Real-time interactive streams and temporal objects language

Mauricio Toro, Myriam Desainte-Catherine, David Janin, Yann Orlarey

October 4, 2018

Abstract

We introduce a real-time interactive streams language based on the model of temporal objects in order to (1) study various typing features for timed streams and (2) define a hierarchical calculus for stream processus. The present document deals only with formalisation, and introduces the typing features.

In the sequel, we consider a discrete notion of time where time values are taken in $\mathbb{N}$. At the abstract level this is just enough. At a more concrete level, these time values can either be interpreted as samples numbers, or frame numbers, or whatever else...

1 Basic stream features

A stream is a partial map $S : \mathbb{N} \rightarrow \mathbb{R}^n$. The domain of such a stream S , written $dom(S)$ is assumed to be a segment of $\mathbb{N}$ of the form

$$dom(S) = [s; s + d[= \{m \in \mathbb{N} : s \leq m < s + d\}$$

where $start(S) = s \in \mathbb{N}$ is the *starting delay* of stream S , $duration(S) = d \in \overline{\mathbb{N}}$ is the *duration* of stream S - with $\overline{\mathbb{N}} = \mathbb{N} + \{\infty\}$ - and $dim(S) = n \in \mathbb{N}$ is *dimension* of stream S .

Such a (finite) stream, with an implicit time line going from left to right, can be drawn as follows:

As far as semantic is concerned, we assume that any stream belongs to a program that has been fired at time 0. We must also keep in mind that when a stream is undefined, this *does not mean* that the stream programs cannot handle it.

More precisely, when $t \notin \text{dom}(S)$, we write $S(t) = \perp$ to denote the fact that $S(t)$ is undefined. But, this undefined value can be used in computation, as illustrated, for instance, by the sequential composition below.

The somehow orthogonal problematic raised by *computational availability* or *knowledge* is nevertheless of a crucial importance especially in interactive manipulation of streams where some streams can be external inputs. It is thus discussed in Section 3.

For every $x \in \mathbb{R}$, and $n \in \mathbb{N}$, we shall write x^n to denote, with a little abuse of notation, the vector $x^n = (x, x, x, \dots, x)$ of length n .

For the sake of completeness of the stream algebra, we allow stream S of dimension 0 hence x^0 is also defined as a special zero-length vector written ϵ .

A zero-dimensional stream is thus a mapping from $\text{dom}(S) \rightarrow \mathbb{R}^0$ where $\mathbb{R}^0 = \{\epsilon\}$. It is thus completely determined by its domain.

In the sequel, for every $u \in \mathbb{R}^m$ and $v \in \mathbb{R}^n$ we shall write $uv \in \mathbb{R}^{m+n}$ for the vector obtained from u and v just by concatenating its values. With such a notation, we have, for every $u \in \mathbb{R}^m$, $\epsilon u = u \epsilon = u$ hence this concatenation of vectors of floats really just behaves like concatenation of finite words.

The set of streams is denoted by $\mathcal{S}$.

2 Operators on streams

Let S_1 and S_2 be two streams with

$$\text{dom}(S_1) = [s_1, s_1 + d_1[\text{ and } \dim(S_1) = n_1$$

and

$$\text{dom}(S_2) = [s_2, s_2 + d_2[\text{ and } \dim(S_2) = n_2$$

2.1 Primitive operators

Sequential composition. The sequential composition $S_1.S_2$ of streams S_1 and S_2 with $\dim(S_1) = \dim(S_2) = n$ and $\text{duration}(S_1) < +\infty$ is defined by $\text{dom}(S_1.S_2) = [s_1, s_1 + s_2 + d_1 + d_2[^1$, $\dim(S_1.S_2) = n$ with, for every $x \in \text{dom}(S_1.S_2)$,

$$S_1.S_2(x) = \begin{cases} S_1(x) & \text{if } s_1 \leq x < s_1 + d_1, \\ 0^n & \text{if } 0 \leq x - (s_1 + d_1) < s_2, \\ S_2(x - (s_1 + d_1)) & \text{if } s_2 \leq x - (s_1 + d_1) < s_2 + d_2. \end{cases}$$

