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# The convergence of Grandi's series

Open Mathematics Collaboration<sup>\*†</sup>

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## Abstract

The infinite series  $1 - 1 + 1 - 1 + \dots$  converges to  $1/2$ .

keywords: Grandi's series, convergence, Cesàro sum

## Introduction

1. Infinitely many “divergent” series *converge*, depending on the summation method [1].

## Definition

2. Let

$$S_n = \sum_{i=0}^n (-1)^i = 1 - 1 + 1 - 1 + \dots$$

## Parity

3.  $(n \text{ is odd}) \rightarrow (S_n = 1)$ .
4.  $(n \text{ is even}) \rightarrow (S_n = 0)$ .

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## Associativity

5.  $1 + (-1 + 1) + (-1 + 1) + \dots = 1$

6.  $(5) \equiv (n \text{ is odd}) \equiv (3)$

7.  $(1 - 1) + (1 - 1) + \dots = 0$

8.  $(7) \equiv (n \text{ is even}) \equiv (4)$

## Convergence

9. Let

$$S_g = \lim_{n \rightarrow \infty} S_n.$$

10.  $S_g$  is known as the *Grandi's series* [2].

11.  $S_g = 1 - 1 + 1 - 1 + \dots$

12.  $1 - S_g = 1 - (1 - 1 + 1 - 1 + \dots)$

13.  $1 - S_g = 1 - 1 + 1 - 1 + \dots$

14.  $1 - S_g = S_g$

15.  $S_g = \frac{1}{2}$

## Final Remarks

16. Since infinity is not a number, it is neither odd nor even.

17. From (3), (4), (16),  $S_g \neq 1$ , and  $S_g \neq 0$ .

18. Therefore,

$$S_g = 1 - 1 + 1 - 1 + \dots = \frac{1}{2}. \quad \square$$

# Open Invitation

*Review, add content, and co-author this article. Join the Open Mathematics Collaboration.* Send your contribution to mplobo@uft.edu.br.

## Ethical conduct of research

This original work was pre-registered under the OSF Preprints [3], please cite it accordingly [4]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

## References

- [1] Bromwich, Thomas John I'Anson. *An introduction to the theory of infinite series*. Macmillan and Company, limited, 1908.
- [2] Kline, Morris. "Euler and infinite series." *Mathematics Magazine* 56.5 (1983): 307-315.
- [3] COS. *Open Science Framework*. <https://osf.io>
- [4] Lobo, Matheus P. "The Convergence of Grandi's Series." *OSF Preprints*, 4 Jan. 2020. <https://doi.org/10.31219/osf.io/aq5xz>

## The Open Mathematics Collaboration

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