

Atlas holds apart sky and earth, figuratively filling a gap in the causal chain.



# 10 Causal Models for Life

**Physics, Life Science, Urbanism, Humanities**

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## Abstract

Creativity is found in artworks as well as in the colorful feathers of paradise birds. Diversity is found in ecosystems as well as in cities. Digital signals are found in nerve cells as well as in computer systems. Can causal models explain why life is at once creative and diverse, and why it uses digital systems? This present text builds on common empirical observations as well as long accumulated modeling experience to develop a unified framework for causal modeling that applies to all sciences including physics, biology, and cultural studies. In this framework, life can be diverse, creative, and digital all at once.

## Introduction

Life, in the physical sense of the word<sup>1</sup>, begins in quantum void space where virtual quarks, electrons, positrons, and other virtual particles are spontaneously created as pairs of matter and antimatter<sup>2</sup>. Some of these particles disappear almost instantly, yet other versions of the same elementary particles are observable, and hence real. Among them, two types of quark, "up" and "down", are almost always found together in groups of three. They form nucleons, i.e. protons and neutrons<sup>3</sup> that are heavy enough to gravitate towards each other. If you could follow their traces, you would find them tightly packed in inhospitable stellar objects where they create massive chain reactions from which they are stable enough to re-emerge, recombined into dozens of different atomic kernels<sup>4</sup>. In turn, the atomic kernels serve as alphabet for a world of countless chemical compounds<sup>5</sup>. Thus, the very first physical signs of life are initially virtual, then, they are quarks; yet, they eventually discover life's creativity and diversity.

**Physics**

**Chemistry**

Life, in the biological sense of the word, restarts or continues with two genetic base pairs, "AT" and "GC", that are chained together to form DNA. In neat sequences of three, these base pairs encode for amino acids and entire proteins<sup>6</sup>. Every biological organism synthesizes tens of thousands of proteins<sup>7</sup>. Evidently, the genetic base pairs succeed in facilitating enough creativity for life to continue.

**Biology**

And finally, life, in yet other senses of the word, restarts or continues in void space, this time in computer memory. "0" and "1", only two bits combined in sequences of eight, encode for letters<sup>8</sup>. You can use the letters to write prepositions, conjunctions, word roots, prefixes, suffixes, entire words, and entire stories<sup>9</sup>. Exchange the stories, and you get cultural life.

**Culture**

Cultural life is a great integration of all stories that people exchange; yet, it is much more than that. In addition to writing letters, you can also use the bits to encode for musical notes and for other things such as lines and colors that can be employed to draw architecture and build cities. Zettabytes of computer data have been written only in 2021 or are yet to come.

**Cities**

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<sup>1</sup> Bejan 2016.

<sup>2</sup> Feynman 1949.

<sup>3</sup> Rutherford 1920, Chadwick 1932.

<sup>4</sup> Eddington 1920, Hoyle 1946.

<sup>5</sup> Mendeleev 1869.

<sup>6</sup> Nirenberg et al. 1965.

<sup>7</sup> Wilkins 2009.

<sup>8</sup> Alvestrand 1998.

<sup>9</sup> Nowak & Komarova 2001.

These three or more stories about life are probably best told together not only because they are part of a great causal chain of life, but also because they share a common theme: A small number of elements (call them 0 and 1, AT and GC, or up and down) are creatively combined and recombined at higher and higher levels of complexity<sup>10</sup>. Through this activity, life gains increasing access to the energy that flows through it<sup>11</sup>, and it opens up worlds of limitless possibilities. This idea is graphically rendered in **Sketch 1**: Life repeatedly goes from digits to diversity, and back to digits.

**Digits, diversity**

If you wish, life is a very rewarding type of story. It begins with bunches of zeroes, yet these zeroes end up excellently playing their role in billions of successful things in which they are absolutely needed. I have studied life. I studied its digits, its creativity, and its diversity. I tested my thoughts in experiments; and I have written shorter and longer pieces about the results<sup>12</sup>.

The purpose of my present article is to review common empirical observations. Building on the observations as well as on long-accumulated modeling experience<sup>13</sup>, I develop a unified framework for causal modeling that applies to almost all sciences. Naturally, this framework describes first and foremost how life repeatedly and creatively proceeds from digits to diversity and back to digits.

**This article**

The text is organized in two main sections. The first of these sections reviews common empirical observations. The second section develops the causal framework. Overall, each section reads like a demonstration that is best witnessed in one go.

**Chapters**

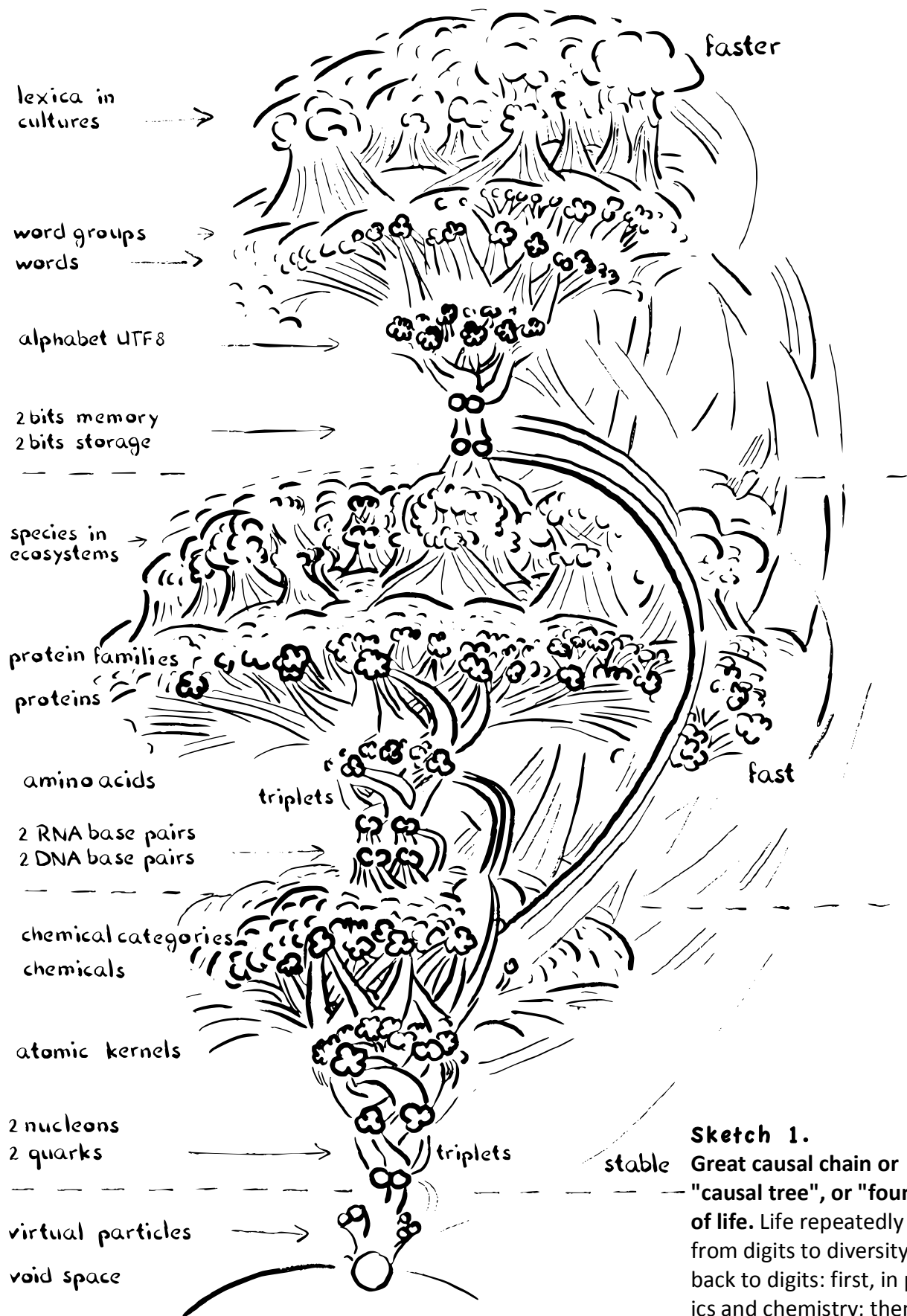
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<sup>10</sup> Baciú 2021.

