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Number Concepts Are Constructed through Material Engagement

A Reply to Sutliff, Read, and Everett

Karenleigh A. Overmann

School of Archaeology, University of Oxford, Keble College, Parks Road, Oxford OX1 3PG, United Kingdom, and Center for Cognitive Archaeology, University of Colorado, Colorado Springs, Colorado, USA (karenleigh.overmann@keble.ox.ac.uk). This paper was submitted 3 III 16 and accepted 3 III 16.

Mathematical ideas—including numbers—are not innate, as Sutliff (2016, in this issue) avers, but culturally constructed concepts (De Freitas and Sinclair 2014; Núñez 2009; Read 2016, in this issue) created through the interactivity of brains, bodies, and materiality (Malafouris 2013). Certainly, the natural numbers (i.e., the numbers used in counting sequences) demonstrate significant cross-cultural commonality, both as concepts and in their overall method of invention. Such commonality is entirely predictable, given that the natural numbers are concepts of cardinality shared by sets of objects (Russell 1910, 1920) realized through human psychological, physiological, and behavioral interactions with the same material world and motivated and conventionalized by similar social requirements and transactions.

However, even the natural numbers are not identical across number systems. Within a particular system, they may change over time, from being equivalences (“that many”) to collections (“two tokens and two tokens are four tokens”) to entities related numerically to similar entities (“two plus two is four”; Gowers 2008). Across number systems, the natural numbers also differ in such matters as what they can be applied to (animate beings are not always included); how they are applied (through numeral classifiers; to single or pairs of objects); who learns and uses them within a society (they can be privileged knowledge); whether they comprise an open (extensible) or closed system; the materiality used to realize, represent, and manipulate them (which informs how they are conceived); and how they are manipulated, their degree of manipulability, and the types of operations they can be used with (discussed below).

Nor is the realization of number concepts a function of “the consciousness of self in time” (Sutliff 2016). Brouwer’s introspectionist view of the natural numbers, which he “derived from the old metaphysical view that individual consciousness is the one and only source of knowledge” (Ferreirós 2008:148–149), may suffice to explain numerical origins for those who agree that numbers are innate (though Brouwer’s

theory seems self-contradictory in this regard; he was asserting, against mathematical realism, that mathematical ideas were explicitly constructed, which one might think would exclude numbers not only from being real but also from being innate as a matter of definition). Regardless, as an explanation of numerical origins, Brouwer’s theory was not grounded in the psychological, ethnographic, and linguistic data on numerical cognition developed in the past fifty years.

Psychological data show that human infants and non-human species perceive quantity as small-set cardinalities, as well as large-set quantity differentials above a noticeability threshold, an ability that is considered to be innate (Carey 2009) and independent of language (Brannon 2005; Varley et al. 2005). The (alinguistic/anumeric) ability to perceive quantity, however, is not the same as having number concepts, or all species demonstrating the former would possess the latter—and this is not thought to be the case. Furthermore, the acquisition of numeracy is an ontogenetic process (Booth and Siegler 2006, 2008; Siegler and Booth 2004) characterized by specific neural reorganizations that differ across cultures (Dehaene et al. 2008; Grabner et al. 2009; Piazza et al. 2013; Zamarian, Ischebeck, and Delazer 2009). Were numbers innate, developmentally inflected neural change in response to numeracy enculturation becomes inexplicable, as does the neurofunctional variation associated with cross-cultural differences in numeracy.

Ethnographic data show number concepts varying across cultures (including whether there are any numbers) and describe peoples with few numbers comparing the cardinality of sets of objects through pairing and one-to-one correspondence. These behaviors require neither concepts of nor language for numbers (Russell 1920) but comprise the mechanism for occasioning both (Overmann 2015a). As Rips, Bloomfield, and Asmuth (2008) noted, the interaction of cardinality (a property of sets of objects) and numerosity (the psychological ability for appreciating the cardinality of sets, as previously described) is not enough to form number concepts. Here Russell’s (1910, 1920) definition of number as cardinality shared by sets of objects provides insight into the ethnographically observed behaviors. Number concepts are the categorization and abstraction of cardinal similarities between sets of objects placed into comparative arrangements; one (the reference set) is then used to represent the concepts (Overmann 2015a). Cardinal similarities become tangible, explicit, and manipulable when expressed materially (Coolidge and Overmann 2012; Malafouris 2010); they may (and typically, but not invariably, do) receive subsequent lexical names that further facilitate their social transactability (Overmann and Coolidge 2013). Such behavioral-material-psychological interactions would be unnecessary, and indeed rather strange, were numbers not explicitly constructed concepts.