The result of a stream sequential product of stream S_1 with stream S_2 can be illustrated as follows:

¹with, for every $m \in \mathbb{N} + \infty$, $m + \infty = \infty + m = \infty$

Parallel composition. The parallel composition $S_1 || S_2$ of streams S_1 and S_2 is defined by $dom(S_1 || S_2) = [min(s_1, s_2), max(s_1 + d_1, s_2 + d_2) - min(s_1, s_2)[$, i.e. $dom(S_1 || S_2)$ is the least segment that contains $dom(S_1) \cup dom(S_2)$, with $dim(S_1 || S_2) = n_1 + n_2$. For every $x \in dom(S_1 || S_2)$,

$$(S_1 || S_2)(x) = \begin{cases} S_1(x)S_2(x) & \text{if } x \in dom(S_1) \cap dom(S_2), \\ S_1(x)0^{n_2} & \text{if } x \in dom(S_1) - dom(S_2), \\ 0^{n_1}S_2(x) & \text{if } x \in dom(S_2) - dom(S_1). \end{cases}$$

The result of the parallel product of S_1 and S_2 as above with $s_1 < s_2$ and $s_2 + d_2 < s_1 + d_1$ can be illustrated as follows:

Synchronized parallel composition. The synchronized parallel composition $S_1 | S_2$ of streams S_1 and S_2 is defined by $dom(S_1 | S_2) = [s_1, max(s_1 + d_1, s_2 + d_2) - s_1[$, i.e. $dom(S_1 | S_2)$ is the least segment that contains $dom(S_1) \cup dom(S_2)$, with $dim(S_1 | S_2) = n_1 + n_2$, such that S_2 is retarded or advanced to start at the same time than the S_1 . Let S'_2 such that $start(S_2) = s_1$, $duration(S'_2) = d_2$, be the stream resulting when retarding or advancing S_2 to start at the same time than S_1 .

$$(S_1 | S_2) = S_1 || S'_2$$

The result of the parallel product of S_1 and S_2 as above with $s_1 < s_2$ and $s_2 + d_2 < s_1 + d_1$ can be illustrated as follows:

Restriction. The restriction composition S_1/S_2 of streams S_1 and S_2 is defined essentially in the same way $S_1||S_2$ is defined but with a reduced domain $dom(S_1/S_2) = dom(S_2)$.

In particular, when S_2 is a zero-dimensional stream, the restriction stream S_1/S_2 is essentially as stream S_1 restricted to domain $dom(S_2)$.

The result of a the restriction product of S_1 and S_2 with the same hypothesis as above can be illustrated as follows:

2.2 Streams operators

Transformation. Given a stream transformation F and a given stream S we want to define $F(S)$.

Given $s = start(S)$ and $d = duration(S)$ we assume that $start(F(S)) = s + \delta_F(d)$, i.e. the extra start delay $\delta_F(d)$ of $F(S)$ compared to S only depends on the duration d of S , and that $duration(F(S)) = d \cdot \alpha_F(d)$, i.e. the duration of $F(S)$ is a factor of the duration of S with coefficient factor $\alpha_F(d)$ that also only depends on d .

When $\delta_F(d) = 0$ and $\alpha_F(d) = 1$, the result of the application of a stream transformation can be illustrated as follows:

Reverse. Reverse corresponds to *audio reverse*. Let us consider the two following reverse operators : $reverse(S)$ computes the reverse stream by accessing the samples of S from left to right. Reverse does not change the duration nor the start of the original stream.

Stream syntax vs stream semantics. In the remainder of the text, we use the same notation for both stream expressions and stream themselves. This is done on purpose in order to keep some reasonably simple notations.

