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Numerosity Structures the Expression of Quantity in Lexical Numbers and Grammatical Number

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CA+ Online-Only Material: Supplement A

Using data from the World Atlas of Language Structures and other sources, this study analyzed 905 languages for the presence of grammatical number (GN) and lexical numbers (LNs) to investigate what the distribution of these linguistic features might suggest about the relationship between language and numerosity, the perceptual system for quantity. Nearly 7% of the sample had LNs but lacked GN, and GN never occurred without LNs, implying that LNs may develop first and that GN is neither necessary nor sufficient for developing LNs, despite its role in helping children acquire number concepts when present as a feature of language. The geographic-temporal distribution of the two linguistic features additionally supported the idea that LNs may emerge prior to GN. Furthermore, the “one-two-three-many” structure of both LNs and GN, along with the failure of historic artificial intelligence modeling to converge on real-world number system solutions, suggested that numerosity may structure the expression of quantity in both linguistic domains. The role of the hand in numbers (the interaction of numerosity with cognitive processes such as finger gnosis, haptic perception, and neural reactions to tools) implies that LNs may originate in tactile engagement with material structures that may subsequently extend to nontactile domains, such as GN.

In the quest to understand how numeracy, the uniquely human ability to reason with numbers, is elaborated from numerosity, the perceptual system for quantity that humans share with other species, linguists have assigned a preeminent role to language. For example, Hurford claimed that “Without language, no numeracy” (Hurford 1987:305) and “Without using language, we still can’t go [higher than the subitizing range of about four]” (Hurford 2007:91–92). Similarly, Everett (2013) asserted that language has a “fundamental role . . . in shaping basic human numerosity” (81), implying that numeracy may be the result of language acting upon numerosity.¹ The observation that grammatical number (GN; the morphosyntactic category for quantity) helps children to acquire number concepts and lexical numbers (LNs; quantity terms used in counting) through mechanisms like bootstrapping and semantic mapping (Barner 2012; Carey 2009) implies that it may be

a feature of language, rather than language itself, that influences numerosity.

In acquisition, however, such mechanisms entail numerate cultures whose languages have both GN and LNs (conditions that presumably would not obtain in original contexts or apply to numerate cultures whose languages lack GN, like Cantonese [Pellatt 2007]). These circumstances are further complicated by a general lack of insight into how such features develop (e.g., order and conditions). Another potential complication is the observation that language and numerosity are independent as cognitive systems, based on characteristics like the double dissociability of language and numeracy (Brannon 2005) and the presence of numerosity but not language in other species (reviewed in Coolidge and Overmann 2012). Finally, the commonality of their patterning (i.e., one-two-three-many) suggests that numerosity may structure expressions of quantity in language (as GN and initial LNs), implying a bidirectional influence between the two cognitive systems. These possibilities are explored in this study.

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1. Alternatively, the engagement of material structures may critically inform the elaboration of numerosity into numeracy (Coolidge and Overmann 2012; Malafouris 2010; Overmann 2015), with the influence of both language and material engagement being necessary to realizing numerical concepts (Overmann 2016) and material engagement preceding linguistic access to them (Overmann and Coolidge 2013).

GN is a morphosyntactic category that expresses quantity to distinguish singular from multiple entities through prefixes, suffixes, stem changes, tones, reduplication, clitics, and words (Corbett 2000; Dryer and Haspelmath 2013).² GN is complex in that it need not be marked, expressed, restricted to nominals, or behave in a predictable manner; however, GN also shows a decided hierarchy, with terms for larger quantities generally being found only where there are complementary terms for smaller quantities (Corbett 2000; Greenberg 1972, 1978). In comparison, LNs are also expressions of quantity, with the most common and intuitive being the natural numbers (the ordinary numbers used for counting and ordering) and the integers (which express whole quantities). LNs are represented with linguistic labels that may entail the use of rules (algorithms that create expressions for higher quantities), structured with positional values and bases and organized into useful sets (e.g., natural numbers, integers, rational numbers, and real numbers). Basic numeracy, then, is the ability to use LNs in simple mathematical operations (addition, subtraction, multiplication, and division). However, LNs are necessary for basic numeracy; it is not possible to perform operations with numbers without having numbers in the first place. LNs are not necessary for quantifying with material devices, as the ethnographic literature documents.

The possibilities that language is fundamental to developing LNs at all, developing LNs beyond the limit of about four, or shaping numerosity raise the question of whether it is language itself or some feature of language, such as GN, that provides a necessary or sufficient condition. Language is clearly not necessary for numerosity, or numerosity would never occur without language; on the contrary, numerosity is well documented in prelinguistic human infants (Ansari et al. 2005; Cantlon et al. 2006; Feigenson, Dehaene, and Spelke 2004; Van Marle and Wynn 2011) and a list of nonlinguistic species that includes mammals, birds, fish, amphibians, and perhaps even insects (e.g., Bar-Shai, Keasar, and Avi Shmida 2011; Beran 2012; Bisazza et al. 2014; Jaakkola et al. 2005; Krusche, Uller, and Dicke 2010; Piazza 2011; Rugani et al. 2013). Numerosity has, in fact, been found in so many species to date that it might now be exceptional to find one that lacked it.

The notion that language is necessary to numeracy, the ability to reason with numbers, is in a sense trivial, because humans have language generally, and the availability of language in some form or another characterizes all modern human societies. As Newmeyer (1998) observed, “What human activity *isn’t* language a central part of?” (1; emphasis in original). Language certainly provides names for lower quantities

and rules for naming higher quantities, although it is possible in principle (however awkward in practice) to name higher quantities uniquely in the way that lower quantities are named. Subsequent to the practice of forming unique names for lower quantities, lexical rules emerge to permit the formation of a potentially infinite number of names for higher quantities (Hurford 1987; Menninger 1992). LNs also facilitate the learning of number concepts by children, as does GN (Barner 2012; Carey 2009), a scenario in which the existence of number concepts and their labels is implicit (as is the presence of GN in the language), and the prior development of these conditions is implied. However, in original contacts, quantity concepts must surely precede their lexical labels, or there would be nothing to name (Overmann and Coolidge 2013). As Malafouris (2013) aptly noted, a method of invention cannot presuppose that which it invents.

Not all extant human languages have developed LNs, which are necessary to numeracy, or GN, which can support the acquisition of number concepts in numerate societies whose languages also have GN. The World Atlas of Language Structures (WALS; Dryer and Haspelmath 2013) lists 20 languages with restricted LNs (big-small or one-two-three-many), and this status might be construed for another 92 languages on the basis of ethnographic observations of restricted LNs (10 of which were included in this analysis). The Pirahã of Amazonian Brazil, who have been said to lack LNs beyond what is typical of most societies with restricted LNs, as they have been described as having neither LNs nor GN (Dryer and Haspelmath 2013; Everett 2005), are especially pertinent here: their case suggests that language in toto is not a sufficient condition for developing LNs, or the Pirahã would have developed them, since they have language. The Pirahã case also admits the possibility that it is some feature of language, rather than language per se, that may provide the sufficiency condition. Since the Pirahã lack “morphosyntactic instantiations of number such as nominal plurality” (Everett 2013:81), it is reasonable to examine whether GN in the form of nominal plurality may indeed be a feature of language necessary or sufficient for developing LNs.