<sup>11</sup> Bejan 1996.

<sup>12</sup> Baciú 2015-2021.

<sup>13</sup> Bejan 2000, Hofbauer & Sigmund 1998, Nowak 2006.



**Sketch 1.**  
Great causal chain or  
"causal tree", or "fountain"  
of life. Life repeatedly goes  
from digits to diversity and  
back to digits: first, in phys-  
ics and chemistry; then, in  
biology; and many more  
times in cities and cultures.



## Empirical observations

Consider the great causal chain or "causal tree of life" rendered in Sketch 1. As already mentioned in the introduction, the tree shows a pattern that repeats itself over and over again: Life begins, and it repeatedly restarts with few building blocks that it combines and recombines at increasing levels of sophistication. Along the way, life becomes increasingly diverse, and it gains increasing access to energy. At the uppermost levels of sophistication, ecosystems stand next to cities. Each of them is diverse, and each is competitive.

When thinking how life organizes itself in ecosystems, cities, and cultures, it may be surprising to return to the observation that, at the bottom of all this, one finds digits that are typically stable and that offer little room for flexibility. This insight can be gained from many observations across physics, biology, and culture.

For example, among all quarks known to exist, the up and down quarks are the only stable ones<sup>14</sup>. They are also not particularly versatile, given that they are never found in isolation<sup>15</sup>.

Similarly, DNA is a chemical that is more stable than RNA or proteins; and complex biological organisms have found ways to make DNA even more stable than it naturally is. They store it in better ways, and they correct errors that occasionally occur<sup>16</sup>—although the genetic code remained otherwise mostly unchanged.

Similarly, too, many contracts that people have chiseled in stone have survived several millennia, yet, they have survived only in their written form. Today, libraries and archives, where texts are stored, are typically quiet places with strict rules. Information is always the most stable and the safest when it is written down and stored. In computer systems, information on a hard drive is safer and more stable than it is in working memory.

The idea that such stable digits exist is not new. The ancient Greek philosopher Leucippus already theorized that the world consisted of indivisible elements he and his pupils called atoms<sup>17</sup>. The atoms were believed to be invisible, but once they were combined and recombined, they gave rise to all perceptible physical matter.

**Digits are stable:**

**quarks,**

**genetic base pairs,**

**bits.**

**Atomism.**

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<sup>14</sup> Particle decay.

<sup>15</sup> Due to color confinement.

<sup>16</sup> Lindahl, Modrich, Sankar 2015.

<sup>17</sup> Diels 1903.

It is uncertain to what extent the alphabet with its re-combinable letters inspired atomists in their philosophy. They could not have seen any atom, but they recombined letters on an everyday basis. What is certain that the atomic principle applies not only to the physical world but also to biology and culture where new types of atoms re-emerge to provide a certain stability necessary for life. What Leucippus called atoms, is known to science today as quarks, DNA base pairs, digits. In ancient philosophy, they also had many names. Initially, atomists believed that there were countless types of atoms, but logical thinking and accurate observation led them to realize that atoms existed in only very few distinct shapes<sup>18</sup>.

Next to the atomic principle, the school of Leucippus also brought the concept of void space, which symbolizes freedom to change, move, and to recombine small atomic units into larger units. This idea, too, applies to all fields of study. Anywhere in physics, biology, or culture, life combines and recombines smaller units into larger units; which is often described as creativity. Indeed, creativity is one of the most visible characteristics of life.

Life's creativity has been observed by many. Ancient Roman philosopher and poet Lucretius beautifully portrayed how life advances through species that procreate, each in its own surprising way. Like the atomists, Lucretius, too, held fast to the idea that void space was needed to explain creativity; and, by his time, atoms in void space were already imagined to "swerve" and combine and recombine themselves in unpredictable yet creative ways<sup>19</sup>.

Two millennia later, Alfred Russel Wallace, when he was catching butterflies and paradise birds in Indonesia, marveled much at variations that he observed in their wings and colors<sup>20</sup>. What he saw inspired him to propose the theory of evolution and to send it to London for publication. According to this theory, life spontaneously creates variants that move further and further away from the original type.

Wallace was not alone to marvel at variation. Francis Pascoe, one of his contacts in London, cataloged his and his colleagues' insects into species and occasional quasi-species<sup>21</sup>, recognizing,

**Creativity is freedom to combine and recombine:**

**species,**

**varieties,**

**quasi-species,**

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<sup>18</sup> Lucretius explains that otherwise some would be large enough to be seen.

<sup>19</sup> Greenblatt 2011.

<sup>20</sup> Wallace 1869.

<sup>21</sup> Pascoe 1866.

as proposed by Wallace<sup>22</sup>, that the concept of a "species" involves a great amount of variation.

