Explanation of black hole evaporation by applying the Lense-Thirring Effect and extending the Bardeen-Petterson Effect

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Abstract

The purpose of this publication is to demonstrate that there may be a relativistic explanation for black hole radiation. This explanation consists of an extension of the Bardeen-Petterson Effect, an application of the Lense-Thirring Effect in the case of black holes, in order to explain the inclination of the accretion disk with respect to the inclination of the gaseous clusters entering and leaving the accretion disk. The Lense-Thirring Effect is a relativistic phenomenon, due to the speed of propagation of the gravitational field (the speed of light), causing a space-time "dragging". The Atropic Effect (named after one of the Moirai of Greek mythology: Atropos, "the inevitable") consists of an extension of the Bardeen-Petterson Effect to the internal structuring of the black hole and an application of the Lense-Thirring Effect to describe the "gravitational bath" of the black hole, in order to make possible the evacuation of quanta of energy and, thus, the evaporation of black holes.

Keywords: Astrophysics, Atropic Effect, Bardeen-Petterson Effect, Black hole, Black hole evaporation, Black hole radiation, General Relativity, Gravitation, Lense-Thirring Effect.

Introduction

The idea that black holes can evaporate initially comes from a thermodynamic approach to their modeling. This hypothesis of black hole radiation generated by the quantum fluctuations of the vacuum, or hawking radiation, will here be replaced by a relativistic cause to the evaporation of the black holes. The black hole radiation will then be closer to the relativistic jets possibly generated by the Penrose process, but this concept will be expanded to allow the extraction of all the energy from the rotating black hole.

In order to explain the evaporation of black holes, it will be argued that black holes lose mass by emitting radiation. The relativistic explanation for these radiations is an application of the Lense-Thirring Effect and an extension of the Bardeen-Petterson Effect.

Thus, the evaporation of the black holes would be due to their speed of rotation relative to their mass. It is then argued that all black holes that evaporate are rotating. Although strictly non-rotating black holes (Schwarzschild and Reissner-Nordström black holes) are theoretically possible, they are very unlikely in the natural state, as their formation conditions do not correspond to the regular formation conditions of black holes, according to no-hair theorem. It is therefore inconceivable that a black hole existing in nature does not evaporate, if the Atropic Effect is correct. Also, it is argued here that a singularity is theoretically assimilated to the black hole. However, the heterogeneity of the black hole, which we will argue here, makes highly improbable the formation of a perfect singularity, thus indicating, in application of the Lense-Thirring Effect and extension of the Bardeen Petterson Effect, that light can escape from the black hole itself. The term "singularity" will then be used without corresponding to a perfect gravitational singularity, as usually described as the "heart" of a black hole.

It will be explained, on the one hand, the application of relativistic principles to the internal structure of black holes and, on the other hand, how this structuration could explain the evaporation of black holes.

Part I – The precession

Let's start with a quick explanation of the Lense-Thirring Precession and the Bardeen-Petterson Effect.

Subpart 1 – The Lense-Thirring Effect

In General Relativity, when a body is in rotation on itself, in addition to the gravitational effect that modifies the space-time, its rotation modifies the geometry of the gravitational field that it emits (propagating with the speed of light): this is the Lense-Thirring Effect [2], [3] (LTE).

Precession is calculated by first determining the gravitomagnetic field of the object. It is an analogy between electromagnetism (Maxwell's equations) and gravitation, due to the

behavior of the gravitational field moving like a wave at the speed of light. This field is expressed by:

$$B_{GEM} = \frac{3}{5} R^2 q \left(\omega r \frac{r}{r^5} - \frac{1}{3} \frac{\omega}{r^3} \right)$$

Simplified in:

$$B_{GEM} = - \frac{4}{5} \frac{\omega m R^2}{r^3} \cos \theta$$

Allowing us to determine the precession of Lense-Thirring:

$$\Omega_{LT} = - \frac{2}{5} \frac{\omega m G}{c^2 R} \cos \theta$$

Where Ω_{LT} is the precession, ω is the angular velocity, R is the radius, and θ is the latitude.

