Sum of Binomial Coefficients and its Lemma

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Abstract: This paper presents a lemma on binomial coefficients. This idea can enable the scientific researchers to solve the real life problems.

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1. Introduction

When the author of this article was trying to develop the multiple summations of geometric series, a new idea stimulated his mind to create a combinatorial geometric series [1-9]. The combinatorial geometric series is a geometric series whose coefficient of each term of the geometric series denotes the binomial coefficient V_n^r. In this article, binomial identities and multinomial theorem is provided using the binomial coefficients for combinatorial geometric series.

2. Combinatorial Geometric Series

The combinatorial geometric series [1-9] is derived from the multiple summations of geometric series[10-19]. The coefficient of each term in the combinatorial refers to the binomial coefficient V_n^r.

$$\sum_{i_1=0}^{n}\sum_{i_2=i_1}^{n}\sum_{i_3=i_2}^{n}\cdots\sum_{i_r=i_{r-1}}^{n} x^{i_r} = \sum_{i=0}^{n} V_i^r x^i \;\;\&\;\; V_n^r = \frac{(n+1)(n+2)(n+3)\cdots(n+r-1)(n+r)}{r!},$$

where $n \geq 0, r \geq 1$ and $n, r \in N = \{0, 1, 2, 3, \cdots\}$.

Here, $\sum_{i=0}^{n} V_i^r x^i$ refers to the combinatorial geometric series and

V_n^r is the binomial coefficient for combinatorial geometric series.

Lemma 2.1: $V_k^{n-k} + V_{k+1}^{n-k-1} = V_{k+1}^{n-k}$.

$Proof$: $V_k^{n-k} + V_{k+1}^{n-k-1}$

$$= \frac{(k+1)(k+2)(k+3)\cdots(k+n-k)}{(n-k)!} + \frac{(k+2)(k+2)\cdots(k+n-k)}{(n-k-1)!}$$

$$= \frac{(k+2)(k+3)\cdots(k+n-k)}{(n-k-1)!}\left(\frac{k+1}{n-k}+1\right)$$

$$= \frac{(k+2)(k+3)\cdots(k+n-k)}{(n-k-1)!}\left(\frac{k+1+n-k}{n-k}\right) = V_{k+1}^{n-k}.$$

Hence, the lemma is proved.

For examples,

$$V_0^n + V_1^{n-1} = V_1^n;\ V_1^{n-1} + V_2^{n-2} = V_2^{n-1};\ V_2^{n-2} + V_3^{n-3} = V_3^{n-2}.$$

Similarly, we can constitute more examples using Lemma 3.1.

3. Conclusion

In this article, a lemma on binomial coefficients was constituted. This idea can enable the scientific researchers to solve the real life problems.

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