Another century later, physical chemist and Nobel laureate Manfred Eigen further developed the idea of quasispecies, turning it into a causal model with solid mathematical foundations<sup>23</sup>. Eigen studied how bacteria combine and recombine genetic base pairs. He observed that, as a consequence of this activity, DNA was always found in populations of similar genetic variants. Many collaborators followed on his path. From this work, one can understand that life evolves in heterogeneous populations<sup>24</sup>. These heterogeneous populations have also been termed "mutant swarms", "mutant spectra", or "creative clouds".

**genetic quasispecies,**

**mutant swarms,  
spectra,**

**viral quasispecies,**

In 2021, COVID-19 variants vividly render the idea that life evolves in heterogeneous populations. There are so many clearly distinct variants, that many people lost track of them<sup>25</sup>. Even in the people's heads, the unit of evolution is not the individual variant but the group of interrelated variants.

The names of viruses are not the only names that people memorize and share. People can share many other ideas, emotions, thoughts, and stories. All of this information evolves in groups of related variants. The existence of such groups has long been known. Fashion designers call them fashions; trend scientists call them trends; literature professors call them literary genres; social scientists call them social axes; and artists and architects call them styles. The list of names could go on. All of the names stand for creative units of evolutionary selection, as they are present in human culture<sup>26</sup>. Each such creative unit is a group of interrelated variants. The unit of evolutionary selection is not the individual variant but the group of interrelated variants. The unit of selection is a heterogeneous, adaptable population.

**cultural quasispecies,  
fashions,  
social axes,  
trends,  
genres,  
styles.**

**Creative groups.**

The co-existence of so many units of evolutionary selection—and so many names for them, too—is yet another of those common observations about life that I wish to consider in this article. This co-existence is usually referred to as "diversity"<sup>27</sup>.

**Diversity is the co-  
existence of multiple  
creative groups:**

Countless researchers across physics, biology, and cultural studies have described and studied diversity. It is found as a recurring and fascinating theme in their texts that cover centuries of research and explorations.

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<sup>22</sup> Wallace 1858.

<sup>23</sup> Eigen 1971.

<sup>24</sup> Domingo & Schuster 2016.

<sup>25</sup> WHO 2021.

<sup>26</sup> Baciú 2020.

<sup>27</sup> Baciú 2020.

You can find diversity in the microscopically small "animalcula" captured by Antony van Leeuwenhoek in lakes and sea near Delft<sup>28</sup>. Looking through his hand-held microscope, it became evident to him that life has many more species than are visible to the naked eye. The immensely difficult endeavor undertaken by researchers such as Roy Anderson and Robert May around the turn of the 21st century to develop an analytical framework of how viruses, bacteria, and protozoans interact with their host populations illustrates how diverse life is at the scale of microbes<sup>29</sup>.

**microbes,**

Next to the discovery of microscopic worlds, traveling and exploration also opened the eyes of researchers onto how diverse life really is. You can find beautiful descriptions of diversity in the texts of Alexander von Humboldt, who re-introduced the term "cosmos" to English, German, and other languages. Among other, Humboldt was among the first Europeans to travel up the Amazon. His descriptions everywhere speak of biological diversity. Diversity is found even so deep in the jungle, when he wrote about the nightly tropical canopy and the exotic shrieks that ran through it. Small disturbances woke up species after species to create avalanches of more and more noise.<sup>30</sup>

**cosmos,**

**the Amazon.**

Today, large-scale mapping of biodiversity as practiced in the 21st century with remote sensing tools<sup>31</sup> substantiates that the Amazon is a center of biodiversity. If you look at such maps, you will probably notice that places where there is both water and sunshine light up brightly on the maps<sup>32</sup>. In such places, there is both high density and diversity.

**Ecosystems are both dense and diverse.**

This observation is no happenstance. It is one of the most common observations that one can make about diversity. Diversity radiates out of centers of density. Where there is loads of life, there also is loads of diversity<sup>33</sup>. The same rule also holds true for human culture. Cities are centers of diversity.

In the context of cities and human culture, the concept of diversity has fascinated people for millennia. For example, one can trace such fascination to the story of the Babylonian languages; and already back then, density and diversity were found together: Babylon not only was a great city with a great tower, but it also stunned people through its linguistic diversity.

**Dense cities have almost always been centers of diversity.**

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<sup>28</sup> Van Leeuwenhoek 1677.

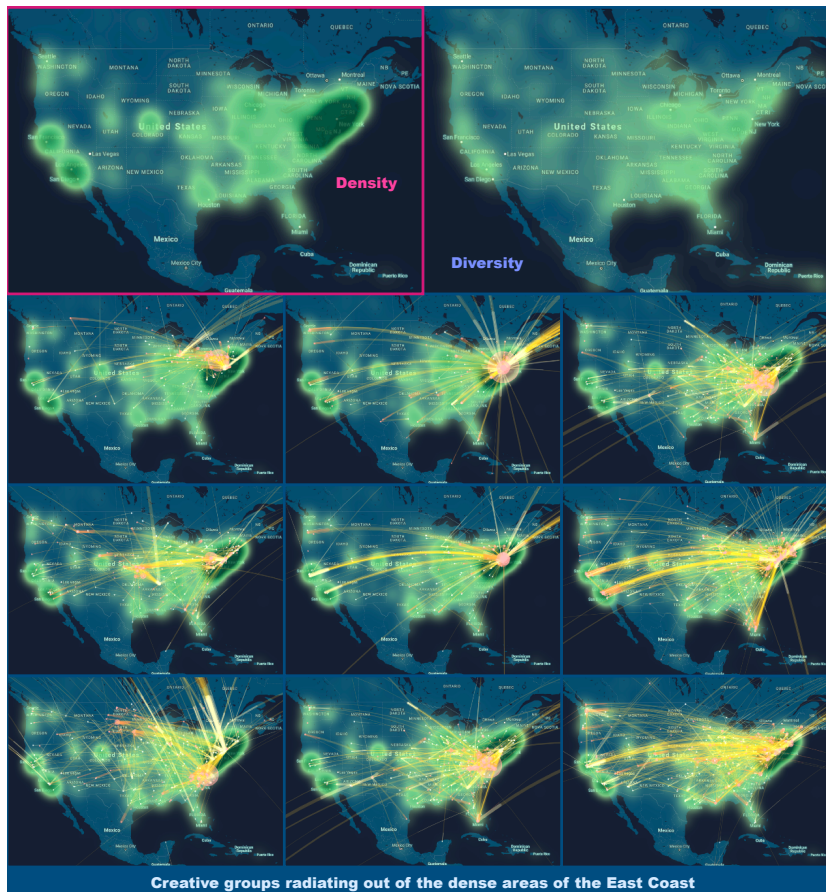
<sup>29</sup> Anderson & May 1992.

