Is there a π solution for $\sum_{i=0}^n i^{(-i+1)}$?

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Abstract

The sum $\sum_{i=0}^n i^{(-i+1)}$ converges very fast as compared with $\sum_{i=0}^n i^{(-i)}$ but its value is very close to Euler's solution for the Basel Problem. This begs the question, can there be a solution involving π or other identities ?

Problem Statement

The famous Basel problem has a solution of 1.6449... or $\frac{\pi^2}{6}$, [1].

A peculiar thing however is that infact $\sum_{i=0}^n i^{(-i+1)} \approx \frac{\pi^2}{6.0606...}$

Hypothesis 1

$\zeta(2) - \sum_{i=0}^n i^{(-i+1)} = c$ with c as a constant.

Hypothesis 2

$\sum_{i=0}^n i^{(-i+1)} = d$

with $d = 1.6284...$ as another constant

Can there be another generalization of the irrational number(if at all it is) ?

Other values for $\sum_{i=0}^n i^{(-i+k)}$ for $k > 1$

These are somehow unremarkable because the first few elements are sums of power integers before $i = k$ where there's a transition to fractions.

References

Sandifer, C. E. (2007). Euler's solution of the Basel problem—the longer story. Euler at, 300, 105-117.