Backtracking-free Interactive Music Scores with Temporal Relations over Rhythms

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1 Introduction

Constraint programming is a declarative programming paradigm wherein relations between variables are stated in the form of constraints. Applying constraint programming to classical music is a very natural idea because harmonization rules are easily stated. This is also true for many contemporary music. The reader may see, for instance, *Situation*, a constraint solving library that has been used in several compositions [18].

An interesting kind of musical problems solved with constraint programming are those concerning rhythmical structures. As an example, to find n rhythmical patterns of fixed lengths, each pattern played on one voice, such that two different voices never play the beginning of a note simultaneously. Such problem was proposed by composer Mauro Lanza and it has been extensively studied by Truchet et al., among other music problems where a constraint programming was found to be very pertinent [38].

Constraint satisfaction problems (CSPs) are mathematical problems defined as a set of objects whose state must satisfy a number of constraints or limitations. Formally, a CSP is defined as a triple $\langle X, D, C \rangle$, where X is a set of variables, D is a domain of values, and C is a set of constraints. A solution is an evaluation of the variables that satisfies all constraints with values from their domains. CSPs can be solved with complete methods such as forward checking, used by *Situation*, but they are NP-complete problems in general.

Another alternative to solve CSPs is to use (incomplete) local search algorithms to find iteratively an approximative solution. Local search is a metaheuristic for solving computationally hard optimization problems. An example of local search is *adaptive search*: A computational method that optimizes a problem by iteratively trying to improve a candidate solution with regard to the problem constraints (and possibly an optimization function). Truchet et al. found appropriate this method for multiple problems in music composition [38].

In spite of the advances in constraint solving for music compositions problems, as far as we know, there is little research on solving CSPs during real-time music

interaction. In this paper we will focus on *interactive scores*, a formalism for the design of interactive scenarios closely related to constraint programming. Interactive scores has been a subject of study since the beginning of the century [9]. A recent model [3] has inspired two applications: *iScore* to compose and perform Electroacoustic [1] music and *Virage* to control live spectacles and for interactive museums [2].

Scenarios in interactive scores are represented by *temporal objects*, *temporal relations* and *interactive events*. Examples of temporal objects are sounds, videos and light controls. Temporal objects can be triggered by interactive events: The user can choose a date for the start or end of a temporal object. In addition, several temporal objects can be active simultaneously. A temporal object can contain other temporal objects: this hierarchy allows us to control the start or end of a temporal object by controlling the start or end of its parent. Temporal relations provide a partial order for the execution of the temporal objects; for instance, temporal relations can be used to express precedence between two objects.

All temporal and hierarchical relations can be easily stated by the means of constraints. In fact, there is a CSP associated to an interactive score. In such problem, the variables are the starting and ending dates of the temporal objects, the domains are finite for practical applications, and the constraints are the temporal and hierarchical relations of the problem. We will formally present such CSP and the formal definition of interactive scores in section 2. Later, in section 3, we will show that under certain conditions, such CSP can be solved in polynomial time.

Obviously, we do not have a polynomial algorithm to solve all instances of the problem. Consider, for instance, the case where the durations of the temporal objects are not single values nor intervals, but discrete values. Durations represented as sets of discrete values are useful to model rythmical problems as the one presented in Figure 1. They are also useful when modeling loops and conditional branching on the lines of [29]. Nonetheless, the problem of finding if there is a solution for the CSP associated to an interactive score, in the general case, is NP-Complete by encoding from the *subset sum problem*, as we will describe in section 3.

Process calculi has been applied to the modeling of interactive music systems [27, 26, 32, 37, 25, 36, 4, 33, 28, 12, 23, 20, 22, 24, 5, 31, 21, 29, 30, 19] and ecological systems [34, 14, 35, 15]. In addition, research on algorithms [13, 11, 17] and software engineering [16] also contributes to this field.