Analysis of Data from WALS

To examine whether GN was either necessary or sufficient for developing LNs, an analysis was performed on languages for which it was possible to identify the status (presence or absence) of nominal plurality (WALS categories 33A/34A).³ These data were coded dichotomously (no nominal plurality; nominal plurality in any form). Languages in the sample were also characterized in terms of highest number counted, as

2. As an example, Corbett (2000) offers one from Bayso, an Afro-Asiatic language spoken in Ethiopia that distinguishes a singular (“I watched a lion,” “lubán-titi foofe”) from a paucal with a range from two to six (“I watched a few lions,” “luban-jaa foofe”) and a large plural with a range larger than six (“I watched [a lot of] lions,” “luban-jool foofe”) (11).

3. In most cases in which WALS categories 33A and 34A conflicted, resolution was provided by Matthew Dryer (personal communication) and the languages were included in the study; if WALS categories 33A and 34A could not be reconciled, the languages were omitted.

either not counting at all, counting no higher than three or four, or counting to five or higher, selected because these limits were applicable to the Pirahã or represented subitization or its transcendence, respectively. For the categorization of highest number counted, WALS data were supplemented with data from Rosenfelder (2011) and observations from Yale's electronic Human Relations Area Files database (Bieseke et al. 2013).

Highest number counted was interpreted as characterizing whether the subitization constraint had been transcended. Subitization is the component of numerosity that enables the rapid, unambiguous recognition of quantities up to three or four (Kaufman et al. 1949). Subitization engages attentional resources for object recognition with a limited capacity for identification and enumeration, a limitation also found in the visual tracking of moving stimuli (Alvarez and Franconeri 2007; Burr, Turi, and Anobile 2010; Carey 2009). Transcending the subitization constraint may be an outcome of material engagement, the interaction of psychological abilities with material structures, including the body (Coolidge and Overmann 2012; Malafouris 2010, 2013). Thus, languages without any numbers or in which the counting limit was no higher than three or four were interpreted as not having transcended the subitization constraint, and languages counting to quantities higher than five were interpreted as having transcended it.

The supplemented data were sufficient to identify the status of both nominal plurality and highest number counted for 905 languages.⁴ Seventy-seven languages were identified as lacking GN in the form of nominal plurality, whereas 828 had nominal plurality. Regarding highest number counted, one language (Pirahã) did not count at all, nine languages counted no higher than three or four, and 895 languages counted to five or higher. These data were used to assess the hypotheses that (1) the presence of LNs was associated with presence of GN and (2) the absence of LNs was associated with the absence of GN. As shown in table 1, languages with nominal plurality ($n = 828$) significantly outnumbered those without it ($n = 77$), and languages counting to five or higher ($n = 895$) outnumbered those not counting at all or counting no higher than three or four ($n = 10$), suggesting that most languages develop both LNs and GN. Two languages without GN either did not count at all (Pirahã) or counted no higher than three or four (e.g., Kayardild, an Australian language). Notably, most languages lacking nominal plurality were nonetheless able to count higher than three or four

Table 1. Grammatical number by lexical numbers

Lexical numbers (highest number counted)	Grammatical number (nominal plurality)		Total
	None	Any	
None	1	0	1
Up to three or four	1	8	9
Five or higher	75	820	895
Total	77	828	905

Note. Distribution of data for lexical numbers (LNs; no LNs, LNs up to three or four, or LNs of five or higher) and grammatical number (GN; no nominal plurality, or nominal plurality in any form).

($n = 75$), with some counting at least as high as 20 without loan words. Of the 828 languages with nominal plurality, 820 counted to five or higher; the remaining eight languages (three each from Australia and Papua New Guinea and two from South America) had GN but counted no higher than three or four.

These results, although not adjusted for language-family relatedness, nonetheless suggest that nominal plurality is neither necessary nor sufficient for developing LNs, the former because, if it were true, no language lacking nominal plurality would develop any LNs at all or would develop none higher than three or four (and there were 75 cases that did) and the latter because all languages with nominal plurality would develop LNs (and there were eight cases with only restricted LNs). The presence of GN did not guarantee the development of LNs beyond the subitizing range nor did the absence of GN ensure an absence of LNs. Conversely, the possibility that LNs are necessary for the development of GN cannot be discounted, since there were no languages in which GN occurred in the absence of LNs.

It must be emphasized that languages lacking LNs (e.g., Pirahã) and languages counting only to quantities within the subitizing range (e.g., Kayardild) are distinct, although WALS categorizes both as restricted-number languages. In the present analysis, Pirahã was recategorized as having neither GN nor LNs. This suggested the addition of another category: languages with GN but lacking LNs as completely as the Pirahã. However, no language with these characteristics was identified in the sample. The overall pattern—LNs occurring both with and without GN, and GN occurring only with LNs—is consistent with the idea that LNs may emerge prior to GN in original contexts (see fig. 1). If the null category and Pirahã were removed from the analysis (the former because it contained no exemplars, the latter on the basis of continued debate over how truly anumeric Pirahã might be), the remaining four categories still support the idea that LNs may emerge prior to GN in original contexts. That is, although languages with GN and restricted LNs are uninformative as to priority, because both are structured as one-two-three-many, languages that lack GN but have LNs suggest that LNs may emerge first. Whether four or six logical categories are considered, what would be needed to disprove the hypothesis

4. The unsupplemented WALS data ($n = 167$, drawn July 13 2015) were as follows: Of the languages with GN ($n = 154$; 92.2%), 140 had LNs, 14 had restricted LNs, and no language lacked LNs. Of the languages without GN ($n = 13$; 7.8%), nine had LNs, three had restricted LNs, and one lacked LNs. A similar analysis of ordinal numbers and numeral classifiers is provided in tables A1 and A2 (CA+ online supplement A).

Figure 1. Pattern of emergence suggested by feature distribution. Lexical numbers (LNs) occur both with and without grammatical number (GN), whereas GN occurs only with LNs. The pattern is consistent with the idea that LNs emerge before GN in original contexts lacking any ability to express quantity. A language with GN but no LNs would show a second path to quantity expressions, and a third potential path might be an intermediate form of quantity expression giving rise to both LNs and GN. Should languages with GN but no LNs be identified, the question would change to whether one path was more typical than any other.

that LNs emerge first is the identification of a language with GN but lacking any LNs whatsoever (that is, languages with GN and restricted LNs would not falsify the hypothesis).

However, should a language with GN but no LNs whatsoever be found, it would merely show that there were two paths to quantity expressions in language: in addition to the one currently suggested by the WALS data (LNs first, GN second), it would add GN first, LNs second (and possibly a third, in which both LNs and GN emerge together from an intermediate form). The inquiry would then shift to the question of whether one path was more typical than any other; this might require additional and more granular data than is presently supported by WALS.