Linguistic data show number concepts emerging in a way that is initially consistent with quantity perception (i.e., as indexed by lexical terms for subitizable numbers and quan-

tivity differentials; Menninger 1992), through the use of materiality (e.g., as the employment of the fingers informs the development of terms like “digit”), and in material combinations that vary across cultures. A term for “six,” for example, might be “three plus three” (Waimiri; Brazil), “five plus one” (Berik; Indonesia), “four plus two” (Mianmin; Papua New Guinea), or an unanalyzable term, like the one it has in English (Rosenfelder 2011). Even subitizable numbers may be formed by such productive grouping (as “three” might be “two plus one”), which suggests that, although they may undergo the process first as a function of their perceptual salience, subitizable quantities must nonetheless be realized as concepts in the same way that nonsubitizable numbers are (Overmann 2015a).

Rather than supporting innateness, these phenomena demonstrate that number concepts are explicitly constructed through the interactivity of human psychological, physiological, and behavioral abilities with materiality, a process affording both overall commonality and cultural variability. Given Brower’s general constructivist orientation, one might think he would have been sympathetic to the idea that this applies to numbers too.

In reply to Read, human development of number concepts is undoubtedly grounded in psychological abilities and behaviors shared by other primates (e.g., numerosity, categorizing, and pairing). Great apes (other than humans) can form categories of relations (sameness and difference in multiple elements; Christie and Gentner 2007). For example, AA and bbbb share the quality of containing identical elements, whereas Aa, %\$, and II share the quality of containing the same quantity of elements; the latter is a categorization of shared cardinality. For nonhuman great apes, forming relational categories is neither intuitive nor effortless, as it is for human children; it requires specific training, whereas it is spontaneous in humans, and even with training, it never appears to become as powerful as it does for humans (Christie and Gentner 2007; Thompson and Oden 2000). Nor do great apes appear to form concepts of cardinality, ordinality, or succession analogous to the concepts humans form, even if they can (given sufficient repetition) learn to match the cardinality of sets (within a fairly restricted range). Absent the profound material adaptation that characterizes the *Homo* lineage, other species appear to lack the capacity and motivation to engage materiality to form and use number concepts.

Russell’s (1910, 1920) definition of number allows for an analytical separation of the perceptual experience of quantity (the cardinality of sets of objects) from the shared cardinality apprehended by comparing sets of objects (numbers) and the development of relations between the members of a set of numbers (a counting sequence). As described in Overmann (2015a), the first is shared with other species and is prior to concepts of numbers, language for numbers, and language itself; the second is the realization of number concepts through the active manipulation of materiality, making them available for material representation and subsequent lexical

labeling; and the third is an elaboration gained through continued material engagement. However, where Russell might have envisioned number concepts as multilevel mental representations (i.e., sequestered in the brain and increasing in abstractness), number concepts, as considered here, are (1) constructed through interactivity among the psychological, behavioral, and material components of numerical cognition; (2) neither wholly external (real) or internal (innate/representational) but extended to include brain, body, and materiality; and (3) their content changes as the components interact and change through their interactivity.

The questions the cultural construction of number raises are these: How and why do number concepts vary across number systems, and why is the variability so difficult to discern? As “how they vary” was addressed previously in the reply to Sutliff, the “whys” are addressed here, in reverse order. Variability is obscured because comparative lexical numbers and numerical notations highlight the conceptual component shared across systems. In all number systems, the natural numbers are concepts of discrete quantity arranged in order of increasing magnitude. For example, “*ūnus, duo, trēs*” and “I, II, III” in Latin; “*ichi, ni, san*” and “一, 二, 三” in Japanese; and “*tip^na, tipnarip, bumrip*” and “right-hand thumb, right index finger, right middle finger” in Oksapmin all mean “one, two, three” and “1, 2, 3,” as they do in English. Yet the Roman numeral system famously lacked a zero and the algorithmic manipulability that it can facilitate, Japanese numbers are attached to the objects they enumerate with numeral classifiers in a language spoken by a highly numerate society, and Oksapmin numbers reach their limit at twenty-seven and are relatively unmanipulable, qualities that have changed under Western influence (Saxe 2012; further discussed below in my reply to Everett). By comparison, Western numbers have been elaborated over millennia to include concepts like zero, algorithmic manipulability, negativity, fractions, areas and lengths, axioms, proofs, set theory, and so on and to discard extensional conditions and restrictions (Bender and Beller 2011; Gouvêa 2008; Ifrah 1981, 2000; Selin 2000).