<sup>30</sup> Humboldt 1845, Wulf 2015.

<sup>31</sup> Gould 2000.

<sup>32</sup> Jenkins 2013.

<sup>33</sup> See also Annex 1.



**Figure 1.**  
Diversity radiates out of centers of density in urban environments.

The heat map labeled **Density** represents density of US-news items that contain the term "science". Green means dense. The densest area is around New York.

The data used in the density map can be analyzed and split into multiple creative groups of news. Nine such groups are displayed below. They were chosen because they all radiate out of the dense zone along the East Coast.

The map labeled **Diversity** is a diversity map created by calculating Simpson's diversity index  $1/D$  on 300 creative groups. The highest diversity found is in the centers of density.

Source: The density map together with the maps of the creative groups was first published in Baciú 2020. The diversity map was first published in Baciú & Birchall 2021. Interactive versions of both visuals are available online.

Today, many demographical studies show that diversity and density are commonly found together<sup>34</sup>. Together, density and diversity are the defining characteristics of cities<sup>35</sup>. National and cross-national boards define cities as centers of both density and diversity<sup>36</sup>. This observation is illustrated in **Figure 1**. In urban centers, there is high urban and cultural diversity<sup>37</sup>.

While diversity is a common phenomenon of life, the amount of diversity present in a system is rarely stable. Ernst Haeckel, who initially coined the term "ecology", graphically rendered this idea in his tree of life<sup>38</sup>. Haeckel's tree features ages of fishes, reptilians, and mammals. Each of the tree's major branches diversifies, and Haeckel also chose to show that diversity could decay. Haeckel's work was inspired by the paleontological record that speaks of massive historical extinction events. Today, biodiversity is again in a downward trend, as many researchers warn<sup>39</sup>.

<sup>34</sup> Vertovec et al. 2021.

<sup>35</sup> Baciú & Birchall 2021.

<sup>36</sup> Dijkstra & Poehlman 2012.

<sup>37</sup> Baciú & Della Pietra 2021, Baciú & Birchall 2021.

<sup>38</sup> Haeckel 1866.

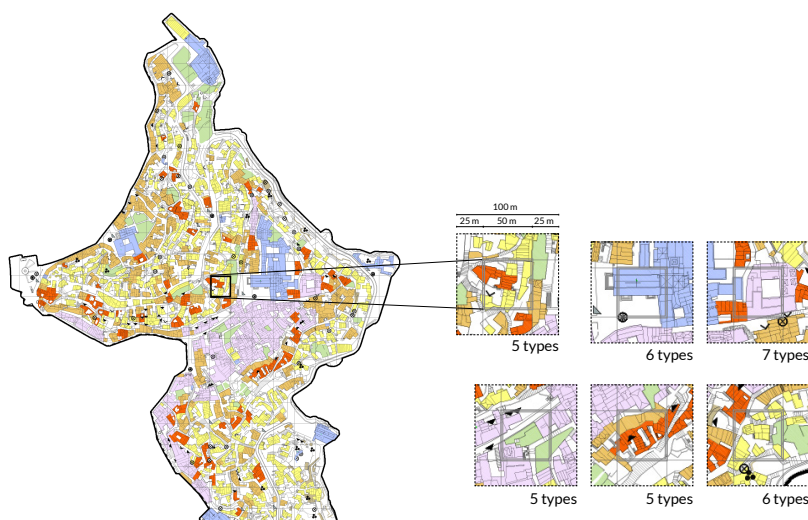
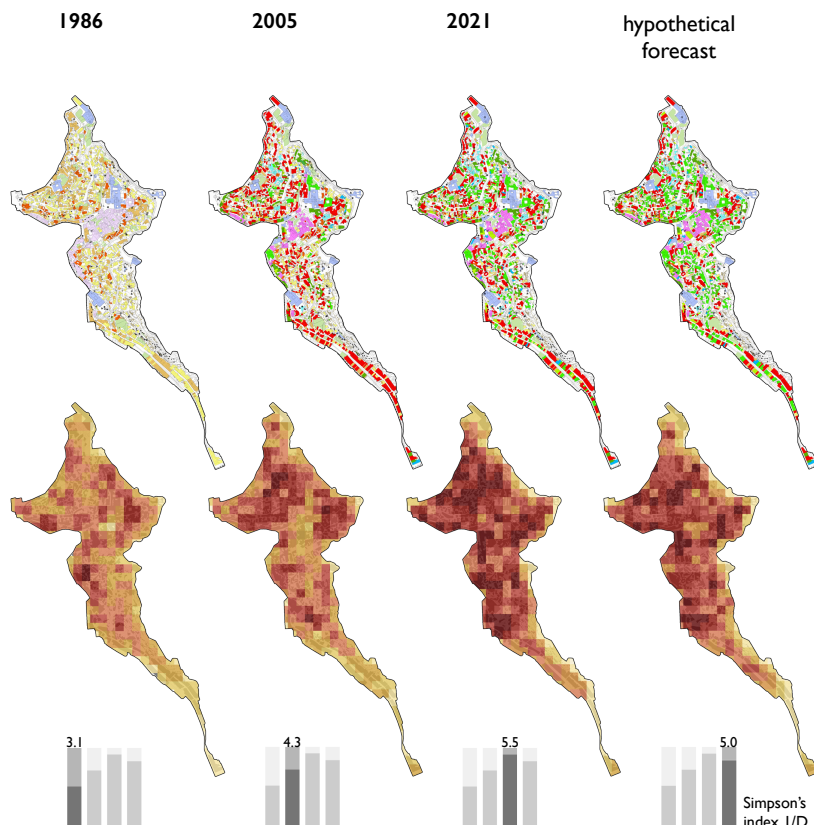
<sup>39</sup> Wilson 1999.