Language and Numerosity/Numeracy Are Both Independent and Related

Against the strong claim that language fundamentally shapes numeracy or the weaker claims that GN is either necessary or sufficient for developing LNs beyond about four, it has been argued that language and numerosity, the innate sense of quantity, are independent, because the latter can be demonstrated in prelinguistic human infants and many nonlinguistic nonhuman species (reviewed in Coolidge and Overmann

2012). Language and numeracy appear to be independent as well. First, they are doubly dissociable⁵ (Brannon 2005): language can be impaired while numeracy is spared (a condition for which Varley et al. 2005 provides clinical examples), and numeracy can be impaired while language functions remain intact, as in acalculia, the inability to perform simple numeric operations (Ardila and Rosselli 2002). Additional evidence from ethnography supports the idea that language and numeracy are independent: it is possible to understand, manipulate, and communicate concepts of quantities that exceed the lexicon (e.g., numerous studies describe nonverbal quantities being understood, manipulated, and communicated by means of fingers and toes, gestures, or material devices; Chagnon 2013; Da Silva 1962; Karsten 1935; Murphy and Quain 1955; Staden 1928; Turrado Moreno 1945).

The independence of language and numerosity/numeracy, however, is clearly not the whole story. Historical linguistic evidence suggests that the initial emergence of LNs (one-two-three-[possibly four]-many) is constrained to discrete quantities no higher than three or four (including languages that express quantity dichotomously as big-small or one-many; Menninger 1992). This early stage is consistent with the perceptual experience of quantity rather than the number of fingers on a hand or the cognitively prepotent association of fingers and counting (reviewed in Overmann 2014). Simply, concepts of and words for subitizable quantities may emerge first because subitizable quantities are more perceptually salient.

The structure of GN (singular-dual-trial-[possibly quadral]-many) not only resembles that of numerosity and LNs (table 2), it is similarly universal, so much so that Perkins (1992) declined to incorporate it into his study of deixis, finding GN to be constant rather than variable. Most plurality systems encompass some combination of singular (one real-world entity), dual (two entities), trial (three entities), and plural (many entities), with a minority of languages including a quadral (four entities; Corbett 2000; Menninger 1992). An upper limit of three or four distinct entities for GN is intriguing. Some languages have nominal number distinctions of singular (one entity), few (the paucal; e.g., two to seven entities), and many (a greater number of entities), or lesser and greater plurals; other languages distinguish between lesser and greater plurals (Corbett 2000). These structures might also be construed as roughly consistent with early forms of LNs emergence (e.g., one-many and one-

5. Double dissociation, an experimental technique commonly used in neuroscience and cognitive psychology, requires demonstrating that a lesion in one brain structure impairs function X but not function Y and that a lesion in a second brain structure impairs function Y but not function X. When both conditions are true, the two functions are interpreted as being independent of one another. The double dissociation technique has been used to argue for the relative independence of two major language centers in the brain, Wernicke's area for language comprehension and Broca's area for language production.

Table 2. Comparison of structure in numerosity, lexical numbers (LNs), and grammatical number (GN)

Numerosity	Initial LNs	GN
Subitization (quantities up to three or four as individuals)	One, two, three, and sometimes four	Singular, dual, trial, and occasionally quadral
Magnitude appreciation (quantity differentials of bigger and smaller above three or four)	Many; "big many" and "small many"	Plurals; greater and lesser plurals

Note. As shown in the comparison, the structure of numerosity, initial LNs, and GN are similar and occur cross-culturally, suggesting patterning by an underlying cognitive process. The perceptual system for quantity may influence the structure of both linguistic domains for quantity expression.

two-many). Corbett (2000) also noted that "dual and trial forms originate from numerals 'two' and 'three', and . . . the plural comes historically from 'four'" (21), values suggestive not just of subitization but of temporal precedence as well, with the development of LNs informing the development of GN, a hypothesis also offered by Greenberg (1972). Menninger (1992) painted a more complex picture in which adjectival number forms detach from their referents to become LNs, representing an original form that is perhaps intermediate between GN and LNs.

LNs and GN may be similar in structure because one influences the other or because a third variable, such as numerosity, influences both. Because numerosity (including its characteristic structure) is shared with species lacking language, its structure is unlikely to originate in language for humans. Nonetheless, the hypothetical case can be considered in which LNs take their structure from language. It would not be feasible (or ethical) to develop LNs in natural language without numerosity, since numerosity is a perceptual capability demonstrated to precede language acquisition. However, at least one artificial model (Artificial Life or "A-Life") has attempted the experiment. The results are instructive. A-Life generally did not converge on solutions resembling natural number systems: ". . . a numeral system resembling the dominant type found in the world's languages [i.e., the decimal system] can emerge only very rarely, though it is not actually impossible" (Hurford 1999:37).

Hurford (1999) ascribed the failure to produce results consistent with real-world observations of number system structure to limitations of the linguistic parameters programmed into the model: grammar size, lexicon/syntactic rule ratios, semantic primitive and semantic operation weightings, and fitness functions such as coverage (no gaps in numerals), redundancy (few instances of multiple terms for identical quantities), and complexity (less complexity was judged to be better than more complexity). An alternative interpretation might be that A-Life produced artificial results because it excluded numerosity, the canalizing effect of embodiment with penta-

dactylic limbs,⁶ and material engagement (Malafouris 2013). As is, the A-Life results do not challenge the idea that GN and LNs both take their structure from numerosity. How we perceive quantity may govern how we initially express quantity in language (although LNs can be, and in most cases are, elaborated further).

Distribution of Linguistic Features and Human Migration

If LNs emerge before GN, then the predicted geographic distribution would be for the youngest languages to lack both features, older languages to have LNs but not GN, and the oldest languages to have both features. As shown in figure 2, this appears to be the case. Languages lacking one or both quantity features are distributed across continents whose peopling is relatively recent (e.g., Australia, peopled 60,000 years ago, and South America, peopled 12,000–20,000 years ago; Davidson 2013). Although languages along each migration arc are related, languages across the two migration arcs are not, except in the most distant sense. The majority of languages with LNs but lacking GN in the form of nominal plurality are found along the two major swaths passing from Southeast Asia to Australia and from North to South America, a distribution evoking the major migration trajectories peopling these land masses (e.g., Atkinson 2011). Languages lacking GN with restricted LNs are concentrated at the migration end points, with Pirahã, the only language said to have an absence of both, also located near the end of the American migration.