And to make an approximation in order to establish the metric [6], [7], [11] (obtained by Lense and Thirring), describing the deformation of the geometry of the Space-Time taking into account its "dragging":

$$ds^2 = \left(1 - \frac{2GM}{rc^2} \right) c^2 dt^2 - \left(1 + \frac{2GM}{rc^2} \right) d\sigma^2 + 4G \varepsilon_{\mu\nu\varphi} S^\varphi \frac{x^\mu}{c^3 r^3} c dt dx^\nu$$

Subpart 2 – The Bardeen-Petterson Effect

The precession rate varies with distance. Thus, an accretion disk around a black hole increasingly close to the black hole will be more and more confused with the plane perpendicular to the axis of rotation of the object. This is the Bardeen-Peterson Effect [12], [13], [14] (BPE).

Part II – Conceive a black hole interior

In order to apply this to the singularities, it is necessary, on the one hand, to consider their rotation [6] and, on the other hand, to consider the effect of the rotation on the curvature of the space-time [2], [3], [12] and the consequence of this alteration of the deformation at the poles of the black hole, crossed by the axis of rotation of the black hole, inclined perpendicularly to the accretion disk. First, we need to determine what constitutes the heart of our black hole, what is on the other side of the event horizon.

Now, imagine that the internal structure of the black hole, which is on the other side of the event horizon, is not a compact object, but another vortex of quanta of energy * - given their speed - whose light could not really escape, which we will call the heart. Indeed, taking into account how matter and energy of the accretion disk enters the black hole, and by

application of LTE and BPE, we are led to conceive that the disc continue on the other side of the event horizon and that it actually constitutes a vortex of quanta in rotation. So, considering the precession :

$$\Omega_{LT} = - \frac{2}{5} \frac{\omega m G}{c^2 R} \cos \theta$$

We will have to replace the mass m by his equivalent $\frac{E_p}{c^2}$ and get :

$$\Omega_{LT} = - \frac{2}{5} \frac{\omega E_p G}{c^4 R} \cos \theta$$

By applying LTE, the geometry of the Space-Time would be altered by the rotation of the black hole heart and, according to the BPE, this “dragging” would result in a higher concentration of quanta on a plan perpendicular to the axis of rotation and less important to the poles traversed by said axis of rotation. We will call this disk corresponding to the plan the critical accretion disk (CAD), given that such an accretion of quanta would have the consequence, of allowing the propagation of an intense gravitational field, in the same way as for matter, which would be, given the heterogeneity of the “gravitational bath”, not a perfect plan. In this way, we would have a vortex full of extremely tight particles, strongly deforming the geometry of Space-Time. The average speed could not exceed that of light, the particles not all evolving on a trajectory perfectly integrated with the plane of rotation.

Graphical representations of "dragging" tori inside the black hole

Graphical representations of central (A) and peripheral (B) “dragging” tori inside the black hole, behind the event horizon (C), including the sense of rotation (D) of the singularity. For each torus, we represent the interior (red) and exterior (blue) "dragging".

* In order to consider that this set of quantities of energy can curve Space-Time geometry, we must of course consider it as an object, whose characteristics satisfy the description of Kerr's metric. Thus, considering that each quantum goes at the speed of light and spirals, because of the curvature of Space-Time and the continuity of the accretion disk, we obtain a compact set of energy quanta, whose average rotational speed, considering the heterogeneity that will be explained below, is not equal to that of light, all quanta not moving perfectly parallel to the plane perpendicular to the axis of rotation. We also get a black hole whose average motion speed is the result of the motion speed is the set of quanta in space. So we have a compact object, consisting of quantities, whose speed of rotation and motion are lower than the speed of light. If the black hole is almost motionless and almost homogeneous, the Kerr metric will be sufficient to describe the curvature of the space-time geometry of such an object.

Part III – From homogeneous singularity to heterogeneous singularity

Without the Lense-Thirring Effect and therefore without the Bardeen-Petterson Effect, the singularity is homogeneous and the CAD is not formed. Thus, the poles are not discharged and the force $F_G^{i'}$, as represented below, is higher, which makes it more difficult for the energy to be evacuated by the poles. Thus, by increasing the rotation speed of the singularity, we obtain a more important Lense-Thirring precession, discharging in E_p the poles in favor of the CAD, thanks to the Bardeen-Petterson Effect.

