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IM

[knowledge base]
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RINGS: Almost a ring, semiring, zero, integral domain

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Abstract

RING, commutative ring, almost a ring, semiring, zero ring, zero property, zero divisors, domain, integral domain, and their underlying definitions are presented in this white paper (knowledge base).

keywords: ring, zero, domain, abstract algebra, knowledge base

The most updated version of this white paper is available at

https://osf.io/bzugr/download

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Definitions

1. **RING**

$$(R, +, \cdot)$$

   (a) $(R, +) :=$ commutative group
   (b) $(R, \cdot) :=$ monoid
   (c) Multiplication is distributive over addition in R

   $R :=$ set
   $+, \cdot :=$ binary operations on R (addition and multiplication)

   [1, 2]

2. **Commutative Ring** := multiplication is commutative in R

   [1, 2]

3. **Almost a Ring (rng)**

$$(R, +, \cdot)$$

   $(R, +) :=$ commutative group
   $(R, \cdot) :=$ semigroup

   [1, 2]

4. **Semiring (rig)**

$$(S, +, \cdot)$$

   $(S, +) :=$ commutative monoid
   $(S, \cdot) :=$ monoid

   [1, 2]

5. **Zero Ring**

$$(\{0\}, +, \cdot)$$

[1, 2]

6. **Zero property**

$$\forall x \in R : 0 \cdot x = x \cdot 0 = 0$$

$R :=$ set
$\cdot :=$ binary operation on R

[1, 2]

7. **Zero divisors** := nonzero elements whose product is zero

[1, 2]

8. **Domain** := a ring that does NOT contain any zero divisors

[1, 2]

9. **Integral domain** := commutative domain

[1, 2]

Prerequisites

10. **Ordered pair**

$$(a,b) = \{\{a\},\{a,b\}\}$$

$a :=$ first coordinate
$b :=$ second coordinate

[1,3]

11. **Cartesian product**

$$A \times B = \{(a,b) \mid a \in A,\ b \in B\}$$

$A, B :=$ sets
$A \times B :=$ Cartesian product
$(a,b) :=$ ordered pair

[3]

12. **Binary operation**

$$\star : S \times S \to S$$

$S :=$ set
$S \times S :=$ Cartesian product

[1]

13. **Group**

$$(G, \star)$$

(a) Associativity: $\forall x, y, z \in G,\ (x \star y) \star z = x \star (y \star z)$

(b) Identity: $\exists e \in G : \forall x \in G,\ e \star x = x \star e = x$

(c) Inverse: $\forall x \in G\ \exists y \in G :\ x \star y = y \star x = e$

$G :=$ set
$\star :=$ binary operation

[1]

14. **Commutative group (Abelian)**

$$G_b$$

$$\forall g_1, g_2 \in G_b,\ \ g_1 g_2 = g_2 g_1$$

$G_b :=$ group

[2]

15. **Associative binary operation**

$$\forall x, y, z \in S : x \circ (y \circ z) = (x \circ y) \circ z$$

$\circ :=$ binary operation

[4]

16. **Semigroup**

$$\mathcal{S} = (S, \circ)$$

$S :=$ non-empty set
$\circ :=$ associative binary operation

[4]

17. **Monoid** := a semigroup $\mathcal{S} = (S, \circ)$ that contains an identity $e \in S$ such that

$$\forall x \in S : e \circ x = x \circ e = x$$

[4]

18. **Distributive**

$$\forall x, y, z \in R : x \cdot (y + z) = x \cdot y + x \cdot z \qquad \text{left distributive}$$

$$\forall x, y, z \in R : (y + z) \cdot x = y \cdot x + z \cdot x \qquad \text{right distributive}$$

[1,2]

Open Invitation

Review, *add* content, and **co-author** this white *white paper* [5,6].
Join the **Open Mathematics Collaboration**.
Send your contribution to `mplobo@uft.edu.br`.

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The **latex file** for this *white paper* together with other *supplementary files* are available in [7].

Ethical conduct of research

This original work was pre-registered under the OSF Preprints [8], please cite it accordingly [9]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

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  `https://cos.io`

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  `https://osf.io`

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