1.1 Related work

Satisfaction (SAT) is the problem of determining if the variables of a given Boolean formula can be assigned in such a way as to make the formula evaluate to true. Many CSPs can be easily translated into SAT instances. A class of algorithms called SAT solvers can efficiently solve a large enough subset of SAT instances to be useful in various practical areas; however, no current such methods can efficiently solve all SAT instances.

Figure 1: In the score above, temporal objects b , c and d are preceeded by a . The possible durations of a are given by a rythmic pattern that can be representd by the set $[0, 2, 4, 8, 11]$, where each a eighth note lasts one discrete time-unit. Since b, c, d are preceeded by a , they have to be executed zero, two, four, eight or 11 time units after f starts.

We believe that none of constraint solving techniques described above nor SAT solvers are appropriate to compute the satisfacibility of an interactive score during a real-time performance. (1) Forward checking has exponential time and space complexities. (2) The iterative nature of adaptive search will cause the starting and ending times of the objects to fluctuate along time and may cause inconsistencies with values already chosen in the past. In addition, local search does not guarantee a solution, for instance, it may cause that two temporal objects that are supposed to finish at the same time finish within a noticeable interval of time, which is unacceptable in most compositions. (3) Finally, we conjecture that SAT solvers are not appropriate for real-time interaction: they will not allow to interact the computer with an user without noticeable delays.

We believe that using space-efficient backtrack-free representation of constraint satisfaction problems [6] is appropriate for real-time interaction. This approach has never been used in real applications as far as we know although the results with random generated CSPs are quite promising. Such representation consists on removing values from the domain of the variables that lead to dead-ends, trading the need of backtracking by a preprocessing of the problem. The algorithm is parametric on a *seed*: a solution that is already known and must be preserved after reducing the domains.

A particular case of these algorithms is 1-BFR, which guarantees a space-efficient representation but also solution lost. This algorithm sadds *notequals constraints* ($\neq$) to the original problem, which is equivalent to remove values from the domains. The 1-BFR algorithm is presented in section 4. A generalisation of 1-BFR is n -BRF, where n is the number of variables. This algorithm adds n -ary constraints

and guarantees no solution lost but space requirements may be exponential in the size of the problem, thus is not of practical use for interactive scores.

1.2 Contribution

In this paper we propose an extension of the 1-BFR algorithm proposed in [6] to allow multiple seeds. Since the 1-BFR requires a known variable ordering, we propose a static variable ordering for the CSPs of a subclass of interactive scores. We also present results on the complexity of those CSPs to identify the tractable (polynomially solvable) cases and the cases where the 1-BFR algorithm is of practical use in section 3. We show the pertinence of 1-BFR in a score whose temporal objects' durations are represented as sets of pair numbers in section 4. Finally, we discuss results, conclusions and future work in section 5.

2 Interactive Scores

Interactive scores are composed by *temporal relations* and *temporal objects*. A temporal object has two *points identifiers*: for its start and for its end. In the implementation, points have an external action associated (e.g., turn on the lights or play a sound).

Let Σ be set of names, P a set of all point identifiers, and let $\mathcal{H} = (V, E)$, $V \subseteq \Sigma, E \subseteq V \times V$, be a directed tree called the *object hierarchy*. Temporal objects must have a unique object identifier to guarantee that the hierarchy is always a tree. Interactive points are a special case of temporal objects with zero-duration. Interactive points must be temporal objects instead of point identifiers (ids). Consequently, point ids are either the start or the end of an object. Interactive points control the starting or ending time of so-called *static temporal objects*.