Whether a given streams expression is interpreted as itself or as the stream it denotes shall always remain clear from the context or will be explicitly mentioned.

The set of streams expressions is denoted by $\mathcal{E}$.

2.3 Derived operators

For all $n \in \mathbb{N}$, $s \in \mathbb{N}$ and $d \in \overline{\mathbb{N}}$, let $0_{s,d}^n$ denote the stream with domain $[s, s + d[$ and with constant value 0^n .

Delays. The first delay operator one can define on a stream S with $dom(S) = [s, s + d[$ and $\dim(S) = n$ by some constant δ is given by

$$delay(S, \delta) = 0_{\delta,0}^n.S$$

with $dom(0_{\delta,0}^n.S) = [s + \delta, s + d + \delta[$.

Extract operator. This operator extracts a finite segment of a (finite or infinite) stream.

$$extract(S, s, d) = S/T$$

with $T = 0_{s,s+d}^0$ a zero-dimensional stream with domain $[s, s + d[$.

Sum operator. This operator merges two streams S, R . Sum is a particular case of $F(S \parallel R)$ in which F takes a bidimensional stream, $S : \mathbb{N} \rightarrow \mathbb{R}^2$ and returns a unidimensional stream, $S : \mathbb{N} \rightarrow \mathbb{R}$. In the case of sum, $\delta_F(d) = 0$ and $\alpha_F(d) = 1$.

Bandpass filter.

3 Temporal properties

In this section, we want to investigate different temporal properties of streams like *computational availability* and *computational necessity* of streams. These notions appear as soon as we handle various type of streams : from *input streams* that are known sequentially in real time to *file streams* that are known at any time.

3.1 Input and buffered streams

Let us first review the characteristic of input or buffered streams.

Input stream An input stream is a stream denoted by an expression of the form I_p where p is an input port. For such a stream, we necessarily have $\text{dom}(I_p) = [0, \infty[$. At any time t , provided the input stream is buffered, $S(t')$ is known only for $t' \in [0, t]$.

Buffered stream A file stream is a stream denoted by an expression of the form B_f where f is a file descriptor. For such a stream, we necessarily have $\text{dom}(B) = [s, d[$ with $d < \infty$. At any time t , $S(t')$ is known (though possible with undefined value) for arbitrary $t' \in \mathbb{N}$.

3.2 Computational availability and knowledge

As already mentioned, handling streams in real time context poses problems of *computational availability* of a stream S : at a given time t , for which t' the value $S(t')$ is available for computation.

Implicitly, we assume here that streams are buffered as much as needed. The minimal size of these buffer is handled by the orthogonal notion of *computational necessity* presented below.

We have two different definitions : *computational availability* and *computational knowledge* of streams. Let S be a stream :

- $\text{avail}(S, t)$ is the set of index of samples of S that are available at execution time t .
- $k(S, t)$ is the interval containing the dates of execution at which $S(t)$ is known. For example, if we assume that we bufferize until the end of execution all the streams that are handled by the program, we can state that $k(I, t) = [t, \infty[$ with I denoting an input stream (in real-time).

Note that there is a relation between *avail* and *k*. In a future version of this writing, only one should remain (the simplest one to express all the operators).

$$avail(s, t) = \{t' | t \in k(s, t')\} \quad k(s, t) = \{t' | t' \in avail(s, t)\}$$

We also want to define the notion of computational necessity. We thus define the *computational necessity* of a stream *S* used in a program *F(S)*. We propose the following definition : at a given time *t* during execution, the computational necessity is the set of dates *t'* such that the values of *S(t')* must be known to compute *(F(S))(t)*.

The notion of stream availability goes in pair with the somehow dual notion of *computational necessity* of a stream *S* used in a program *F(S)* : at a given time *t*, for which time *t'* the values of *S(t')* *must* be known to compute *(F(S))(t)*.