Relative to the peopling of the respective landmasses, languages concentrated at the migration end points are the most recent, and languages along each migration arc are unrelated. The interesting question is whether the similar distribution of linguistic features along each migration arc arose as a matter of coincidence, common descent, or convergent evolution, which is a matter for further investigation. The distribution of the features, considered in light of migration time lines, is consistent with the idea that LNs precede the emergence of GN: the geographic areas where the peopling is the most re-

6. The dominance of the decimal system may reflect another aspect of quantity appreciation in humans: its embodiment with five fingers on two hands. As LNs begin to form in languages, the subitization constraint limits quantities to numbers smaller than the number of fingers on a hand (Coolidge and Overmann 2012), but as counting develops through pairing or one-to-one correspondences between sets of entities, the fingers form a convenient reference set that influences the development of base numbers (Menninger 1992). The role of fingers in expressing quantity concepts is cognitively prepotent and cross-culturally ubiquitous, a function of collocated regions of the parietal lobe that integrate numerosity with precise motor control of the fingers (Coolidge and Overmann 2012; Orban et al. 2006; Orban and Caruana 2014; Overmann 2014). However, the fact that a numeric base need not be decimal demonstrates the influence of the perceptual system for quantity, which does not think in base 10.

Key	GN	LNs	Nr	Example
○		Yes	139	Persian
▽	Yes	Restricted	14	!Xóõ
		No	0	None
▲		Yes	10	Pohnpeian
■	No	Restricted	1	Kayardild
◆		No	1	Pirahã

Figure 2. Map showing distribution of lexical numbers (LNs) and grammatical number (GN) for languages in the World Atlas of Language Systems (WALS). Map and data adapted from Dryer and Haspelmath (2013). As shown in the figure, languages listed in WALS with no GN and restricted LNs were concentrated at the end points of the major migration arcs into Southeast Asia/Oceania and North/South America. Languages with LNs but no GN follow the migration arcs. The geographic distribution, when coupled with the temporal distribution, is consistent with the idea that LNs emerge first, followed by GN. Of the four languages listed in WALS as having restricted LNs and no GN, three were reclassified as follows: Pirahã was categorized as lacking both GN and LNs because of reports highlighting the language's anumeric qualities (e.g., Everett 2005), and Yidiny and Imonda were classified as lacking GN and having LNs because of WALS annotations documenting counting up to five. These changes left a single language, Kayardild, in the no-GN, restricted-LNs category. Only the subset of the sample contained in WALS is presented here, to aid replicability of the results. Languages in WALS that are not included in the sample because of incomplete data have the following characteristics: languages lacking GN are concentrated along the two major migration arcs, and languages with restricted LNs are concentrated at the end points, especially in isolated areas (shown in CA+ online supplement A, figs. A1 and A2, respectively). Nr = number of languages. A color version of this figure is available online.

cent contain languages lacking both quantity features or with restricted LNs and no GN, areas where the migrations traversed contain languages that lack GN only, and the areas where the migrations originated contain languages with both quantity features.

As is the case with any *rara* (any feature or property that most languages do not possess; Cysouw and Wohlgemuth 2010), the lack of LNs, GN, or both can be interpreted in several ways: as mere coincidence, as the survival of an an-

cient linguistic form (common descent), or as a more recent linguistic development (convergent evolution). Mere coincidence, while not logically impossible, seems nonparsimonious and will not be further treated. The idea that the absence of either linguistic feature represents a persisting primal state is challenged by the question of why any particular feature would survive diachronic or synchronic change, processes to which all languages are subject, especially over long time spans. This question has been asked of other *rara*, like the click phoneme

found in several African languages (Güldemann and Stoneking 2008).

A potential influence on the pattern of geographic distribution is the serial founder effect, the decreasing variation that occurs when new populations are established by very small numbers of individuals from larger original populations (more specifically, the decrease in variability from an original population as a percentage of the total variability in a new population). Because individuals bring with them only a subset of the diversity inherent in an original population, the net effect of repeated founding events is a significant decrease in diversity. The effect has been demonstrated in genetics (e.g., Betti et al. 2009), technology (Movius 1948), and phonemes (Atkinson 2011). Decreased diversity has the potential to reduce the knowledge of LNs directly through the loss of knowledgeable individuals, as was documented for the Yuki octonary system (Foster 1944); it might also indirectly influence the need for GN through demographic effects, consistent with the idea that population size correlates with cultural innovation (Powell, Shennan, and Thomas 2009; Shennan 2001). The smallest phonemic diversity, in fact, is found in South America and Oceania (Atkinson 2011), the same migration end points where the lack of GN, restricted LNs, or both are concentrated.

LN may emerge first simply as a function of how the perceptual sense of quantity (shared by many species, including humans) becomes accessible and explicit as concepts of discrete quantity (possessed by humans alone). Discretization has been argued to be an emergent property of the interaction of numerosity with material structures (both the physical body and cultural artifacts; Coolidge and Overmann 2012; Malafouris 2010, 2013). Material structures provide affordances (Gibson 1979) such as tangibility and manipulability that help to scaffold quantities beyond the subitizing range by enabling their identification as distinct tactile and visual entities. The ability to exploit material structures for this purpose may be enabled by regions of the parietal lobe that functionally integrate quantity appreciation, precise motor control of fingers and held objects, and three-dimensional vision; collocate quantity appreciation with key language centers in the brain;⁷ and share neuronal habituation to quantity stimuli with adjacent neurons to ground symbolic understanding (Bruner 2010; Dehaene 2007; Harnad 1990; Malafouris 2010, 2013; Orban et al. 2006; Orban and Caruana 2014; Pica et al. 2004). If the material engagement required to develop LNs is motivated by social processes, such as the need to manage complexity in large groups or conduct trade between groups (e.g., Lean 1992; Saxe 2012), it follows that restricted LNs might characterize small, isolated groups (as appears to be

the case), despite the presence of numerosity (e.g., Butterworth et al. 2008; Gordon 2004; Pica et al. 2004; recent work by Everett and Madora [2012] shows that the Pirahã can subitize).

The hand has been positioned as a primary mechanism in developing human agency and cognition (Tallis 2003, 2011; Wilson 1998). As Malafouris (2013) noted,

. . . the hand is not simply an instrument for manipulating an externally given objective world by carrying out the orders issued to it by the brain; it is instead one of the main perturbatory channels through which the world touches us, and it has a great deal to do with how this world is perceived and classified. (Malafouris 2013:60)

Regarding the role of the hand in language, Cushing (1892) noted an interesting distinction in Zuñi between “verbs relating to acts performed with the hands, as, with something, to something, at something, in something, out of something, from something, etc.; and . . . verbs relating to acts performed (or actions occurring) without the agency of the hands” (310) and suggested that the involvement of the hands might subsequently influence grammatical distinctions.⁸ Although Cushing’s work is dated, a distinction based on hand involvement that subsequently influenced grammar would support the emergence priority of LNs. Cushing’s Zuñi distinction additionally recalls the phenomenon, remarked on in several ethnographies (e.g., Hilger 1957; Von den Steinen 1894), concerning the difficulty of counting with the eyes alone rather than involving the tactile sense. Given the involvement of numerosity with finger gnosis (yielding the somatosensory knowledge of finger quantity) and its role in finger counting and reference sets (reviewed in Overmann 2014), touch may be important in both forming numeric concepts and informing proximal (near) reference. The latter is perhaps underexplored in evolutionary accounts of language origins, which tend to emphasize distal (far) reference. The hand is perhaps involved in both near and far referencing, if Gómez (2007) is correct in suggesting that gestural pointing, a potential embodied precursor to deictic pointing, originated in using the index finger in haptic exploration.