## Urban types

- Cultural institutions
- Palaces
- Courtyard houses
- Palaced houses
- Rural architecture
- Cave dwellings
- Renovated houses
- Renovated palaces
- Productive activities
- Artisan ateliers
- Hospitality (Hotels, B&Bs, room rentals)
- Restaurants, cafes, pubs
- Cultural associations
- ⊕ Relational space (Square)
- ⊕ Fountain
- ▲ Wine cellar
- ^ Oven
- ^ Oil mill
- ^ Wind mill
- ⊕ Rupestrian church

## Diversity analysis

- Urban zones
- Nr. of types 0 1 2 3 4 5 6 7 8
- Diversity ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
- Colour code ■ ■ ■ ■ ■



**Figure 2.**  
**Cycles of diversification.**

Life is diverse, and yet, diversity is not stable; it comes and goes; it goes through entire cycles of diversification. Such cycles can be observed in urban environments, for example in Sassi di Matera, UNESCO world heritage site, Italy.

The maps above show how buildings were used over the course of the last several decades. The row of maps below are diversity maps created based on the method shown in the lower left.

Simpson index 1/D has also been computed, and also shows that diversity is comes and goes.

Source: Baciu & Della Pietra 2021.

From these observations, one can understand that diversity often comes and goes, sometimes fast, other times very slowly. Diversification goes through both short and long cycles. These cycles are illustrated for urban space in **Figure 2**.

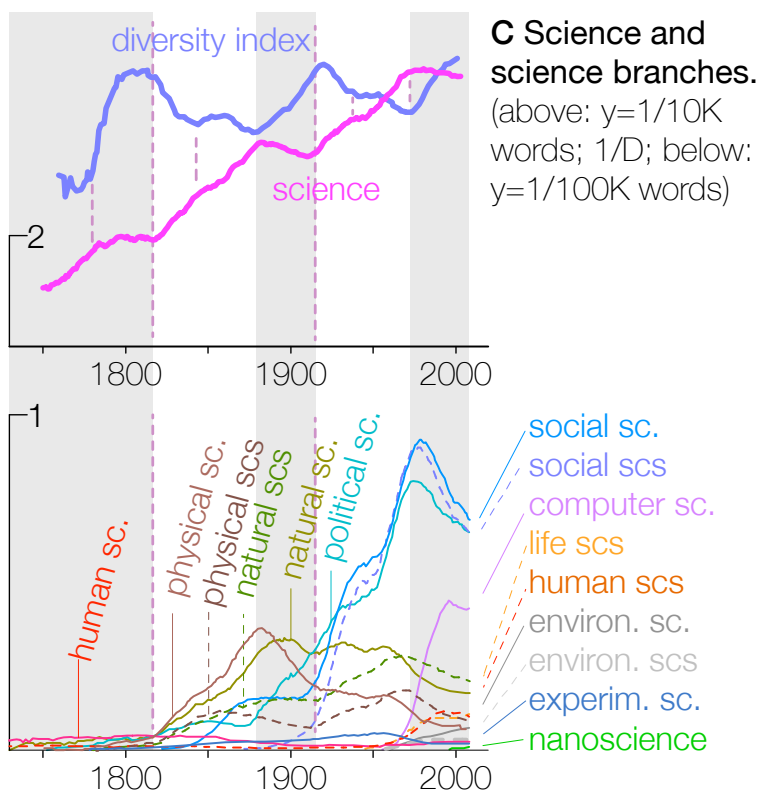
In human culture, too, diversity comes and goes. Culture passes through both ephemeral fashions and century-long cycles of

growth and reform. Fashions may pass within years, whereas long cycles of growth and reform can take much longer<sup>40</sup>.

On a much smaller time scale, changes in diversity are found again, for example, in the progression of persistent diseases. In such a case, the analysis of diversity is performed at the scale of a single human body. In each human body, viruses and cancer cells can diversify, which makes them more dangerous for the patient. This insight supported the development of effective medication for HIV<sup>41</sup>. Active research is underway for cancer<sup>42</sup>.

Cycles of growth and reform are often difficult to quantify ecology and microbiology, but in the study of human culture, big data makes it possible. For example, science and science branches grow and diversify.

Quantifying public interest in science as well as in any scientific field is straightforward. The attention that the public media gives to any science at any point in time can be quantified by looking up how many times different science fields are mentioned in the public media. The data are available online<sup>43</sup>. I have collected the data on science and science branches and calculated the diversity of science branches in **Figure 3**.



**Cycles of diversification:**

**in the human body,**

**in science history.**

**Figure 3.**  
**Cycles of diversification in science and scientific domains.**

Just like cities, culture goes through cycles of diversification. This phenomenon can be observed for example for "science" and different scientific fields.

Science and its many fields go through century-long phases of growth and reform, whereby diversity decays during growth, but it rebounds during reform.

Source: Baciú 2020.

<sup>40</sup> Baciú 2020.

<sup>41</sup> Nowak et al. 1991. Nowak & May 2000.

<sup>42</sup> Reiter et al. 2018.

<sup>43</sup> Michel et al. 2011.



Certainly, Figure 3 is about the perception of science in public media; yet, the observation that matters in this figure is the same as in Haeckel's tree of life. In Haeckel's tree, there were ages of fishes, reptilians, and mammals. Such "ages" are difficult to quantify in the paleontological record; yet, human culture also evolves in "ages". In the history of science, there were three major ages: the age of human science in the 18th century, the age of physical sciences in the 19th, and the age and social sciences in the 20th. We are now moving towards a century of life science and digital science. Public attention given to one branch of science comes at the cost of other branches. This can be seen from the "diversity index" in Figure 3. Great branches of science grow out of the diversity of science at large, only to eventually return into it.

It is uncertain to what extent Haeckel's tree was inspired at least partly from considerations of human history. It is certain that Haeckel, like the atomists, found much interest in human history. His books inspired artists, too. What is also evident is that changes in diversity are found everywhere. The question that remains open for the next section is whether such changes in diversity can be modeled in a unified framework of causality.

Next to all of these observations, a final observation should be relevant in our present context. We already know that life proceeds from digits to diversity. Yet, a close look at human culture can further reveal that the overall tree that links digits and diversity can grow both upward and downward. The tree of life becomes increasingly diverse and increasingly digital—and entire new levels may be created in between.

The evidence is at hand. Human culture did not begin with digital computers. Initially, humans invented pictograms and alphabets. Only later, they stumbled upon digital computers. The point that I am making is that digital computers work with fewer digits, and yet, they process and store both more information and more diverse information than the alphabet and the pictograms could ever have stored together.

In biological life, too, digitization has advanced. It is evident that DNA-replication and protein synthesis have not been invented overnight in one stroke of genius, but they evolved over considerable time.

Similarly, nerve cells exchange digital electrical signals through their axons. However, nerve cells existed before they developed axons, and the myelin sheets covering these axons have become

**Digital systems become increasingly digitized:**

**human digitization,**

**DNA,**

**nerve signals.**



increasingly efficient over the course of history<sup>44</sup>. Thus, in the brain's digital systems, too, digitization has advanced.