4 Computational Availability

4.1 Definition

Formally, given $\mathcal{E}$ the class of *stream expressions* built from elementary streams either constant or of the form I_p or B_f combined by the stream operators defined above, we define a mapping

$$avail : \mathcal{E} \times \mathbb{N} \rightarrow \mathcal{P}(\mathbb{N})$$

that maps any stream expression *S* and time value $t \in \mathbb{N}$ to a set *avail(S, t)* such that for every $t' \in avail(S, t)$, *S(t')* is computable (be it defined or undefined).

In the following for any stream *S*, let us denote by *e(S)* the date of the last sample of *S*, that is $s(S) + d(S)$.

4.2 Availability of constant streams

For an external input stream I_p we have

$$avail(I_p, t) = [0, t]$$

$$k(I_p, t) = [t, \infty[$$

since $s(I_p) = 0$ and $d(I_p) = \infty$. Of course, this implies that the entire input stream from time 0 to time *t* has been buffered.

By contrast, for any file stream B_f , we have

$$avail(B_f, t) = [0, \infty[$$

$$k(B_f, t) = [0, \infty[$$

Of course, this also implicitly implies that the file associated to the stream B_f is kept available until the end of execution.

4.3 Sequential composition

Let S_1 and $S_2 \in \mathcal{E}$ be two arbitrary stream expressions.

$$avail(S_1.S_2, t) = avail(S_1, t) \cup shift(avail(S_2, t), s(S_1) + d(S_1))$$

with, for all $X \subseteq \mathbb{N}$, all $d \in \mathbb{N}$,

$$shift(X, d) = \{n + d : n \in X\}$$

$$k(S_1.S_2, t) = [t, e(S_1)[\cup shift(k(S_2, t), s(S_1) + d(S_1))$$

4.4 Parallel composition

Let then S_1 and $S_2 \in \mathcal{E}$ be two arbitrary stream expressions.

$$avail(S_1 || S_2, t) = avail(S_1, t) \cap avail(S_2, t)$$

Indeed, at time t , the ??

4.5 Delay operator

Let then $S \in \mathcal{E}$ be an arbitrary stream.

$$avail(delay(S, \delta), t) = shift(avail(S, t), \delta)$$

$$k(S, t) = [t + \delta, \infty[$$

4.6 Restriction operator

Let then S_1 and $S_2 \in \mathcal{E}$ be two arbitrary stream expressions.

$$avail(S_1 / S_2, t) = avail(S_2, t)$$

4.7 Transformation operator

Let then $S \in \mathcal{E}$ be an arbitrary stream.

$$avail(F(S), t) = shift(avail(S, t), \delta_F(d)) \cup shift([d, d.\alpha_F(d)[, \delta_F(d))$$

4.8 Reverse operator

Availability of the result of the application of reverse is defined as follows :

$$avail(reverse(S), t) = [e(S) - t, e(S)]$$

5 Computational necessity

5.1 Definition

Given $\mathcal{E}$ the class of *stream expressions*,

$$req : \mathcal{E} \times \mathcal{E} \times \mathbb{N} \rightarrow \mathcal{P}(\mathbb{N})$$

that maps any stream expression S possibly used in a (more complex) stream expression E and any time value $t \in \mathbb{N}$ to an interval

$$req(S, E, t) = [t - pr(S, E, t), t + fr(S, E, t)]$$

defined as the *least* interval such that, for every $t' \in avail(E, t)$, $E(t')$ is effectively computable when we assume (or force) the availability of signal S at time t to be *restricted* to $req(S, E, t)$.

In this definition, we aim at ensuring that for every substream expression S in E , if, for every time value t , we restrict $avail(S, t)$ to be $req(S, E, t)$ then $E(t)$ is indeed computable.

It shall be clear that, in full generality, with arbitrary operators applied to streams, this is even not well-defined. Indeed, all $req(S, E, t)$ for basic streams S occurring in expression E is presumably the solution of a set of interval equations that may not have *least* solution w.r.t. inclusion.