The idea that LNs precede GN also has an implication for extinct languages whose GN status is known but whose spoken LNs (as distinguished from numerical notations) are uncertain and difficult to ascertain through textual analysis or archaeological means. For example, the presence of GN in Sumerian and Akkadian texts (e.g., Woodard 2008) implies that both had developed LNs before the third millennium BC. Although the chronology indicated is relative, GN was ap-

7. For example, the angular gyrus, which has been implicated in metaphor, including embodied metaphors of mathematical concepts (Chiappe and Chiappe 2007; Lakoff and Núñez 2000).

8. A similar distinction is perhaps connoted by the English demonstratives “this” and “that.” The former is more likely to be used with objects within reach that can be touched or grasped, and the latter is more likely to be used with objects outside of reach that can only be pointed to (Gallagher 2013).

parently sufficiently entrenched to be reflected in early writing, suggesting that it was not a recent development and that the time separating it from LNs may have been significant. Numerosity's influence on GN structure in extant languages also implies that similar structure in extinct languages may demonstrate numerosity in ancestral peoples.

Consistency with Linguistic Frameworks

The notion that perceptual systems—such as those involved in color (Berlin and Kay 1969; Kay and McDaniel 1978), space and time (Casasanto and Boroditsky 2008), or, as argued here, quantity—inform linguistic structure is consistent with a cognitive framework for language in which linguistic structure is an emergent property of the interaction between cognitive processes such as categorization, analogy and similarity, memory chunking, and exemplar representation (Bybee 2010). As the perceptual sense of quantity, numerosity is one of several postulated core cognitive capabilities (others are space, time, agency, object recognition, and causality) giving structure and form to the way in which humans and other species experience their environment (Carey 2009), regardless of whether they have language. However, when language is present and operates over perceptual primitives such as quantity (Carey 2009), the relationship between the two does not preclude bottom-up structuring, such as that proposed in this analysis for quantity expressions.

A framework that views linguistic structure as the regularizing byproduct of the interaction of cognitive processes might also incorporate the idea that perceptual systems not only join in that interaction but add unique influences that structure the cognitive domains they represent. Such incorporation would also provide a mechanism for coinfluence between core cognitive domains (for example, both GN and LNs are modulated by animacy hierarchies that are based on the perceived sentience of referent nouns, although in different directions, as animacy in LNs works to exclude high-animacy beings from being counted, whereas in GN, animacy functions to include high-animacy beings). This phenomenon may show the coinfluence of the core domain for quantity with those of agency or object recognition. Furthermore, the idea that perceptual systems or core domains exert structural influence on language might fit more easily with the emergent-property framework than with more traditional views of grammar and syntax postulating innate rules. As the results of the grammar-only model for LNs development suggested (Hurfurd 1999), such rule-based systems might not fully account for all of the influences acting to structure language. The alternative, in which cognitive processes, the body, and materiality interact to create linguistic structure, may be more productive in this regard.

This is not to deny language a role in numeracy, because language provides unique access to numerical intuitions. It maps the semantic meaning of words for numbers onto non-

verbal representations of quantity for children acquiring LNs through enculturation and perhaps for peoples with restricted LNs acquiring numbers through cultural contact (Cantlon et al. 2006; Dehaene 2007). Language also provides a scaffold for numbers so large that manual counting is infeasible, such as hundreds of thousands (Overmann 2013; Overmann and Coolidge 2013), although numerical notations, which are inherently material, have an even greater capacity for representing large numbers (i.e., adding more zeros can create a theoretical infinity of numbers, exceeding the available LNs; Chrisomalis 2010). It is, however, to suggest that language plays the same important role in developing numeracy as it does in conceptual development for any rational endeavor, enabling concepts to be articulated in thought and discourse, expanded and refined, and communicated across time. Regarding numerosity, spoken language may act as a cognitive technology for objectifying and providing access to the sense of quantity so that its properties can be consciously analyzed and designed, a role asserted for writing, numerical notations (Olson 1994), and materiality (Coolidge and Overmann 2012). For numeracy, then, language should be placed in a holistic context with other elements that causally or constitutively contribute to human rationality, including materiality, social structures, and embodiment.

Comments

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Numeracy, the ability to reason with numbers, is unique to human cognition and tightly linked to language; but is language essential for its emergence, or, more precisely, is grammatical number (GN) a precursor for lexical numbers (LNs)? These questions can be approached from a phylogenetic or ontogenetic perspective.

The latter approach has been adopted in most cognitive sciences, with findings pointing toward the affirmative. Although two phylogenetically old systems allow us to exactly represent small sets and to approximate large quantities (Feigenson, Dehaene, and Spelke 2004), the ability to exactly represent larger numbers hinges on the availability of conventionalized counting sequences (Frank et al. 2008; Gordon 2004; Pica et al. 2004). Counting sequences are not constrained to verbal representations (LNs) but include external representations, such as body-based systems (Bender and Beller 2012; Spaepen et al. 2011) or notation (Chrisomalis 2010). Recent work also indicates that the GN system is capitalized when learning LNs; children whose mother tongue distinguishes singular, dual, and plural, for instance, learn the words for one and two earlier than do chil-

dren whose mother tongue makes fewer or no such distinctions (Sarnecka 2014).

If, however, numerical tools such as counting sequences (or GN, for that matter) were indeed necessary for acquiring numeracy, how could anyone ever invent them in the first place? To address this question, Overmann investigates the LNs and GN distribution in contemporary languages, with intriguing results. Although LNs are almost ubiquitous, GN is slightly less widespread and thus neither necessary nor sufficient for LNs emergence. Moreover, languages without GN tend to cluster at the end points of the major migration routes (in Australia/Oceania and South America), which is taken as indicating that LNs predate GN.

Especially the latter finding, though, entails leeway for interpretation. Overmann bases her inference on two assumptions: that languages are younger toward the end points of migration routes and that the younger ones are more likely to lack GN, and probably even LNs. Due to serial founder effects, successive population bottlenecks during expansion reduce diversity in features, which is why, for instance, the phonemic inventory is smallest in South America and Oceania. However, in addition to being one of those areas where lack of GN and/or restricted LNs concentrate, Oceania is, at the same time, home to amazingly extensive number systems (Beller and Bender 2008; Bender and Beller 2014). Obviously, the relationship between the putative age of a language and the presence of numerical tools is intricate, and questions of how early in language evolution these tools were invented loom large. Synchronic data can serve, at best, as proxy for diachronic changes, because features may have existed but dwindled (like the dual in many Indo-European languages) or just recently emerged; and analyses into the evolutionary trajectories of such data need to control for shared inheritance (Levinson and Gray 2012).