Conceptual variability is also obscured when discussions of number concepts focus on the symbolic nature of numerical notations (e.g., Sutliff 2016) rather than on the material nature of numerical representation more generally. This returns us to a primary reason that number concepts vary across systems: the materiality used to realize, represent, and manipulate them. Materiality is integral to concepts in general, where it acts to anchor and stabilize them (Hutchins 2005; Malafouris 2013). Change in material form thus affects conceptual content (as was suggested earlier for the equivalence-collection-entity transition). Materiality is important to number concepts in particular, where it performs functions like aiding visualization, decreasing demand on memory, and extending the manageable complexity of calculations (Dreyfus 1991; Kaput 1987). In this regard, however, numerical notations are only one material form among many, and they are

not even necessarily the preferred form for numerical manipulation, as the centuries-long argument over the relative merits of abaci and algorithms attests (Reynolds 1993; Stone 1972).

The material form(s) used to realize, represent, and manipulate number concepts informs how they are manipulated, how manipulable they become, and hence, the types of operations that can be performed with them. For example, number concepts represented with the fingers, notches on a tally, or body-counting are less likely to be associated with division and subtraction, operations that may require a more manipulable material form for their realization (Overmann 2015*b*). The enumeration of objects but not animate beings likely relates to the origins of number concepts in the manipulation of inanimate objects and the retention of anumeric strategies (e.g., spatial heuristics; see Butterworth and Reeve 2008; Butterworth, Reeve, and Reynolds 2011) for animate beings like divinities, humans, and herd animals, who might not be enumerated because they are known as individuals and are thus not objects. Numeral classifiers may represent a strategy for extending concepts of discrete cardinality to enumerated objects (i.e., sets with the property of cardinality) based on other properties like shape (Overmann 2015*a*). Finally, a number system's conceptual extensibility in terms of highest number counted is related to the material devices incorporated for enumeration (Overmann 2013, 2015*b*). Just as a number concept that has gained manipulability and numerical relations to other numbers differs qualitatively from one that has not, a number concept that disregards animacy or object properties or has no upper limit differs from ones that do.

Undoubtedly, numerical variability has a social aspect as well. As noted in Overmann (2015*a*:650), variability arises "from differing materiality, social needs, and the vagaries of invention." Social groups make different decisions for things like the material form(s) used for numbers, how quantities are productively grouped (cycles of 10 are common but hardly foreordained), what types of objects numbers are used with, who is authorized to know and apply them, and so on, and they have particular circumstances, needs, and priorities motivating the elaboration (or not) of numbers as a cognitive technology. Individuals socially transacting numbers within the group each contribute unique perceptions, comprehensions, decisions, preferences, and mistakes, which inform how social needs are adjudicated and number concepts invented, proliferated, and conventionalized.

However culturally constructed and dependent upon materiality a cognitive system like numbers might be, it nonetheless remains rooted in and governed by the abilities and capacities of the human brain and body. As Read (2016) notes, human numeracy has a strong biological (psychological, physiological, and behavioral) aspect. For example, the interaction between the parts of the parietal lobe that perceive quantity and "know" the fingers (respectively, the intraparietal sulcus and angular gyrus; reviewed in Coolidge and

Overmann 2012) likely explains why finger counting persists even in cultures with highly elaborated number systems (Overmann 2013). Other biological characteristics of human numeracy are acquired through social exposure to cultural materials, behaviors, and language, as is the case with the increasing linearization of the mental number line, an internal resource for estimating numerical magnitude that linearizes in response to numeracy enculturation (Dehaene et al. 2008; Piazza et al. 2013). The cognitive system for numbers is thus the result of interactivity between brains, bodies, and materiality (Malafouris 2013) over cultural spans of time (i.e., longer than ontogenetic and perhaps shorter than evolutionary times). Consequently, number concepts cannot be satisfactorily analyzed and understood as the sole product of any subset of these components.

In response to Everett's critique, being serious about investigating numerical origins means discarding the deeply entrenched assumption of linguistic primacy (the "without language, no numeracy" view [Hurford 1987:305] that Everett deems "prescient" [2013:81]), looking more deeply than broad sociomaterial contexts motivating numbers, and considering longer temporalities than neural reactions and ontogenetic development. It means incorporating the interactions where fingers bridge numerosity with materiality and manipulate it into new concept-generating stimuli and considering psychological-behavioral-material interactivity at temporal resolutions capable of discerning change at both systemic and componential levels. This means adopting a framework (e.g., material engagement theory; Malafouris 2013) that views cognition as comprised of interacting brains, bodies, and materiality and encompasses all of their contributions and characteristics.