Our point is thus to restrict ourselves to some set of stream operators that guarantee the existence and computability of $req(,)$.

5.2 Delay operator

5.3 Reverse operator

$$req(S, rl(S), t) = t$$

$$req(S, rr(S), t) = e(S) - t$$

5.4 Extract operator

6 Study Case

What are $avail(S_{output1}, t)$, $k(S_{output1}, t)$, $req(S_{output1}, t)$?

Process calculi has been applied to the modeling of interactive music systems [18, 17, 23, 28, 16, 27, 1, 24, 19, 4, 14, 11, 13, 15, 2, 22, 12, 20, 21, 10] and ecological systems [25, 6, 26, 7]. In addition, research on algorithms [5, 3, 9] and software engineering [8] also contributes to this field.

7 And here we are for now...any comment welcome

References

- [1] A. Allombert, M. Desainte-Catherine, and M. Toro. Modeling temporal constraints for a system of interactive score. In G. Assayag and C. Truchet, editors, *Constraint Programming in Music*, chapter 1, pages 1–23. Wiley, 2011.

- [2] J. Aranda, G. Assayag, C. Olarte, J. A. Pérez, C. Rueda, M. Toro, and F. D. Valencia. An overview of FORCES: an INRIA project on declarative formalisms for emergent systems. In P. M. Hill and D. S. Warren, editors, *Logic Programming, 25th International Conference, ICLP 2009, Pasadena, CA, USA, July 14-17, 2009. Proceedings*, volume 5649 of *Lecture Notes in Computer Science*, pages 509–513. Springer, 2009.
- [3] J. D. A. Moreno, S. Passos, and M. Toro. On-line assembling mitochondrial DNA from de novo transcriptome. *CoRR*, abs/1706.02828, 2017.
- [4] C. Olarte, C. Rueda, G. Sarria, M. Toro, and F. Valencia. Concurrent Constraints Models of Music Interaction. In G. Assayag and C. Truchet, editors, *Constraint Programming in Music*, chapter 6, pages 133–153. Wiley, Hoboken, NJ, USA., 2011.
- [5] C. Patiño-Forero, M. Agudelo-Toro, and M. Toro. Planning system for deliveries in Medellín. *ArXiv e-prints*, Nov. 2016.
- [6] A. Philippou and M. Toro. Process Ordering in a Process Calculus for Spatially-Explicit Ecological Models. In *Proceedings of MOKMASD’13*, LNCS 8368, pages 345–361. Springer, 2013.
- [7] A. Philippou, M. Toro, and M. Antonaki. Simulation and Verification for a Process Calculus for Spatially-Explicit Ecological Models. *Scientific Annals of Computer Science*, 23(1):119–167, 2013.
- [8] M. Raul, T. Mauricio, and C. Luz. Definicion de la arquitectura de referencia de un dominio: de la elucidacion al modelado. In R. Mazo, editor, *Guia para la adopcion industrial de lineas de productos de software*, pages 193–210. Editorial Eafit, 2018.
- [9] J. M. C. Restrepo, A. F. Z. Palacio, and M. Toro. Assembling sequences of DNA using an on-line algorithm based on debruijn graphs. *CoRR*, abs/1705.05105, 2017.
- [10] M. Toro. Exploring the possibilities and limitations of concurrent programming for multimedia interaction and graphical representations to solve musical csp’s. Technical Report 2008-3, Ircam, Paris.(FRANCE), 2008.
- [11] M. Toro. Probabilistic Extension to the Factor Oracle Model for Music Improvisation. Master’s thesis, Pontificia Universidad Javeriana Cali, Colombia, 2009.
- [12] M. Toro. Towards a correct and efficient implementation of simulation and verification tools for probabilistic ntcc. Technical report, Pontificia Universidad Javeriana, May 2009.