Graphical representation of $F_G^{i'}$

Graphical representation of $F_G^{i'}$ influence (A) on the trajectory (blue) of the particle inside the black hole, behind the event horizon (B), including the axis of rotation (C) of the singularity.

The rotation of the singularity will allow, with CAD formation by extending BPE, the reduction of $F_G^{i'}$ and will be able to allow quanta moving along the axis of rotation not to fall back into the singularity, thus no longer constituting a perfect gravitational singularity. [1], [4], [8]

Part IV – The trek of the particle

A quantum of energy then enters the heart by the CAD plan, it is attracted towards the center of the heart of the black hole (the theoretical singularity), following the direction of rotation of the vortex, and reaches it. At this moment, nothing prevents it to cross it, but the extreme spatial contraction of the CAD would not allow him to leave the singularity if he does not return to the event horizon by following a geodesic that would allow it, and therefore a path not curved by the greater attractiveness of the CAD. The only way for a quantum of energy to leave the heart would be to follow a trajectory extremely close to the axis of rotation of the black hole, where the Space-Time “dragging” would be weak and where it would not “toggle” again to the CAD.

We would then be in a situation of continuous asymmetric “rebound”, where quanta enter the black hole, go through the theoretical singularity (here we study a rotating black holes, and not Schwarzschild black holes, so the singularity is not a perfect singularity), before attempting to redirect to the event horizon.

Thus, by strictly applying LTE and BPE, inducing an increasingly strong concentration of particles towards the plane and increasingly weak towards the poles traversed by the axis of rotation, and taking into account the extreme density of the vortex, we would witness a continuous emission by the poles of radiation having managed to flee the heart by the poles, following the axis of rotation of the singularity.

Graphical representations of trek of the particle

Graphical representations of outgoing particle trajectory (blue) and falling particle trajectory (red) inside the black hole, behind the event horizon (A), including the axis of rotation (B) of the singularity, the CAD approximate area (C), and the strong gravitational attraction force of the CAD (D).

Part V – Black hole evaporation

Moreover, the more the black hole would be massive, the more it would emit energy. We will note this potential emitted energy E_p , corresponding to singularity mass m .

$$\Omega_{LT} = - \frac{2}{5} \frac{\omega m G}{c^2 R} \cos \theta$$

According to LTE, the higher the rotational speed, corresponding to angular velocity ω and mass m , the higher the precession Ω_{LT} would be and thus the LTE and BPE would be higher, [2], [3], [12] which would increase the concentration of particles close to the plane and gradually reduce that of the poles, which would theoretically allow greater energy evacuation, by the method explained above.

That said, it should not be forgotten that the more the black hole is heavy, the greater its radius R is important, which has a negative influence on the final evacuation of quanta of energy, noted E . Indeed, the greater the mass m of the black hole, the more the quantum must approach the axis before leaving it, given the force of attraction F_G^i and the radius R that the quantum must "travel" to leave the black hole. [1] So:

$$E = E_p - \|F_G^i \cdot R\| \cdot (\Xi \left(- \frac{2}{5} \frac{\omega m G}{c^2 R} \cos \theta, m \right))^{-1}$$

Simplified in:

$$E = E_p - \|W^i\| \cdot (\Xi(\Omega_{LT}, m))^{-1}$$

Where $\|W^i\| \cdot (\Xi(\Omega_{LT}, m))^{-1}$ represents the energy that is not evacuated and where $\|W^i\|$ is the energy that should not be evacuated without LTE and BPE applications, expressed by the atropic term $(\Xi(\Omega_{LT}, m))^{-1}$. The older is the black hole, the more regular and important is the precession. The entropy will be more and more important, while the black hole will be more and more stable.

Note: We do not start from the same hypothesis as for the thermodynamic approach of black holes evaporation. I do not know why we come to a similar conclusion about the relationship between stability and entropy.