Definition 2.1 (Temporal object (TO)). *. Let $\mathcal{H} = (V, E)$ an object hierarchy and P the set of point ids. A TO is a tuple $o_{\mathcal{H}} = \langle sp, ep, \Delta, v \rangle$, where $sp, ep \in P$ are the (unique) start and end point ids, $\Delta \subseteq \mathbb{N}$ are the possible durations, $v \in V$ is the object identifier. We assume an injective function $\text{object}_{\mathcal{H}} : V \rightarrow \mathcal{T}$ giving, for each name, the corresponding TO. The set of all TOs is $\mathcal{T}$. An interactive point is an element of $\mathcal{T}$ such that $\Delta = \{0\}$. Functions $\text{sp}(v)$, $\text{ep}(v)$ and $\text{d}(v)$ return, respectively, start point, end point and the durations of $\text{object}_{\mathcal{H}}(v)$.*

A partial order among temporal objects is given by *point-to-point* relations without disjunction nor inequality: Qualitative temporal relations between the start or end of two temporal objects; for instance, the start of a is executed before the end of b , or the start of a is executed at the same time than the start of b [7].

Definition 2.2 (Temporal Relation (TR)). *. A TR is a tuple $\langle p, rel, q \rangle$, where $p, q \in P$ are two different point ids of two different TOs, $rel \in \text{Rel} = \{<, >, \leq, \geq, =\}$. The set of all TRs is $\mathcal{E}$.*

Figure 2: Example of a score. The duration of all temporal objects are pair numbers between 1 and 10.

A score is a tuple consisting of an object hierarchy, a set of temporal relations over those objects and a set of point identifiers, such that two interactive points cannot start at the same time, for simplicity. Figure 4.4 is a example of a score.

Definition 2.3 (Interactive Score.). *An interactive score is a tuple $\langle \mathcal{H}, R, P \rangle$, where $\mathcal{H} = (V, E)$ is an object hierarchy, P is the set of point identifiers, and $R \subseteq (P \times \text{Rel} \times P)$ is given by $\langle p, r, q \rangle \in R$ iff p is the start or end point of an object, q is the start or end point of another object, and they are related by a point-to-point relation r . R must be such that two interactive points in V do not start at the same time. Let $\mathcal{S}$ be the set of all interactive scores.*

2.1 Temporal Constraints of the Score

The temporal specification of a score is the conjunction of constraints over the start and end times of the temporal objects of the score. We use the following notation for temporal constraints: $t + \Delta =_{\text{def}} \{t' | t' = t + \delta, \delta \in \Delta\}$. Function $\text{ct} : P \rightarrow \mathbb{N}$ is a bijective function that associates a point identifier to the a variable that represents the time when it is launched.

Definition 2.4 (Temporal Specification of an Interactive Score.). *The temporal specification $\text{ts}(s)$ of an interactive score $s = \langle \mathcal{H}, R, P \rangle$, $\mathcal{H} = (V, E)$, is the conjunction of constraints:*

1. *For each $a \in V$, $\text{ct}(\text{ep}(a)) \in \text{ct}(\text{sp}(a)) + \text{d}(a)$, where $\text{d}(a)$ are the durations of a .*

For each node x in the subtree rooted at a , $\text{ct}(\text{sp}(x)) \geq \text{ct}(\text{sp}(a))$ and $\text{ct}(\text{ep}(a)) \geq \text{ct}(\text{ep}(x))$.

Figure 3: Temporal Constraints of the Score. For simplicity, two points that must happen at the same time are represented with a single variable and redundant relations induced by the hierarchy are removed. A constraint Δ between a variable x and a variable y means that $y \in x + \Delta$.

2. For each $\langle p, r, b \rangle \in R$, $\text{ct}(p) \text{ } r \text{ } \text{ct}(q)$, where $r \in \{=, <, >, \leq, \geq\}$ ¹.

The playability of a score can be decided by solving a *constraint satisfaction problem* (CSP). This property states that all temporal objects will be *played* during execution; this is desirable because a score can be over-constrained and therefore not playable.