- [13] M. Toro. Structured interactive musical scores. In M. V. Hermenegildo and T. Schaub, editors, *Technical Communications of the 26th International Conference on Logic Programming, ICLP 2010, July 16-19, 2010, Edinburgh, Scotland, UK*, volume 7 of *LIPICs*, pages 300–302. Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2010.
- [14] M. Toro. *Structured Interactive Scores: From a simple structural description of a multimedia scenario to a real-time capable implementation with formal semantics*. PhD thesis, Univeristé de Bordeaux 1, France, 2012.
- [15] M. Toro. Structured interactive music scores. *CoRR*, abs/1508.05559, 2015.
- [16] M. Toro. Probabilistic Extension to the Concurrent Constraint Factor Oracle Model for Music Improvisation. *ArXiv e-prints*, Feb. 2016.
- [17] M. Toro. Probabilistic Extension to the Concurrent Constraint Factor Oracle Model for Music Improvisation. *Inteligencia Artificial*, 57(19):37–73, 2016.
- [18] M. Toro. CURRENT TRENDS AND FUTURE RESEARCH DIRECTIONS FOR INTERACTIVE MUSIC. *Journal of Theoretical and Applied Information Technology*, 69(16):5569–5606, 2018.
- [19] M. Toro, C. Agón, G. Assayag, and C. Rueda. Ntcert: A concurrent constraint framework for real-time interaction. In *Proc. of ICMC '09*, Montreal, Canada, 2009.
- [20] M. Toro and M. Desainte-Catherine. Concurrent constraint conditional branching interactive scores. In *Proc. of SMC '10*, Barcelona, Spain, 2010.
- [21] M. Toro, M. Desainte-Catherine, and P. Baltazar. A model for interactive scores with temporal constraints and conditional branching. In *Proc. of Journées d’Informatique Musicale (JIM) '10*, May 2010.
- [22] M. Toro, M. Desainte-Catherine, and J. Castet. An extension of interactive scores for multimedia scenarios with temporal relations for micro and macro controls. In *Proc. of Sound and Music Computing (SMC) '12*, Copenhagen, Denmark, July 2012.
- [23] M. TORO, M. DESAINTE-CATHERINE, and J. CASTET. An extension of interactive scores for multimedia scenarios with temporal relations for micro and macro controls. *European Journal of Scientific Research*, 137(4):396–409, 2016.

- [24] M. Toro, M. Desainte-Catherine, and C. Rueda. Formal semantics for interactive music scores: a framework to design, specify properties and execute interactive scenarios. *Journal of Mathematics and Music*, 8(1):93–112, 2014.
- [25] M. Toro, A. Philippou, S. Arboleda, M. Puerta, and C. M. Vélez S. Mean-field semantics for a process calculus for spatially-explicit ecological models. In C. A. Muñoz and J. A. Pérez, editors, Proceedings of the Eleventh International Workshop on *Developments in Computational Models*, Cali, Colombia, October 28, 2015, volume 204 of *Electronic Proceedings in Theoretical Computer Science*, pages 79–94. Open Publishing Association, 2016.
- [26] M. Toro, A. Philippou, C. Kassara, and S. Sfenthourakis. Synchronous parallel composition in a process calculus for ecological models. In G. Ciobanu and D. Méry, editors, *Proceedings of the 11th International Colloquium on Theoretical Aspects of Computing - ICTAC 2014, Bucharest, Romania, September 17-19*, volume 8687 of *Lecture Notes in Computer Science*, pages 424–441. Springer, 2014.
- [27] M. TORO, C. RUEDA, C. AGÓN, and G. ASSAYAG. Ntcrt: A concurrent constraint framework for soft real-time music interaction. *Journal of Theoretical & Applied Information Technology*, 82(1), 2015.
- [28] M. TORO, C. RUEDA, C. AGÓN, and G. ASSAYAG. Gelisp: A framework to represent musical constraint satisfaction problems and search strategies. *Journal of Theoretical & Applied Information Technology*, 86(2), 2016.