In holistic context, there's nothing particularly "numberty" about language. Words lack the cardinality and ordinality of objects and the magnitude that yields numerical order. Yes, words can be counted and placed into sets, but this is not how they are meaningful (unlike numbers). Their grouping makes phrases and sentences, not productive cycles; their divisibility generates disconnected sounds, not new quantities related to an original. This is why, by itself, language generates no number systems. Numbers are numberty precisely because of their materiality: They are concepts of cardinality shared by sets of material objects (Russell 1910, 1920) that are explicated, represented, and elaborated by manipulating things like fingers and artifacts. Materiality is why number universals are so similar across number systems and so different from those of nonnumerical language. It is why numerical (but not nonnumerical) signs can be represented without phonetic values in fully glottographic writing and deciphered in otherwise untranslatable scripts (Overmann 2016).

Materiality precedes language in numerical cognition and pervades the full spectrum of its elaboration: Material objects have and mean quantity in virtue of what they are and what humans do with them; their quantity is comprehensible and

communicable without language but not without materiality to instantiate and represent it. Materiality is implicit in behavioral interactions with material objects, which is the mechanism whereby objects acquire meaning: Materiality shapes numerical cognition. Fingers and structures such as tallies impose linearity, and interaction with material linearity increases that of the mental number line. The material grouping necessitated by subitization informs numerical cycles, which language later absorbs as productive rules. Materiality governs the order in which arithmetical operations emerge (Overmann 2015b), enables the visualization that imparts principles like commutation beyond any ability of words (Gowers 2008), and facilitates numerical accessibility and manipulability in ways impossible with paragraphs and discourse (Sfard and Linchevski 1994).

Far from being a “questionable tactic” (Everett 2016, in this issue), placing language in a holistic context highlights its role as one of two interdependent ways of accessing numerical cognition (Clark 2006). Humans “think through concrete things, with words being a special case” that complements and explicates material engagement (Roepstorff 2008:2051), even as language assimilates material characteristics like productive grouping (patterning that emerges from “the dynamical interactions of brain, body, and world” rather than being innate to language; Gibbs and Clark 2012:242–243). Such interactions also potentialize numerical elaboration: in the precontact Oksapmin number system, body-counting and language both provided an ordinal sequence. Their mutual reinforcement, language’s lack of inherent numberiness, and the material form’s relative unmanipulability yielded a highly stable system. Upon incorporating materiality with differing manipulability, new numerical elaboration quickly resulted (Saxe 2012). Language approached nonholistically excludes the very possibility of gaining such insights.

What remain to be addressed are the inaccuracies of Everett’s (2016) critique. Not controlling for language-family relatedness is an appropriate statistical decision with restricted number systems data, where it “drastically reduce[s]” already small samples (Epps et al. 2012:45). Epps et al. found “subsistence practice is not itself a predictor of numeral system type” (2012:72), not a “clearly motivated correlation between numeric language and subsistence strategies” (Everett 2016); furthermore, considered globally rather than areally, the pattern they found is consistent with the one shown in Overmann (2015a). Refuting Atkinson’s (2011) phonemic-reduction hypothesis will not negate the other forms of serial foundation found along the migration arcs that peopled Australia and South America. Indeed, what makes the societies there so interesting is that maximal serial foundation and residential recency have reset their cultural accretion, which is likely why their number systems are still emerging and the sense in which their languages are young.

Regarding my finding that lexical numbers (LNs) temporally preceded and informed the development of grammatical number (GN), Everett asserts an LNs-GN dissociability that he claims renders it “problematic” (2016), even

asking how the two might be related. Against this, while LNs do not guarantee GN (as demonstrated by ~10% of languages), they are associated, because GN development is contingent on prior LNs development. Moreover, the development of GN implies possible differences in material engagement (i.e., with vs. without the hand) that are worth exploring further. Certainly, given the importance developmental psychologists place on GN in LNs acquisition, LNs temporal priority and GN contingency reveal the phylogeny-ontogeny relation to be complex. Given the hesitance of Mesopotamian scholars to embrace numerical signs as evidence for LNs, GN in sequence solidifies the certainty of prior LNs in early writing. Finally, LNs-GN geographical distribution reaffirms the restricted number systems in Australia and South America as emerging and, as such, providing potential insights into the role of both materiality and language in numerical cognition.

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