The CSP associated to an interactive score is given by: a variable x_i for starting, ending or interactive point in the score; the domain $\mathbb{N}_{\mathbb{F}} \cup \{\infty\}$ for each variable, where $\mathbb{N}_{\mathbb{F}}$ is a finite subset of $\mathbb{N}$; and the single constraint $\text{tc}(\varepsilon)$. As an example, the constraint graph for the score in Figure 4.4 is presented in Figure 3.

3 Time Complexity of the Playability of a Scores

In what follows we will show that that the problem is NP-complete in the general case, but there is an interesting subclass that is tractable.

3.1 The NP-complete case

We will show that the decision problem of the subset sum can be encoded as an interactive score. The problem is stated as follows: Given a set of weights $(\{w_1, w_2 \dots w_n\})$ of n objects and an integer W , does any non-empty subset sum to W ? Note that Subset sum can also be thought of as a special case of the knapsack problem.

- $\sum_{i=1}^n w_i \cdot x_i = W, x_i \in \{0, 1\}$, where at least one $x_i \neq 0$.

¹ $r \in \{=, <, >, \leq, \geq\}$ has the usual interpretation over $\mathbb{N}$.

There are several algorithms to solve subset sum, all with exponential time complexity in n , the number of objects. The most naive algorithm would be to cycle through all subsets of k numbers and, for every one of them, check if the subset sums to the right number. The running time is of order $O(n2^n)$, since there are 2^n subsets and, to check each subset, we need to sum at most n elements. The best algorithm known runs in time $O(2^{n/2})$ [10].

In what follow we present the temporal constraints of the score presented in Figure 4². For $1 \leq i \leq n$,

- $e_i \in s_i + \{0, w_i\}$
- $e_s = s_s + W$
- $e_i = e_s$
- $s_s = s_i$

The reader can easily verify that these constraints are equivalent to the constraint

- $\sum_{i=1}^n w_i x_i = W, x_i \in \{0, 1\}$.

Figure 4: Encoding of the subset sum problem into an Interactive Score. Note that the subset sum problem is a variant of the knapsack decision problem where costs are not taken into account and the goal is to find if there is a subset of the elements that fills exactly the knapsack capacity.

It is trivial to show that there is a polynomial algorithm to check whether a solution satisfies the problem. Such algorithm replaces the values of x_i from the solution in the equation above. In conclusion, the subset sum can be encoded as the playability (i.e., satisfiability) of an interactive score and there is a polynomial time algorithm to check if the solution is true, thus the satisfiability of an interactive score is NP-complete.

²Knapsack picture taken from the Wikipedia.

3.2 The tractable subclass

If durations are intervals of integers without holes, the playability can be reduced to compute the path of minimum weight in a graph. Given that there are subtractions to represent the duration of the temporal objects, there will be some negative coefficients. Dijkstra's algorithm is not appropriate because there are negative coefficients. Therefore, a simple solution is to use Floyd-Warshall algorithm, which complexity is $O(n^3)$, where n is the number of points (starting, end and interactive points) in the score.

This problem is equivalent to the well-known Simple Temporal Problem (STP) [8]. A STP consists on a set of point variables $X = \{x_1, \dots, x_n\}$ and binary constraints over those variables $C = \{c_1, \dots, c_n\}$ of the form $c_k : x_i - x_j \in [a, b]$ with $a, b \in \mathbb{N} \cup \infty$. There are better algorithms than Floyd-Warshall for this problem, for instance, *directed path consistency (DPC)* [8] and *partial path consistency (PPC)*. The complexity of DPC is $O(nW * d^2)$, where $W * d$ is the induced length of the graph along a given ordering d .

Arc-consistency (AC) is not equivalent to satisfiability in this problem because the constraint graph usually contains cycles, thus we cannot recursively propagate values without the need of backtracking. Nonetheless, we will discuss in section 4 that there exists a static variable ordering for the CSPs of this subclass of interactive scores that allows us to recursively propagate values using arc-consistency, even in presence of cycles, without the need of backtrack. Complexity of arc-consistency is $O(ed^2)$ for AC-7, where e is the number of arcs and d the maximum domain among the variables. Conjecture: Its performance is usually better than DPC ?

