



[microresearch]

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Logical Loop

Open Mathematics Collaboration*†

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Abstract

We discuss the dubbed term “logical loop” and its implication regarding provability in undecidable theorems.

keywords: logic, provability, undecidable theorems, Gödel’s incompleteness theorems

The most updated version of this paper is available at

<https://osf.io/c5ezm/download>

Introduction

1. T = tautology
2. C = contradiction
3. A, B = formulas
4. Let $A \overset{t}{\leftrightarrow} B$ be true only when both A and B are true.
5. $P \vdash Q$ means Q is derivable from P in the system.

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Self-Recursion and Equivalences

6. $A \equiv (A \leftrightarrow T)$
7. $A \equiv ((A \leftrightarrow T) \leftrightarrow T)$
8. $A \equiv (((A \leftrightarrow T) \leftrightarrow T) \leftrightarrow T)$
9. $A \equiv (((((A \leftrightarrow \dots) \leftrightarrow T) \leftrightarrow T) \leftrightarrow T) \leftrightarrow T)$
10. $C \equiv (T \leftrightarrow C)$
11. $C \equiv (T \leftrightarrow (T \leftrightarrow C))$
12. $C \equiv (T \leftrightarrow (T \leftrightarrow (T \leftrightarrow C)))$
13. $C \equiv (T \leftrightarrow (T \leftrightarrow (T \leftrightarrow (\dots \leftrightarrow C))))$
14. $(A \equiv (A \leftrightarrow B)) \rightarrow (B \equiv T)$

Semigroup

15. semigroup = closed (A) + associative (B)
16. $\mathcal{S} \equiv (S \text{ is a semigroup})$
17. $D \equiv (A \leftrightarrow B)$
18. $E \equiv (A \overset{t}{\leftrightarrow} B)$.
19. D does not prove (16) because both A and B can be false.
20. E proves (16) because both A and B are true.
21. Note that E is a particular case of D .

A proves B and B proves A

22. Let $L \equiv (A \vdash B \vdash A)$.

23. L stands for a *logical loop*.

24. Suppose L is true.

25. Does (24) prove (16)?

26. What is the difference between $A \leftrightarrow B$ and L ?

Final Remarks

27. This paper is meant to be the first piece of argument towards solving this puzzle [1].

Open Invitation

Review, add content, and co-author this paper [2, 3].

Join the **Open Mathematics Collaboration**.

Send your contribution to `mplobo@uft.edu.br`.

Open Science

The **latex file** for this paper together with other *supplementary files* are available [4].

Ethical conduct of research

This original work was pre-registered under the OSF Preprints [5], please cite it accordingly [6]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

Acknowledgement

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