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Proof of the inseparability of maximal entanglement

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Abstract

We proof the inseparability of two maximally entangled qubits. Next we analyze four separability criteria for a general two-qubit system.

keywords: maximal entanglement, qubit, separability

The most updated version of this white paper is available at
https://osf.io/aejm3/download
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Introduction

1. [1–5]

2. A quantum bit (qubit) is represented by $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$.

3. α and β are complex numbers, satisfying $|\alpha|^2 + |\beta|^2 = 1$.

4. $|0\rangle$ and $|1\rangle$ are quantum states.

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5. (4) can account for spin (up/down), polarization (horizontal/vertical), whatsoever.

6. A two-particle system is described by the tensor product, $|0\rangle \otimes |1\rangle \equiv |0\rangle |1\rangle \equiv |01\rangle$.

7. In (6), particle 1 is in the state $|0\rangle$, and particle 2 is in the state $|1\rangle$.

Maximal Entanglement

8. The following Bell state represents maximal entanglement

$$|\psi\rangle = \alpha |01\rangle + \beta |10\rangle ,$$

where: $\alpha, \beta \in \mathbb{C}$; $\alpha \neq 0$, $\beta \neq 0$; $|\alpha|^2 + |\beta|^2 = 1$.

9. The system (8) is composed by two particles, each can be measured in one of the two states, $|0\rangle$ or $|1\rangle$.

Inseparability

10. Suppose $|\psi\rangle$ in (8) is separable, i.e.,

$$|\psi\rangle = |A\rangle |B\rangle .$$

11. Particle A is in the quantum superposition

$$|A\rangle = a |0\rangle + b |1\rangle .$$

12. And particle B is in the state

$$|B\rangle = c |0\rangle + d |1\rangle .$$

13. $a, b, c, d \in \mathbb{C}$ are the amplitudes for the quantum states $|A\rangle$ and $|B\rangle$, with $|a|^2 + |b|^2 = 1$, and $|c|^2 + |d|^2 = 1$.

14. The following conditions must hold in order for A and B to be in a superposition,

$$a \neq 0,\ b \neq 0,\ c \neq 0,\ d \neq 0.$$

15. Inserting (11) and (12) in (10),

$$|\psi\rangle = (a|0\rangle + b|1\rangle)(c|0\rangle + d|1\rangle).$$

16.

$$|\psi\rangle = ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle.$$

17. Due to (10), (8) is equal to (16),

$$\alpha|01\rangle + \beta|10\rangle = ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle.$$

18. Comparing the terms multiplying $|01\rangle$ in (17), the result is $\alpha = ad$.
19. Doing the same for $|10\rangle$, $\beta = bc$.
20. There are no states $|00\rangle$ and $|11\rangle$ in the left side of (17); therefore, $ac = 0$, and $bd = 0$.
21. (20) contradicts (14).
22. So, our **assumption** (10) is **incorrect**.
23. This proves that (8) is a quantum *inseparable* state,

$$|\psi\rangle \neq |A\rangle|B\rangle.$$

Superposition of two qubits

24. Consider $|\psi_2\rangle$ in the following superposition,

$$|\psi_2\rangle = m|00\rangle + n|01\rangle + p|10\rangle + q|11\rangle,$$

where $m, n, p, q \in \mathbb{C}$, and $m, n, p, q \neq 0$.

25. In the next section, we analyze whether $|\psi_2\rangle$ is separable.

Conditions for the Separability of a two-qubit system

26. Let $|\psi_2\rangle$ be in the superposition

$$|\psi_2\rangle = m\,|00\rangle + n\,|01\rangle + p\,|10\rangle + q\,|11\rangle\,,$$

where m, n, p, $q \in \mathbb{C}$ with $m, n, p, q \neq 0$.