A recent option is "Incrementally Solving STNs by Enforcing Partial Path Consistency". Discuss this in the conclusions.

4 Backtrack-free Interactive Scores

In this section we present the original seeded algorithm to find a 1-BFR form of a CSP. We also present our extension of such algorithm using multiple seeds, and finally, we show an example of how to use 1-BFR algorithms in the CSP of an interactive score.

4.1 The seeded algorithm

It was introduced in [6] an algorithm to find a 1-BFR form of a CSP that guarantees that a known solution of the problem (a seed) will be part of the solutions of the 1-BFR form of the CSP. Its space complexity is polynomial in the number of variable because it only needs to represent the domains of each variable.

What is P_i ?

What is PP_i ?

How does the algorithm works...

Its correctness was also proved in [6]. The argument is as follows. The only place where the algorithm could fail is in line 8 if it is impossible to choose a such that $a \notin s^*$. ?? ARGUMENT THIS. A failfure in line 10 enforcing arc-consistency is neither possible. To see that this cannot happen, consider that we never remove a value that is the seed, thus there is always at least one value that can be chosen in the domain of each variable.

Algorithm 1 Non-deterministic algorithm for Seeded-1-BFRs

```

1: function Seeded-1-BFR( $n, s^*$ )
2: if  $s^*$  is NIL then
3:   return Failure
4: end if
5: enforce consistency on  $P_n$ 
6: for  $i \leftarrow n \rightarrow 2$  do
7:   for all solutions  $s$  to  $PP_i$  that do not extend  $P_i$  do
8:     choose  $a = x_j$  s.t.  $a \in D_j, a \in s$  and  $a \notin s^*$ 
9:     remove  $a$  from  $D_j$ 
10:    enforce consistency on  $P_n$ 
11:   end for
12: end for
13: return Success
14: end function

```

4.2 Multi-seeded Algorithm

A simple extension to the 1-seeded algorithm into a multi-seeded algorithm. We present a deterministic version of the algorithm, which closer to the implementation than the 1-seeded algorithm presented in [6]. Function MultiSeeded-1-BFR enforces consistency on $\mathcal{P}_n$ and delegates the rest of the computation to MultiSeeded-1-BFR-sons that recursively reduces a value from the domain from each solution of $\mathcal{P}_i$ that do not extend $\mathcal{P}\mathcal{P}_i$. This function delegates the traversal of each solution that do not extend $\mathcal{P}\mathcal{P}_i$ to MultiSeeded-1-BFR-brothers.

Algorithm 2 Obtains a 1-BFR for a CSP using a set of seeds

```

1: function MultiSeeded-1-BFR( $domain, n, seeds$ )
2: if  $seeds$  is NIL then
3:   return Failure
4: end if
5: Reduce  $domain$  by enforcing consistency on  $\mathcal{P}_n$ 
6: return MultiSeeded-1-BFR-sons( $domain, n, seeds$ )
7: end function

```

Algorithm 3 Computes backtracking for MultiSeeded-1-BFRs for P_i

```

1: function MultiSeeded-1-BFR-sons(domain, i, seeds)
2: if  $i == 1$  then
3:   return domain
4: else
5:   dontExtend = solutions to  $PP_i$  that do not extend to  $P_i$ 
6:   if dontExtend is NIL then
7:     return MultiSeeded-1-BFR-sons(domain,  $\text{index}(PP_i)$ , seeds)
8:   else
9:     return MultiSeeded-1-BFR-brothers(domain, 0, dontExtend, i, seeds)
10:  end if
11: end if
12: end function