Although Overmann's work is only a first step in unraveling the evolution of numerical tools, it is nonetheless an important step, because it brings existing data to bear on the question of origin. Of even greater value is the explanatory alternative she proposes, which is based on an integrated view on numerical cognition as comprising brain/mind, body, and material artifacts. Rather than one generating the other, both LNs and GN may have been generated by the embodied (visual and tactile) systems of quantity perception. This approach accommodates the view that cognition may be distributed across internal and external representations, thereby extending the mind beyond boundaries of skin and skull (Clark and Chalmers 1998; Hutchins 1995). It could be made even more fruitful by extending the notion of language beyond the verbal, to incorporate all types of number representations regardless of their modality. Each type of counting sequence, regardless of modality, not only affects how numbers are represented and processed but feeds back on the brain itself. For this very reason, dissociations of language and numeracy in people with respective lesions are uninformative with regard to evolutionary issues, because the lesions occurred only after the patients had already acquired

both numeracy and language (and potentially the former through the latter).

The importance of cultural tools for cognitive development is also highlighted by the observation that, although LNs may predate GN, children take advantage of the latter for learning the former. This inversion of phylogenetic processes in ontology should caution us against simply interpreting developmental observations as accounts of evolution. Taking a fresh look from a different angle might also enable us to solve some of the persevering questions: If the human body scaffolds numeracy, how can we account for the incredible diversity even in body-based number representations (Bender and Beller 2012)? Can today's diversity in counting systems be traced back to just one origin, or were these tools invented several times independently? And how has using such tools shaped human cognition and culture more generally?

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Numerosity Structures Lexical and Grammatical Number: Overmann's Definitive Argument against Hurford's "Without Language, No Numbers" Hypothesis

In 1987, linguist James Hurford proposed that there was no sense of numbers without language. Twenty years later, in the face of overwhelming empirical evidence that human infants and monkeys can apparently subitize (immediately discriminate between one, two, and three things), Hurford (2007) modified his stance to say there is no number sense without language except in the case of subitization. However, even the latter position is inadequate, as there is now ample empirical support that number sense, even beyond subitization, requires neither words, symbols, nor language, and provocatively, it may even be independent of experience (e.g., Kant 2003 [1871]). This nonsymbolic ancestral ability to compare quantities of items is shared by human neonates and infants, older children and adults, animals, and many other species. As Overmann duly notes, the latter include fish, birds, reptiles, and perhaps even insects. The origin of these abilities obviously lies in natural selection for reproductive success of organisms that can perform quantitative evaluation of their environments. These ancestral abilities may also span nonnumerical dimensions, such as time, magnitude (size, height, and weight), speed, brightness, and acoustic pitch and amplitude. Although both numerical and nonnumerical judgments appear to be ancestral and independent of language, their relationship to each other remains

undetermined. Recently, however, de Hevia and Spleke (2009, 2010) have provided evidence from neonates (mean age = 52 hours) that number and magnitude may share a common biological and systematic relationship that may be independent of language and experience. Indeed, modern studies appear to support Kant's original contention that space, time, and number representations may not only precede experience but structure it.

It is often proposed that numerosity has two core functions: subitization, whose upper limit appears to be three or four things, and analog comparison, that is, discrimination of smaller sets from larger sets, where both sets have an indeterminate number of members. It may even be suggested that both core functions may conform to Weber's law, where the ability to discriminate between two objects or sets of objects depends on the ratio between them instead of the absolute value of each (i.e., a just noticeable difference). For example, in the subitization discrimination between one and two things, the latter is 100% greater than the former. In the subitization discrimination between two and three things, the latter is 50% greater than the former. The least commonly proposed upper limit of subitization, four, is 33% greater than three, and this is the point at which discrimination often fails. Thus, a logarithmic judgment may underlie both core functions of numerosity (e.g., Dehaene et al. 2008).

Overmann begins her hypothesis that lexical numbers (LNs) are responsible for the development of grammatical number (GN) by defining LNs as the terms usually used for counting quantities and GN as a morphosyntactic category of words or word forms that distinguish singular and plural entities. In light of the previously reviewed studies, Overmann's contention appears to be undeniable and highly logical. She further discusses the possibility of an interactive scaffolding among fingers and haptic perception with materiality as perhaps an origin for LNs, and it is here, particularly, that one wishes the article was replete with more examples of how this material engagement might have evolved. (Alas, the article is too short!)

Recent studies of the neurological foundations for numerosity give further gravitas to Overmann's ideas about LNs preceding GN. Harvey et al. (2013) have shown that the parietal lobes topographically represent numbers, both in the subitization range and analog comparisons, and two areas in the superior parietal lobes have received particular attention, with evidence for neuronal specialization for numbers, the intraparietal sulcus (IPS) and the precuneus (e.g., Coolidge 2014; Krause et al. 2014). Interestingly, the former region may have special relevance for Overmann's basic contention. In functional brain imaging studies involving children, Emerson and Cantlon (2014) found differential response of the right and left IPS to numerical quantity. It appeared that number-related neural responses occur early in childhood in the right IPS. As children's familiarity and acuity with LNs increases over childhood, the left IPS becomes activated. They also found a longitudinal continuity of right IPS numerical processing coupled

with later childhood left IPS numerical processing, which suggests at least two conclusions: (1) the IPS plays a fundamental role in numerical cognition, and (2) mastery of LNs builds upon more basic numerosity systems (which are independent of language) and interacts with language in a manner that scaffolds with material culture to produce greater linguistic number acuity over time.

In summary, it may be that, just as Overmann contends, "language plays the same important role in developing numeracy as it does in conceptual development for any rational endeavor." Also, as Overmann has astutely reasoned, if, in her analysis of 905 languages, GN never occurred without LNs, but 7% of the languages developed LNs without GN, then GN may require the same (or greater?) numerical discrimination acuity as LNs. Perhaps, then, maturity of the left IPS and the lateralization of many of language's functions to the left hemisphere are not merely coincidental in the ultimate acquisition of GN. Furthermore, development of GN may indeed rely on more cognitively sophisticated operations allowed by language, whereas LNs may be acquired earlier with only some of basic necessities of language's operations (e.g., associating arbitrary sounds to specific things).

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I am not well positioned to evaluate all of the claims in Overmann's article, and I will make only a few comments pertaining to the evolution of grammatical number (GN) and lexical numbers (LNs) in the early textual cultures of ancient Mesopotamia. GN is poorly developed in Sumerian, particularly in Sumerian texts from the early third millennium BC. The system for GN presented in standard grammars of Sumerian (e.g., Edzard and Thomsen) is synthetic, or represents the latest stage of the language only (e.g., noun base duplication for totality, suffixes for simple plural, duplication of verbal base for plurality of object, and use of secondary verbal stem and/or suffixes for subject plurality). No GN can be detected in the earliest texts from Mesopotamia, the so-called Late Uruk and Jemdat Nasr texts (ca. 3,300–3,000 BC), as the linguistic affiliation of these texts remains unclear. Basically, very early Sumerian texts (2,900–2,350 BC) would express GN through duplication of the sign (for object plurality), and later Sumerian texts can express GN in the same way or by the use of very simple suffixes (GN can also be expressed in the verbal base by use of auxiliary verbal base). Akkadian, which is attested a few centuries after Sumerian (from 2,600 BC), exhibits full GN. It is possible that the Late Uruk and Jemdat Nasr texts exhibit the same kind of simple GN as early Sumerian texts (simple duplication for plurality), but this cannot be verified, nor can it be ascertained

Figure 3. *a*, Oksapmin 27-body-part counting system. *b*, Suffix *fu* to communicate a double of a body part's value.

whether this would have related to a linguistic reality or only a scribal convention.