Taken together, some seven key empirical observations can frequently be made:

1) Life uses digits, which it creatively combines and recombines to open up worlds of limitless possibilities. Along the way, multiple levels of sophistication emerge.

2) The digits are few and stable.

3) When life combines and recombines the digits, it creates spectra of creativity that can be understood as units of evolutionary selection.

4) Diversity is a common phenomenon of life. It can be understood as the co-existence of many units of evolutionary selection.

5) High diversity is often associated with high density in physical space.

6) Diversity is unstable, going through short and long cycles. Growth is often associated with decreasing diversity.

7) Overall, life may become increasingly digitized. It can at once develop more efficient digits and offer more room for diversity.

With these key-observations in mind, let us now proceed to the development of a framework for causal modeling that explains these and other observations while also building on past modeling experience.

**Seven common observations:**

**multi-level tree,**

**digits,**

**creative groups,**

**diversity,**

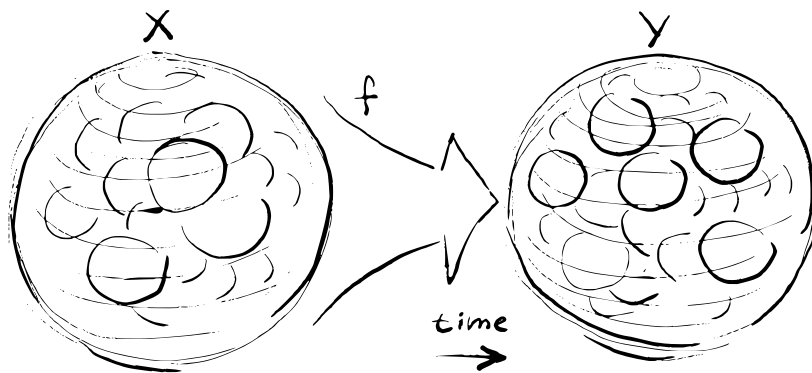
**diversity and density,**

**cycles of diversification,**

**the tree grows.**

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<sup>44</sup> Waxman & Koczis 1995.



The present is caused by the past .

$$Y = f(X).$$

### Causal Models

Consider the statement, "The present is caused by the past." This statement can also be formulated as mathematics; and one can also draw a flow diagram to graphically render the idea behind the mathematics. With such a flow diagram, the equation becomes a causal model. Equation and flow diagram are shown, playfully annotated, in **Model 1**.

Some of the choices made in Model 1 are helpful, but they are arbitrary. For example, by choosing the passive voice ("the present is caused by"), we can take into account that the existence of the present is always dependent on the past. By choosing the function notation, we make any connection between past and present possible. As long as the same set of causes always leads to the same set of outcomes, anything goes.

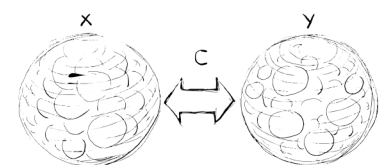
Causality can also be interpreted as a relation, in which case the same set of causes could lead once to this, another time to that set of outcomes. A notation of this interpretation is shown in the margin below Model 1.

### Model 1.

#### Causality as a function or relation.

Causality can be interpreted as a function or relation that continuously maps the past onto the present. This interpretation is the most general imaginable.

If a function is chosen, causes always have the same outcomes. If a relation is chosen (shown below), causes can have once this once that outcome.



Causality is the relation between  
 $C = X \times Y.$   
 past and present.

Causality is a key mode of reasoning in all sciences<sup>45</sup>, and it is hard to imagine a universe whatsoever in which causality does not work.

Already in void space, causality applies. Void space creates virtual particles. The particles occur in accordance with Heisenberg's uncertainty principle, which is an important pillar of quantum mechanics. In one of its multiple versions, the uncertainty principle states that one cannot know the exact amount of energy present in any system, and so, one knows that the amount of energy must fluctuate, which also applies to vacuum: The amount of energy present in vacuum fluctuates. Thus, vacuum and vacuum fluctuations together give birth to virtual particles<sup>46</sup>.

Causality begins not only in void space, but also in some of the earliest days of scientific reasoning. The statement "No thing comes out of nothing without cause" is the first principle on which already the aforementioned Lucretius based his atomic philosophy.

**Origins of causal thinking.**

Causal thinking began but did not stop here. Already Lucretius imagined different types of causes, different effects, and different causal mechanisms; and he mentioned that people were often stunned when they did not recognize a causal mechanism behind an effect they observed, but this did not mean that there was no causal mechanism and no cause.

From this mode of reasoning, one can already begin to understand that causal thinking, to a great extent, is a work of categorizing the world into causes, effects, and causal mechanisms. There can be many different causal mechanisms, many different causes, and many different effects. Causality is the link between them.

**Causality is system of classification into different types of causes:**

Let us take these ideas into account and write a new formula. This new formula will keep track of the causal mechanisms involved. On the other hand, when no causal mechanism is involved, there is no change to think about. Therefore, we can decide to focus less on those situations. We focus on change, and we focus on causal mechanisms that lead to change.

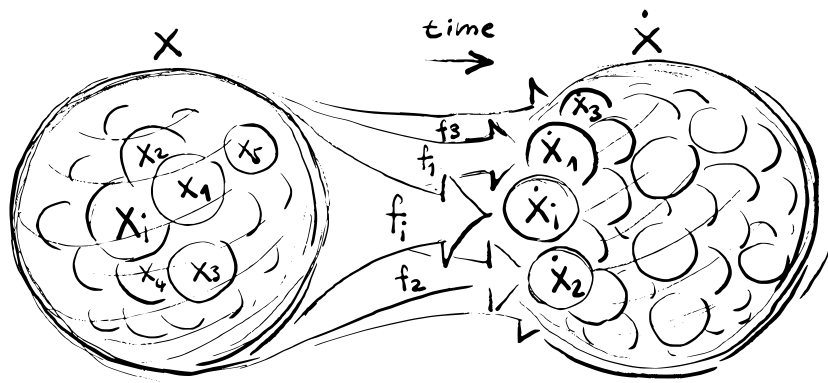
These considerations lead us to the statement "Anything new is caused in specific ways by the past." To complete the formula, we must also state what the past is, and according to our logic, the past can be interpreted as a set of causes. This formula is graphically rendered in **Model 2**.