The atropic term $(\Xi(\Omega_{LT}, m))^{-1}$ is a dynamic factor depending on the distributions of LT precession and mass in the Space-Time inside black hole.

These equations only work in a rotating black hole. In a perfectly non-rotating black hole (Schwarzschild black hole), the quantum could not cross the singularity. Indeed, by analysing these equations, we could conclude that a non-rotating black would allow an exit, but these equations cover the exit of the quantum from the passage through the black hole "heart", passage that would be impossible if it was a perfect singularity, as in a non-rotating black hole.

So:

$$E \leq E_p$$

And for:

$$\omega > 0$$

We get:

$$0 < E \leq E_p$$

And when:

$$E = E_p$$

The black hole evaporates. We will finally get the evaporation rate:

$$0 < \rho = \frac{E}{E_p} \leq 1$$

In addition, it is also necessary to take into account two additional elements: The CAD progressive formation time, inducing that LTE and BPE will have more effect as the black hole will gain in "age"; And the importance of the rotation with respect to the mass, and therefore to the gravitational attraction of the black hole, inducing that the more the speed of rotation is important compared to the mass, the more the final energy evacuated E will be important. These two elements make older black holes "less black".

The amount of energy emitted would therefore be the result of all these factors. Now, the quantity of energy of the emitted radiation is relative to its frequency, and therefore its wavelength. In other words, the rotational speed and the mass would influence the wavelength of the black hole radiation. In addition, the heterogeneity of the "gravitational bath" in which the black hole is located would also induce heterogeneity within the vortex, which would

allow the particles leaving the heart following the axis of rotation to cross the event horizon in different points, although always close to the poles so that they do not "fall back" in the singularity, which would have the particular effect of widening the jet emanating from the pole, not being precisely confused with the axis.

Part VI – Graphical representations of different cases

Now try to represent different examples of black holes. The spherical shape shown here is only a schematic representation, so as not to extrapolate the non-spherical appearance of the black holes. In the reality described by this model, rotating black holes are, in fact, not spherical. It is also necessary to include the ergosphere, coupling with the deformation of the structure of the black hole by the Atropic Effect, and to consider the Penrose Process, [9], [10] in order to have a more complete representation.

Case 1 – Moderately rotating black hole

Graphical representation of a moderately rotating black hole

Graphical representation of black hole (black) with his accretion disk (purple), his opaque torus (blue), his radiation jets (A), his sense of rotation (B), and his space-time "dragging" sense (C).

Graphical representation of a moderately rotating black hole interior

Graphical representation of black hole interior, behind the event horizon (D) with his CAD (purple), his detention area (red), his escape area (E), his axis of rotation (F), the plan perpendicular to his axis of rotation (G), and his radiation jets (A).

Characteristics of the case represented:

- High mass.
- Average rotation.
- Presence of an accretion disk outside the event horizon.

Due to its high mass, the potentially emitted energy is important. However, this emission potential is offset by the strong gravitational attraction of the heart of the black hole that results. As seen above, its rotation speed will however allow the evacuation of radiation at the poles, the escape zone remaining relatively narrow. The Atropic Effect is then average.

Case 2 – Slightly rotating black hole

Graphical representation of a slightly rotating black hole

Graphical representation of black hole (black) with his radiation jets (A) and his sense of rotation (B).

Graphical representation of a slightly rotating black hole interior

Graphical representation of black hole interior, behind the event horizon (D) with his CAD (purple), his detention area (red), his escape area (E), his axis of rotation (F), the plan perpendicular to his axis of rotation (G), and his radiation jets (A).

Characteristics of the case represented:

- High mass.

- Low rotation.
- No accretion disk outside the horizon.

Due to its very high mass, the potentially emitted energy is important. However, this emission potential is offset by the strong gravitational attraction of the heart of the black hole that results. The importance of its mass in relation to its rotational speed will have the consequence of reducing the Atropic Effect. Indeed, as seen above, its rotation speed will allow the discharge of radiation at the poles, the escape zone remaining very narrow, due to the weakness of said rotational speed. In addition, a high mass induces a long Schwarzschild radius of the black hole, from the theoretical singularity to the event horizon, [1] making longer the distance a quantum will have to travel to reach the horizon without relapsing into the singularity. The Atropic Effect is then weak, as long as the black hole does not lose mass while preserving its speed of rotation.