Perhaps the best evidence for LNs in Late Uruk and Jemdat Nasr texts, as well as in very early Sumerian texts, comes from the ordinal numbers detectable in some texts; however, apart from two texts, possibly from Uqair, both recording a period of 8 years, only “first” through “fifth” are attested (see Englund 1998:125–126 with examples), possibly suggesting restricted LNs. Ordinal numbers in the Late Uruk texts are not grammatically different from cardinal numbers, and the distinction is made solely on syntactical grounds. LNs are otherwise poorly attested in texts from the third millennium BC, which for the most part have only notations with number signs and no evidence of LNs. From Ebla in Syria comes the most complete evidence for the pronunciation of Sumerian number words (ca. 2,350–2,400 BC), and only 1–10 are attested. To what extent my comments on the development of GN and LNs in early Sumerian and Akkadian force the author to change her arguments is unclear to me, but a fuller study of the chronology of both LNs and GN in the ancient world is, despite the work of Friberg and Damerow in the 1980s and 1990s, a desideratum.

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Based upon cross-linguistic data, Karenleigh Overmann argues that, in the (socio)geneses of linguistic representations for number, the emergence of lexical numbers (LNs) is not rooted in grammatical number (GN). Rather, LNs and GN are both structured by a nonlinguistic process of subitizing, a process that supports the discrimination of very small numbers of objects.

Overmann's argument is cogent. After all, prelinguistic infants and nonhuman animal species discriminate small numbers, and thus these ontogenetic and phylogenetic patterns provide one plausible cognitive ground for early semiotic acts of numerical reference. However, in my remarks that follow, I raise a cautionary note about conjectures concerning the origins of semiotic systems when observations of human activity are lacking. I support my concern by pointing to findings from fieldwork (1978, 1980, 2001) in remote Papua New Guinea communities, the Oksapmin (Saxe 2012). My focus is on a critical element missing in Overmann's account: interlocutors' communications about number in the context of shifting collective problems with which people are engaged in everyday practices.

The Oksapmin traditionally use a 27-body-part counting system (see fig. 3*a*), and there is no evidence that Oksapmin used the system for arithmetic computation in prehistory (the first missionaries arrived in the early 1960s). Body parts were simply for counting, tallying contributions to bride price, and communicating about cardinal and ordinal values. However, in fieldwork conducted in 2001, I documented a striking development. When an Oksapmin used the suffix *fu* after a body part, the speaker conveyed that the value of the body part should be doubled (see fig. 3*b*), and I initiated an investigation of the emergence of this multiplicative body part function for *fu*. My research focus paralleled Overmann's concerns. Both Overmann and I pursue questions concerning the origins of new semiotic forms for number. Overmann focuses on cognitive resources. I focused on a sociohistorical analysis of the use of *fu*, situating my investigation in the history of Oksapmin semiotic activities through time with a focus on collective practices of economic exchange.

First, with respect to the premultiplicative functions of *fu*, the early meaning of *fu* was a “complete group of things” (e.g., a string bag full of sweet potatoes). The meaning of *fu* was also incorporated into the body counting system; a complete round

Figure 4. Woman completing all 27 body parts in the counting system, exclaiming “*fu!*” with fists raised (see <http://www.culturecognition.com/tit-fu/>).

of all 27 body parts was referred to as *fu*, exclaimed with fists raised at the end of a count (see fig. 4).

Second, a new position for *fu* emerged with the introduction of Australian pounds and shillings. With the introduction of Australian currency, people counted shillings with body parts and generated equivalence relations in the context of exchanges, with 20 shillings (inner elbow; i.e., 20) being the equivalent of 1 pound. Eventually, a pound equivalent became a complete group of 20 shillings, or *fu*, represented by the inner elbow (20). The shift in position of *fu* is depicted in figure 5.

Third, the introduction of the Papua New Guinea kina and toea was accompanied by a shift in arithmetical function of *fu*. With independence in 1975, Papua New Guinea issued its own currency: the 2-kina note, which was equivalent in value to 1 pound, and the 10 toea coin, equivalent to 1 shilling. *Fu*, referring to 1 pound, was then extended to refer to a 2-kina note. This shift led to two alternative ways of counting 2-kina notes (see fig. 6). One could count three 2-kina notes as “middle finger (3) *fu*,” (3 pounds) or the equivalent of 6 kina, with one count a double of the other.

Fourth, the doubling function of *fu* was eventually generalized. For instance, an individual might refer to the sum of a 5-kina note and a 1-kina coin as middle finger (3) *fu*, although there are no 2-kina notes to count (see fig. 7).

In the Oksapmin case, numerical functions of *fu* emerged through individuals’ communicative efforts in collective practices of economic exchange. Historical shifts in properties of currency and in collective practices of economic exchange led to shifting communicative problems; in their efforts to accomplish recurring and also shifting problems in communication, earlier semiotic developments seeded subsequent ones. If we were to extend my approach to the Overmann case, we would frame questions beyond the cognitive resources that predate semiotic developments in number. We would ask, for example: What are the communicative problems involving num-

ber that emerge in everyday practices, and how are such problems shifting through historical time?

Reply

In providing Coolidge more detail on the material origins of numbers, how I understand what numbers are informs my view of how they are realized through material engagement (Malafouris 2013). For, understanding what numbers are, I find Russell’s (1910, 1920) conception of number useful, despite its now-outdated representationalist flavor, because it allows various aspects of the realization process to be distinguished. In his theory of logical types, Russell differentiated individuals (type 1), sets of individuals (type 2), sets of sets of individuals (type 3), and sets of type 3 (type 4). Material objects are type 1 entities; their cardinality is type 2. Cardinality shared across sets—or numbers—is type 3. Collections of numbers (e.g., the natural numbers; the subsets that are counting sequences) are type 4. In original contexts (i.e., with numerosity, material culture, and language,⁹ but no numbers), subitizable quantities cannot be one, two, and three, as numerate individuals experience them, because such lexical terms assume existing number concepts (type 3). Rather, subitizable quantities might be experienced as cardinal qualities (type 2) and, if expressed linguistically, take the adjectival forms postulated by Menninger (1992).