**distinct causal mechanisms,**

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<sup>45</sup> Blalock 2017, Ryall & Bramson 2013, Ried 2016. Nowak & May 2000.

<sup>46</sup> Kane 2007.



Any thing new is caused in its own way by the past .

$$\dot{x}_i = f_i(x)$$

$$x = x_1, x_2, x_3 \dots x_n$$

The past can be divided into causes .

## Model 2.

A world of many causes,  
many causal mechanisms,  
and many effects.

Causality is a way to classify the world into many different causes, causal mechanisms, and effects. The causal mechanisms determine which causes have which effects. Not all causal mechanisms are known.

The causes that make up the past can be anything. They can be the species that meet in an ecosystem, or they can be those decisive causes that a historian of art would list in a monograph about an artwork.

When imagining causes and effects in art history, take the example of Michelangelo's frescos on the ceiling of the Sistine Chapel: Michelangelo is one of the causes that led to the creation of the fresco. Other causes are earlier paintings that inspired him to paint each of the scenes the way he did. For example Botticelli's "Birth of Venus" served as a source of inspiration for Michelangelo's "Creation of Adam". A very special causal mechanism is here at work. A famous artist is creatively engaging with an earlier artwork. The outcome is a new artwork.

Another, quite different causal mechanism is predation as found in an ecosystem. One of the causes involved in predation is the predator. The other is the prey. The predator eats the prey. The outcome is that the predator gains while the prey loses.

The starting point of this present article was the tree of life, and so the causes that we will often deal with are those things—species, molecules, ideas, and the like—that are nodes in the tree of life. The nodes in the tree of life are also nodes in our causal models. The causal mechanisms are the lines that connect the nodes.

causes in art history,

causes in ecology,

causes in  
the tree of life.

Our present causal model—i.e. the mathematical formula that we are now applying to link causes, causal mechanism, and effects—is of universal validity. There are always causes, effects, and causal mechanisms that are either known or unknown. The formula always applies.

**Universal validity of  
Model 2:**

This universal validity finds itself reflected in the formula's history. It may surprise readers that some of the earliest versions of this formula stem from the field of art and architectural history. An architect and historian of architecture wrote down such early versions in the 1850s<sup>47</sup>. His name was Gottfried Semper. He was on political exile in London, and he was preparing his notes for a lecture on architecture.

**London 1853,**

This small surprise vanishes when one realizes that architectural history was a truly great field around the time. Before 1800, most European cities were smaller than ancient Rome had been. Cities that wanted to grow could simply look back at the Romans. This situation changed over the course of the 19th century. Architects were forced to think out of the box. In parallel, news reached Europe about ancient Mayan cities in Central America<sup>48</sup>, and, Semper had just witnessed the "Great Exhibition of the Work of Industry of All Nations" held in a giant palace of glass and iron. Everything challenged the dogma that Rome offered the best example for everything. The idea of evolution re-emerged then, in parallel, in architecture and zoology.

**architecture,**

**zoology.**

Maybe Semper, whose name happens to be homonymous with the Latin word for "always", unconsciously searched for a formula that was always valid. He remained uncertain of his success, but he already began to categorize different types of causes that could, in principle, go into the formula. For example, the shape of a cup was determined in specific ways by the availability of materials, the skill of the craftsmen, and several other types of causes. Semper went on to use his formula to theorize about style. For him, style emerged from networks of causes that flowed back and forth and continuously reinforced each other. This conclusion is stunningly correct.

**Style as self-sustaining  
network of causal flows.**

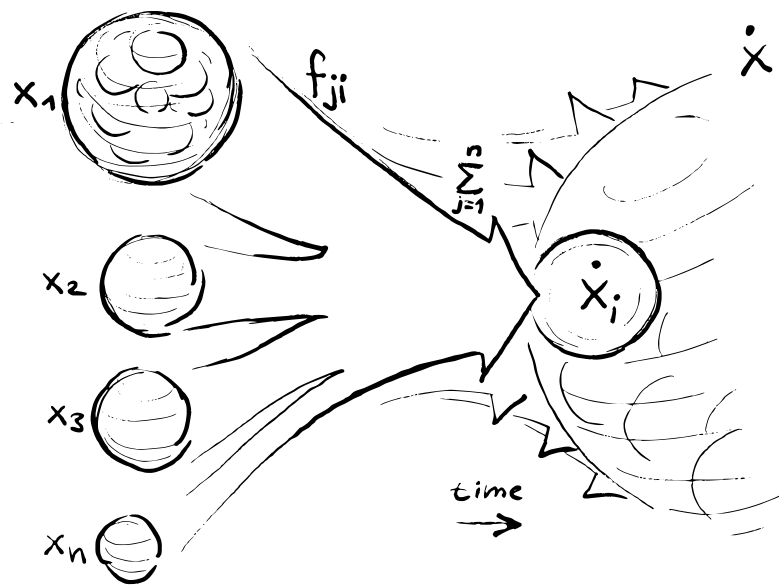
Semper's idea that causes can be classified into different types of causes brings us a step further, all way up to the point that we can make the formula operational. Here is how to proceed: When classifying the world into causes or groups thereof, one can choose to classify in such a way that the causes simply add up. (For example, the support received for a doctorate from parents and from grants likely adds up.) Making this small change in the always-valid formula, we arrive at **Model 3**.

**Causes that add up:**

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<sup>47</sup> Mallgrave 1983-96; Semper 1853-84.

<sup>48</sup> Catherwood 1844.



Any thing new is caused in specific ways by causes

$$\dot{x}_i = \sum_{j=1}^n f_{ji}(x_j) .$$

that add up.

### Model 3. Causes can simply add up.

The world can be subdivided into causes that add up. This division implies that causes flow in many directions between many causes and effects. Such causal flows can dissipate and disappear, but they can also create self-sustaining networks in which causal flows keep flowing, and they can adapt to new conditions. Because these networks keep going, we keep observing them in real life.

Applications of this reasoning include perturbation theory, variation-selection processes, and quasispecies evolution.

At closer inspection, the mathematical formula featured in Model 3 looks familiar: This is the basic skeleton of perturbation theory. The past is divided into causes that add up, and these causes can be listed by the magnitude of their impact onto the outcome.

Perturbation theory applies across all sciences. In physics it is used for example to model the interaction between virtual particles (those quarks, electrons and the like that emerge in void space) and real elementary particles<sup>49</sup>.

In biology and in the study of human culture, the quasispecies equation can be interpreted as perturbation theory. Thus, this formula (of causes that add up) could be a great fit to explain the processes of creativity that we have previously observed. As already described, a quasispecies of variants is a creative group.