Case 3 – Quasar, highly rotating black hole

Graphical representation of a highly rotating black hole

Graphical representation of black hole (black) with his accretion disk (purple), his opaque torus (blue), his radiation and polar plasma jets (A), his sense of rotation (B), his space-time “dragging” sense (C), and his gaseous jets (H).

Graphical representation of a highly rotating black hole interior

Graphical representation of black hole interior, behind the event horizon (D) with his CAD (purple), his detention area (red), his escape area (E), his axis of rotation (F), the plan perpendicular to his axis of rotation (G), and his radiation and polar plasma jets (A).

Characteristics of the case represented:

- High mass.
- Fast rotation.
- Presence of accretion disk out of the horizon.

Due to its very high mass, the potentially emitted energy is important. However, this emission potential is offset by the strong gravitational attraction of the heart of the black hole that results. As seen above, its rotation speed will however allow the discharge of radiation by the poles, the exhaust zone being in this case important, because of the height of said rotational speed. The Atropic Effect is then high. The importance of the speed of rotation, in application of the Atropic Effect, will even allow the gases to escape from the accretion disk without penetrating the horizon, in spite of the gravitational attraction of the quasar. A jet of gas will then be able to complete the polar radiation jet. The importance of polar radiation could, in addition, give rise to a high concentration of particle-antiparticle duos formations.

Case 4 – Black hole similar to theoretical nude singularity

Graphical representation of a black hole similar to theoretical nude singularity

Graphical representation of black hole (grey) with his large radiation jets (A) and his sense of rotation (B).

Graphical representation a black hole interior similar to theoretical nude singularity

Graphical representation of black hole interior, behind the event horizon (D) with his CAD (purple), his weak detention area (red), his strong escape area (E), his axis of rotation (F), the plan perpendicular to his axis of rotation (G), and his large radiation jets (A).

Characteristics of the case represented:

- Weak mass.
- Average rotation.
- No accretion disk outside the horizon.

Due to its low mass, potentially emitted energy is low. However, this emission weakness is offset by the weakness of the gravitational field. As seen above, its rotation speed will allow the evacuation of radiation by the poles, the escape zone being very important, because of the weakness of the mass with respect to the speed of rotation of the black hole. The Atropic Effect is then very important. The black hole cannot effectively contain the quanta of energy and is not quite "black".

Case 5 – Common evolution of black hole

Graphical representation of a relatively young black hole interior

Graphical representation of black hole interior, behind the event horizon (D) with his CAD (purple), his detention area (red), his escape area (E), his axis of rotation (F), the plan perpendicular to his axis of rotation (G), and his radiation jets (A).

Graphical representation a relatively old black hole interior

Graphical representation of black hole interior, behind the event horizon (D) with his CAD (purple), his weak detention area (red), his strong escape area (E), his axis of rotation (F), the plan perpendicular to his axis of rotation (G), and his large radiation jets (A).

Initial characteristics of the case presented:

- Relatively young black hole.
- High mass.
- Average rotation.
- Presence of accretion disk out of the horizon.
- Average Atropic Effect.

An aging black hole loses mass, which then decreases with respect to its rotational speed. The Atropic Effect is then more and more important and the black hole loses more and more mass, until it can no longer effectively contain light. It narrows and finally evaporates.

Final characteristics of the case presented:

- Relatively old black hole.
- Weak mass.
- Average rotation.
- No accretion disk outside the event horizon.
- Important Atropic Effect.

Of course, it is not a sudden change of state. The accretion disk does not disappear in a moment, the area of detention does not become weak from one second to another, the radiation can penetrate the horizon are thus more and more important, in the same way that there are intermediate states between the different types of black holes. Indeed, nothing prevents mathematically that radiation emerges from the singularity by the detention zone, in the case of supermassive and slightly rotating black holes, or that gases escape by polar jets from the accretion disk, in the case of massive and rotating black holes other than quasars.