Because cardinality alone appears to be insufficient for forming number concepts (Rips, Bloomfield, and Asmuth 2008), numbers are realized by comparing the cardinalities of sets. Such comparisons, brain-body-material interactions like pairing and one-to-one correspondence, yield concepts of both subitizable¹⁰ and nonsubitizable number. When linguistically labeled, these would likely be lexical numbers (LNs) and not grammatical number (GN), which, Coolidge speculates, may entail complex cognitive operations, although GN also appears limited to subitizable quantities and magnitude discriminations. Finally, as numbers increase (type 4), greater exemplar availability yields opportunities for numerical elaboration, as scaffolded and structured by the materiality used for numerical manipulation.

Bender raises a critical issue based on the diversity of number systems in Oceania, a region peopled relatively recently, where societies have remained fairly isolated. Serial foundation, detected in various phenomena along the migration arcs into Oceania and the Americas, decreases variability, so the signif-

9. Listed in likely order of occurrence: numerosity, shared with other species; material culture, several million years; language, perhaps several hundred thousand years.

10. Inclusion of subitizable numbers is suggested by both ethnographic and linguistic data. They likely emerge first as a function of their perceptual salience, not because they are intuitive.

Figure 5. Original (a) and new (b) locations for *fu*.

icant variability of the Oceanian number systems is a potential challenge. Such variability is predictable, however, given multiple origins for numbers and the need to realize subitizable number concepts. That is, although initial numbers are one, two, and three across languages and cultures (Menninger 1992), they do not necessarily emerge in consistent order (although magnitude-ordered when expressed in counting sequences). Two might emerge first (cardinality shared by pairs), one might emerge next (dissimilarity of pair and single), and then three might emerge as two plus one or the cardinality of trios; other permutations are suggested by linguistic data. Furthermore, fingers are an initial technology for realizing and representing quantity, but initial numbers are less than the fingers on one hand. Organization into anatomically informed cycles and bases (e.g., 5, 10, and 20, none of which are subitizable) must therefore follow the earlier, more variable phase in which cycles (if any) were based on subitizable numbers (as indeed characterizes some Oceanian number systems; Lean 1992). Such reorganization might reflect social needs for higher numbers and within- and between-group interactions that occasion conventionalization. Numerical emergence, elaboration, and conventionalization are complex, nonlinear phenomena, likely encompassing multiple origins, diffusion, serial foundation, and forgetting; the commonality imposed by shared psychological and physiological attributes (e.g., numerosity and fingers); and the variability arising from differing materiality, social needs, and the vagaries of invention.

Although I alluded to social motivations and interactions, sociality was not my focus, as Saxe critiques. As he notes, social transactions between individuals and groups are where differences are encountered and conventions negotiated; they are the medium of exchange through which linguistic forms like LNs

and GN take shape and, indeed, where material forms are conventionalized. Fortunately, Saxe's example nicely illustrates my point about numerical elaboration being related to sociomaterial conditions. Before contact, Oksapmin numbers appear to have been relatively stable for an unknown but perhaps significant period of time. With contact, change was rapid. Western numerical cycles influenced the value of *fu*, money supplemented or replaced body parts, and arithmetic emerged using both traditional and introduced counting technologies. Notably, the Oksapmin did not simply acquire an introduced system but elaborated their own system as well, which suggests that elaboration may be at least partly facilitated by the availability of multiple, contrasting ways of accessing numerical concepts (Overmann 2015, 2016).

Dahl offers two caveats on extending my hypothesis to extinct languages, like Sumerian, that are known only from early writing. The first is the state of GN in protowriting and early texts. As systems of conventional signs yielded literacy (a matter of centuries; Cooper 2004), writing acquired increasing fidelity to language. Protowriting might not represent GN in any form (even if present in language); later texts might express it poorly, and literate texts might express it with fidelity and conventionality. These are possible limitations of and changes in the expressiveness of writing, however, and not developments in GN. The second caveat concerns the scant evidence for LNs and ordinal numbers (ONs) in early texts; both are numbers used as adjectives or nouns that may include non-subitizable numbers. Like LNs in later texts, ONs provide insight into the Sumerian lexicon for numbers, a matter distinct from saying whether archaic Mesopotamian peoples had LNs. This is the difference between recognizing that nonsubitizable numerals like 十 and 百 (10 and 100) would have associated

Figure 6. *Fu* used as double the value of a body part when counting two kina notes.

Figure 7. *Fu* generalized to contexts that do not include two kina notes.

LN and knowing that they were “shī” and “bāi.” While unambiguous representations (e.g., ONs in later texts) are necessarily privileged in recovering Sumerian vocabulary, it does not follow that LNs were absent or restricted before then.

Central to Dahl’s caveats is the question of whether Sumerian was the language associated with proto-cuneiform. Narrowly, GN implies only that Sumerian had previously developed LNs. On the other hand, earlier LNs are certainly consistent with the possibility that Sumerian may have been the language of proto-cuneiform, as indeed is Sumerian’s close and lengthy association with the numeracy and literacy that developed from it. Furthermore, given their participation in the urbanization and sociomaterial complexity of the late fourth millennium, the period in which proto-cuneiform developed, the Sumerians would most likely have had an unrestricted indigenous number system.¹¹ Similarly, the proto-cuneiform numerals themselves had a level of complexity associated only with relatively mature,¹² not emerging or restricted, number concepts and lexicon (even if somewhat irregular in their earliest use; Englund 1998). Additionally, the approximate time line for developing GN suggested by its current geographic distribution makes it doubtful the Sumerians could have borrowed LNs, complex notations, and numerical accounting—and developed GN—in the few centuries separating the appearance of proto-cuneiform from the ONs of later texts. Granted that arguments for earlier, unrestricted LNs in Sumerian and earlier, unrestricted LNs associated with proto-cuneiform do not entail that proto-cuneiform was Sumerian; nonetheless, this seems more plausible than the alternatives.

Finally, I second Bender’s recommendation for researching numerical representations regardless of modality, as both language and materiality (e.g., bodies, artifacts, and notations) are essential, complementary methods of accessing and elaborating numerical concepts (Overmann 2016). Although structure and variability in number concepts are at least partially tied to the material realizers of those concepts, the role of materiality in numerical cognition is currently understudied, including the ways in which it interacts with language. In such investigations, it will be important to interrogate the things

we know and assume about numbers from numerate cultures (like the dependence of LNs on GN), as these can influence our interpretations of numbers cross-culturally. This is a continuing issue, complicating interpretations of lesion studies demonstrating the double dissociation of numeracy and language, as Bender notes, where the relevance of acalculia (loss of numeracy) to anumerate brains is unclear. That much work yet remains in this area is both exciting and a challenge.

—Karenleigh A. Overmann

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11. As shown in table A1 (CA+ supplement A), ONs appear to follow the development of unrestricted LNs, although again this insight provides only a relative chronology.

12. Exchanging 10 smaller units for one higher unit, understanding a 10-unit sign superimposed on a 60-unit sign as a sign for 600, and recognizing that six circled impressions meant “1/6” presuppose well-developed nonsubitizable numbers.

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