27. We discuss four conditions regarding the separability of $|\psi_2\rangle$.

First case

28. $m = n \neq p = q$

29.

$$|\psi_2\rangle = m\,|00\rangle + n\,|01\rangle + p\,|10\rangle + q\,|11\rangle$$

30.

$$|\psi_2\rangle = m\,|00\rangle + m\,|01\rangle + p\,|10\rangle + p\,|11\rangle$$

31.

$$|\psi_2\rangle = m\,|0\rangle\,(|0\rangle + |1\rangle) + p\,|1\rangle\,(|0\rangle + |1\rangle)$$

32.

$$|\psi_2\rangle = (m\,|0\rangle + p\,|1\rangle)(|0\rangle + |1\rangle)$$

33. Therefore, if $m = n \neq p = q$, then $|\psi_2\rangle$ is separable,

$$|\psi_2\rangle = |A\rangle\,|B\rangle\,.$$

Second case

34. $m = p \neq n = q$

35.

$$|\psi_2\rangle = m|00\rangle + n|01\rangle + p|10\rangle + q|11\rangle$$

36.

$$|\psi_2\rangle = m|00\rangle + n|01\rangle + m|10\rangle + n|11\rangle$$

37.

$$|\psi_2\rangle = (|0\rangle + |1\rangle)m|0\rangle + (|0\rangle + |1\rangle)n|1\rangle$$

38.

$$|\psi_2\rangle = (|0\rangle + |1\rangle)(m|0\rangle + n|1\rangle)$$

39. Therefore, if $m = p \neq n = q$, then $|\psi_2\rangle$ is separable,

$$|\psi_2\rangle = |A\rangle|B\rangle.$$

Third case

40. $m = q \neq n = p$

41.

$$|\psi_2\rangle = m|00\rangle + n|01\rangle + p|10\rangle + q|11\rangle$$

42.

$$|\psi_2\rangle = m|00\rangle + n|01\rangle + n|10\rangle + m|11\rangle$$

43.

$$|\psi_2\rangle = m(|00\rangle + |11\rangle) + n(|01\rangle + |10\rangle)$$

44. Note that $|\psi_2\rangle$ can be written as a sum of two inseparable states,

$$|\psi_2\rangle = |\psi_m\rangle + |\psi_n\rangle,$$

$$|\psi_m\rangle = m(|00\rangle + |11\rangle),$$
$$|\psi_n\rangle = n(|01\rangle + |10\rangle).$$

45. We then conclude that if $m = q \neq n = p$, then $|\psi_2\rangle$ is inseparable.

Fourth case

46. $m = n = p = q$

47.
$$|\psi_2\rangle = m\,|00\rangle + n\,|01\rangle + p\,|10\rangle + q\,|11\rangle$$

48.
$$|\psi_2\rangle = m\,|00\rangle + m\,|01\rangle + m\,|10\rangle + m\,|11\rangle$$

49.
$$|\psi_2\rangle = m\,|0\rangle\,(|0\rangle + |1\rangle) + m\,|1\rangle\,(|0\rangle + |1\rangle)$$

50.
$$|\psi_2\rangle = m(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

51. Therefore, if $m = n = p = q$, then $|\psi_2\rangle$ is separable,
$$|\psi_2\rangle = |A\rangle\,|B\rangle\,.$$

Final Remarks

52. Entanglement is the most important resource of the quantum information theory [1–5].

53. [6] suggests that spacetime itself is entangled.

54. In the special theory of relativity, we know that space and time are bounded by $\tau^2 = t^2 - x^2$ [7, 8].

55. In addition, regarding the uncertainty principle, space and matter are entangled as well, like energy and time.

56. A two-qubit system can be entangled or separable depending on the amplitudes of the individual quantum states.

57. It is worth mentioning that, in addition to the third case shown earlier, the two-qubit system is also inseparable for $m \neq n \neq p \neq q$.

Open Invitation

Review, *add* content, and **co-author** this *white paper* [9, 10].
Join the **Open Physics Collaboration**.
Send your contribution to `mplobo@uft.edu.br`.

Supplementary files

The **latex file** for this *white paper* together with other *supplementary files* are available in [11, 12].

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Agreement

58. All authors **agree** with [10].

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