v_2 is given by:

$$(S10) \quad \|v_2\| = \sqrt{Z}.$$

The cosine of θ_i is equal to $\|v_2\|/\|v_1\|$ (adjacent/hypotenuse):

$$(S11) \quad \cos \theta_i = \frac{\sqrt{Z}}{\sqrt{[1-X(n)]^2 + Z}}, \text{ where } Z = (n-1) \frac{\sum_{j \neq i} [X(n) - a_{ij}]^2}{n-1}.$$

Analogously to the symmetric case in the main text, if Z approaches infinity as n approaches infinity, then $\cos \theta_i$ approaches 1, and θ_i approaches 0, which would imply no tradeoff. The term $(n-1)$ approaches infinity as n approaches infinity; therefore, the scale-invariant term $\frac{\sum_{j \neq i} [X(n) - a_{ij}]^2}{n-1}$ (the average squared difference between $X(n)$ and a_{ij}) must approach 0. Since each $[X(n) - a_{ij}]^2$ term must be positive or zero, $X(n)$ must equal a_{ij} (implying all a_{ij} are also equal to each other) for all j in the limit as n approaches infinity for θ_i to not approach zero (no tradeoff).

Thus, we have shown that, with an infinite number of objectives, the point $\{X(n), \dots, X(n)\}$ will only be found on the tradeoff surface if the tradeoff is symmetric, and in such a case $X(n) = a_{ij} = a$, for all i, j , which is Proposition 1. This completes the proof.

Proposition 2: Consider a case where $\{X(n), \dots, X(n)\}$ is not found on the tradeoff surface, and consider the limit of $\mathbf{X}(n)$ —which does lie on the tradeoff surface, but may be greater than $X(n)$ in some directions—as n approaches infinity. The limit of $\mathbf{X}(n)$ as n approaches infinity is a function of only a_{ij} 's (and thus not a function of the tradeoff-surface curvature).

Proof of Proposition 2:

The proof of this proposition is analogous to the proofs of Proposition 1 and equation (5). Let $X_i(n)$ denote the i th component of the vector $\mathbf{X}(n)$. If we now define θ_i —analogously as in previous analyses—as measuring the angle between the i th corner of the tradeoff surface, the point corresponding to $\mathbf{X}(n)$, and the projection of $\mathbf{X}(n)$ onto the unit hypercube, the cosine of θ_i is equal to [derived analogously to equation (S11)]:

$$(S12) \quad \cos \theta_i = \frac{\sqrt{W}}{\sqrt{[1-X_i(n)]^2 + W}}, \text{ where } W = (n-1) \frac{\sum_{j \neq i} [X_j(n) - a_{ij}]^2}{n-1}.$$

Analogously to the above case and the symmetric case in the main text, $\lim_{n \rightarrow \infty} \theta = 0$ (which would imply no tradeoff), unless $\lim_{n \rightarrow \infty} \frac{\sum_{j \neq i} [X_j(n) - a_{ij}]^2}{n-1} = 0$, or $\lim_{n \rightarrow \infty} X_j(n) = a_{ij}$, for all i, j . This demonstrates that $\lim_{n \rightarrow \infty} \mathbf{X}(n)$ does not depend on the curvature—the main property of interest (for the purposes of Proposition 2 and the paper more broadly)—because it depends only on the a_{ij} 's—completing the proof.

Intuitively, the finding that $\lim_{n \rightarrow \infty} X_j(n) = a_{ij}$ for all i, j implies that differences in $X_j(n)$ values result from differences in a_{ij} in the limit as n approaches infinity—analogously to equation (5) in the symmetric case. Counterintuitively though, this finding also seems to imply that, in the limit of infinite objectives, a tradeoff surface must have the property $a_{ij} = a_{kj} = a_j$ for all pairs, i and k , to be coherent. This would mean that the maximum achievable outcome for each other objective

corresponds to the same fraction a_j of objective j 's single-objective maximum, and this fraction determines the limit of $X_j(n)$ as n approaches infinity. This is somewhat analogous to our finding above that the tradeoff must be symmetric for the point $\{X(n), X(n), \dots, X(n)\}$ to be on the tradeoff surface, but nonetheless this counterintuitive property of the limit of infinite-dimensional tradeoffs merits further study.