NODE-DISJOINT CYCLES IN BRIDGELESS CUBIC GRAPHS

THINH D. NGUYEN

ABSTRACT. We prove the NP-hardness of deciding whether a connected bridgeless cubic graph $G(V, E)$ contains two node-disjoint cycles of equal cardinality $|V|/2$.

Definition 1. (TWO DISJOINT CYCLES problem): As stated in the abstract.

Definition 2. (HAM-CYCLE AVOIDING EDGE): Does a 2-edge-connected cubic graph H contain a Hamiltonian cycle avoiding a given edge e

Claim 3. We have that HAM-CYCLE AVOIDING EDGE $\leq_p$ TWO DISJOINT CYCLES

Proof. Construct G as follows. Take two copies of $H - e$, denoting the endpoints of e in the first copy v_1, u_1 and denoting the endpoints of e in the second copy v_2, u_2 . Add edges e_v between v_1 and v_2 , and e_u between u_1 and u_2 . Observe that G is 2-edge-connected.

Any cycle of G contains both or neither of e_v and e_u , since these edges form an edge cut. Thus if G has two vertex-disjoint cycles of length $|V(H)|$, they must each lie in a separate copy of H . Such cycles exist precisely if H contains a Hamiltonian cycle avoiding e . $\square$

CONCLUSION

As long as we study a mathematical conjecture, we should encourage ourselves of having enough labouring hours on popular maths books. Then, reading some articles on theory of computing like [3] is a good practice. Only after that, could we think of the ultimate final for all mathematics sciences.

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Current address: Department of Mathematics, Moscow State University
Email address: kosmofarmer@yandex.com

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