```

Algorithm 4 Computes backtracking for MultiSeeded-1-BFRs for each solution sol_j of PP_i that do not extend to P_i

```

1: function MultiSeeded-1-BFR-brothers(domain, j, solutions, i, seeds)
2: if solutions is NIL then
3:    $O \leftarrow$  MultiSeeded-1-BFR-sons(domain,  $i - 1$ , seeds)
4: else
5:    $O \leftarrow \text{empty}$ 
6:    $k \leftarrow 0$ 
7:   while  $k < \text{size}(\text{solutions}_j) \wedge O == \text{empty}$  do
8:     ValueToRemove  $\leftarrow sol_j[k]$ 
9:     if ValueToRemove  $\notin seed[k]$ ,  $seed \in seeds$  then
10:      Remove ValueToRemove from domain[k]
11:      Reduce domain by enforcing consistency on  $\mathcal{P}_i$ 
12:       $O \leftarrow$  MultiSeeded-1-BFR-brothers(domain,  $j + 1$ , solutions, i, seeds)
13:       $k \leftarrow k + 1$ 
14:     end if
15:   end while
16: end if
17: return O
18: end function

```

P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
true							
$V_1 \leq V_2$							
$V_2 \leq V_3$							
$V_3 < V_4$							
$V_5 \in V_3 + \Delta_r$ $V_4 < V_5$							
$V_5 < V_6$ $V_6 \in V_4 + \Delta_g$							
$V_6 \leq V_7$ $V_7 \in V_2 + \Delta_l$							
$V_7 \leq V_8$ $V_8 \in V_1 + \Delta_c$							

Table 1: Subproblems for the CSP of the interactive score of Figure 4.4, whose constraint graph is presented in Figure 3. Since there is a total order, the parent of problem i is problem $i - 1$.

4.3 Variable ordering for Interactive Scores

Variable ordering in Music is important and ... [?, ?]

For the CSP of a score, we must choose a variable ordering such that respects the partial order established by the temporal relations and the hierarchy.

Proposition 4.1. *When the durations are intervals of integers and the variable ordering respects the partial order given by the temporal constraints of the score, then the problem after enforcing Arc Consistency is in BFR form for any variable ordering that respects the partial order given by $tc(s)$.*

Proof. ?

□

4.4 Study case

In what follow we study the 1-BFR form of the score in Figure , provided that all the durations must be pair numbers between one and 10. The temporal constraints of the score are presented in Figure 3. The variable ordering is such that it respects the order imposed by the temporal relations and the hierarchy, in this case we have a total order.

Table 1 presents the subproblems P_i for the CSP associated to the score in Figure 4.4. Table ?? presents the results of enforcing arc-consistency on P_n . In the next section we present the results of executing the seeded and multiseeded algorithms for this problem.

5 Results and Conclusions

5.1 Summary

Introduction

Complexity

BFR

Results For non-seeded we got just one solution and preliminary studies in [6] showed that it is slower. For the seeded case: Seed $i = 130$, namely $[0, 0, 1, 3, 5, 7, 8, 10]$, we got 150/603 solutions. With one seed the maximum is 150/603 solutions. With a 2-seeded approach using $i = 130$ and $j = 306$, namely $[0, 2, 2, 5, 6, 7, 10, 10]$, we got 190/603 solutions. Using 3-seeded only made the problem slower and did not ?? more solutions.

5.2 Conclusions

Multiseeded works better than single seeded, but it's slower

Tractable disjunctions, ORD-horn, etc Allen without disjunction is tractable. Disjunction without equality and without ,?? is tractable. ord-horn is tractable. other kinds tractable over reals. most cases in integers and non tractable.

the n voices problem We will need global constraints or disjunctions of temporal objects. We cannot have a static order in this case. The bfr approach does not longer work.

5.3 Future work

We foresee two simple extensions to our approach, already described in [6]. Use *Branch-and-Bound* search to find the 1-BFR form that represents the maximum number of solutions and use the inverse of Geelen's promise heuristic during search (i.e., choose the pair with the most conflicts with values in the domain of neighboring variables).

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