These emanations are however, in these cases, extremely weak in comparison with the polar evacuations and are therefore not particularly prominent. In addition, it seems appropriate to specify that a black hole can be highly rotational relative to its mass, without its accretion disk has already been absorbed, and a black hole can have an accretion disk , while having a high mass compared to its speed of rotation.

Where

$$\rho = \frac{E_p - \|W^i\| \cdot (\Xi(\Omega_{LT}, m))^{-1}}{E_p}$$

Case 6 – Possible neutron star structuration

Theoretical graphical representation of a neutron star

Theoretical graphical representation of neutron star (grey) with his accretion disk (purple), his opaque torus (blue), his high concentration polar jets (A), his sense of rotation (B), his space-time “dragging” sense (C), and his gaseous jets (H).

Theoretical graphical representation of a neutron star interior

Theoretical graphical representation of neutron star interior, behind the radiant neutron star cover (D) with his high accretion area (purple), his escape area (E), his axis of rotation (F), the plan perpendicular to his axis of rotation (G), and his high concentration polar jets (A).

Possible characteristics:

- Average mass.
- Fast rotation.
- Presence of accretion disk out of the radiant cover of the star.

The importance of polar radiation would suggest a strong Atropic Effect. However, there are still modelling problems that could result from a lack of understanding of the composition of neutron stars and their jets. This hypothesis is therefore incomplete in this case.

Part VII – Observations

A series of observations could be conducted on some specimens of each variety of black holes. We would then analyse, for each specimen, the alteration of the deformation of the geometry of Space-Time around the black hole, in order to determine, taking into account the mass of the black hole and its disks, the speed of rotation of said black hole. This would make it possible to establish the amount of energy emanating theoretically from its poles and to establish the frequency corresponding to it. We could finally compare the predictions with the results of the observation of the emitted radiation. If the predictions correspond to the results obtained by the observation, it would mean that the Atropic Effect is indeed at the origin of the radiation of the black holes and their evaporation and determine black hole internal structure. It would be a high precision job, but it would be workable.

Which could thus be verified by observation:

1. The heterogeneity of the black hole and its non-spherical "form" resulting from it.
2. The distribution of the mass in the black hole (a CAD extending the accretion disk on the other side of the event horizon).
3. The fact that all the black holes are rotating in nature.
4. The fact that the black holes emit energy, according to their speed of rotation relative to their mass.

Lense-Thirring precession around observed black holes could be precisely measured to confirm the propositions 1, 2, and 3. Indeed, measuring the Lense-Thirring Effect around a black hole could allow us to know the mass distribution pattern of the black hole and its own rotational speed.

Given the great precision required to arrive at a perfect result and the importance of the factors that can intervene during the experiment, the observations must above all highlight the correlations between the different predictions and their respective results, in order to avoid any bias. The correspondence between the predictions and the results could lead, once all bias has been removed, to demonstrate whether the application of LTE and BPE, the Atropic Effect is at origin of the radiation of black holes and their evaporation.

It would also be interesting to know how the Atropic Effect could be applied to neutron stars.

Conclusion

Finally, if the Atropic Effect is correct, it can provide a relativistic explanation for black hole evaporation. This would be a model that avoids coupling incompatible models of QM and GR. This is not the first relativistic process proposed to explain this theoretical phenomenon, but has the advantages of bringing new LTE and BPE applications to the internal structure of the black hole and to circumvent the fundamental incompatibilities between QM and GR, as well as the black hole information paradox.

Naturally, the model remains limited to rotating black holes and cannot be correct if one of the predictions stated does not correspond to observations, especially concerning the non-spherical black hole “form” and the heterogeneous distribution of black hole mass (more dense in CAD).

Moreover, it could be another explanation for accelerating the expansion of the Universe. Indeed, if the black holes absorb the mass to expel it in the form of radiation, they are responsible for a general decrease in the mass quantity of the Universe and the increase in its quantity of energy, which would not then not subject to conditions conducive to the formation of stable and viable particle clusters. In other words, the Universe would be irreversibly condemned to expansion until definitive evaporation follows, according to, of course, the knowledge we have on the formation of matter, as well as structuring - and even nature, if I may say so - of the Universe itself.

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