A better understanding emerges when the causal model is studied with a basic example at hand. Let us imagine a small world in which there are only two causes. This choice makes it possible to draw all imaginable causal flows between them. Same as in Model 3, all the causal flows will simply add up.

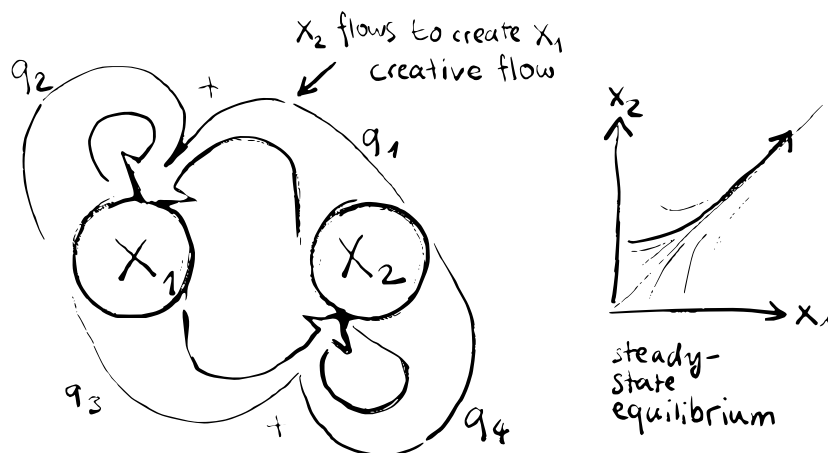
**perturbation theory,**

**creativity.**

**A small world of  
creativity:**

<sup>49</sup> Feynman 1949.

Writing out a mathematical formula for this simplified world, we receive the equations and the flow diagram graphically rendered in **Model 4**.



A first thing is created, grows, or shrinks due to  
 $\dot{X}_1 = q_1 X_2 + q_3 X_1$ ,  
 a second, and it grows or shrinks by itself.  
 The second thing is created, grows, or shrinks due to  
 $\dot{X}_2 = q_2 X_1 + q_4 X_2$ .  
 the first, and it grows or shrinks by itself.

Looking at the flow diagram, one can immediately recognize that X1 can "create" X2. Even if X2 disappears all together, X1 can create it from scratch.

Creativity, as defined here, has found many different terminologies across different sciences. Using other words, one can also say that X1 "generates" X2. In this case, we are in the realm of "generations" and "genetics".

Regardless of what vocabulary one chooses, fact is that X1 and X2 are held together by a creative flow. The creative flow makes their relationship an active one. If a large proportion of X1 flows into X2, then, obviously, they are closely related or closely associated, or any similar term you may choose. Without the causal flow, they could still be related, yet, their relatedness would have no physical effect.

Thus, it is evident that this model applies to things that are close to each other, things that are associated, or things that are held together by active, tight relationships. Occasionally, one finds the term "approximate reproduction" in the literature. Proximity is another word for closeness.

#### Model 4. A small world of creative flows.

Model 3 can be further analyzed by looking at a small world of only two causes. Causal flows can only go back and forth between these two causes.

Analysis of this smaller model suggests that creative flows bring causes together and make them related. The X1 supports X2, the closer they are considered to be related.

Empirically, creative flows occur between genetically related variants as well as between ideas that the human brain creatively associates.

The causal model makes it possible to estimate relatedness from causal flows, or vice versa to estimate causal flows from genetic or other types of relatedness or proximity.

Let us now apply this mathematical model. In genetics, the relatedness between two variants such as X1 and X2 can be determined by studying how much genetic material they share. Once the degree of relatedness has been determined between two or more genetic variants, it can go into the flow diagram, and one can predict the effects of the causal flows. One can predict how the variants spread in any imaginable situation. Experiments have demonstrated that these predictions hold true<sup>50</sup>.

**estimating causal flows  
from genetic relatedness,**

When studying how genetic variants spread, a common mode of action is often observed. As they spread, genetic variants grow in quasispecies of variants. Each quasispecies is a heterogeneous group of variants. Each group forms a unit of evolutionary selection. With each group, creative flows reinforce the group. As a consequence, life tends to grow towards steady-state equilibrium inside the group. This explains our most common observation about creativity: Life comes in self-sustaining creative groups.

**steady-state equilibrium  
in creative groups,**

The same model can also be applied in the humanities. In human culture, there is much data available that records how ideas spread. From these data, one can calculate backwards how large the creative flows between any two ideas must have been, and the value that is estimated can be taken as a measure for closeness between ideas<sup>51</sup>. Thus, from knowing how an idea spreads, one can know what this idea means. This information can be very valuable for example in online search engines.

**observing creative flows  
and estimating similarity  
from them.**

Beyond these two sample-cases, there exist many applications of this formula of creativity. Analytical solutions to the mathematics are particularly revealing. They can be used to explain how virus infections and cancer evolve in any individual patient. In addition, one can explain the evolution of viral mutant swarms during epidemics in entire populations of people. And one can explain how humans think about the viruses, too.

In research on human culture, computer algorithms that can be derived from Models 3-4 have been used to study social groups, fashions, genres, styles, and beyond. Ironically, researchers initially used this or that computer algorithm without knowing why it worked. Somehow, ways were found to demonstrate that the algorithms worked, although the researchers remained unable to develop valid causal models.

**Computer algorithms**

In my research, I not only developed the missing causal model<sup>52</sup>, I also showed that it had been very close at hand<sup>53</sup>: Already in

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<sup>50</sup> Domingo & Schuster 2016.

<sup>51</sup> Baciú 2016, 2017, 2018.

<sup>52</sup> Baciú 2017, 2018.

**Shannon**



the late 1940s, Claude Shannon defined communication as "exact or approximate reproduction of a message at a new place"<sup>54</sup>. Take this definition and imagine that many messages are reproduced in parallel. Some are exactly reproduced, some only approximately. Exact reproduction is one type of causal flows; approximate reproduction is the other type of causal flows. There are causal flows in many directions. All causal flows simply add up, and we have our general model of creativity (Model 3).

Had Shannon developed a model like this, he would have beaten Eigen and his collaborators by two decades. Instead, these life scientists were first to develop and apply such a model in the late 1960s and early 1970s. What brings their case very close to Shannon's is that Eigen knew of Shannon's theories of communication. In his first quasispecies article, he asked, "Can we use this theory to solve our problem?" He also used similar terminology: "digits", "exact reproduction"<sup>55</sup>. With this in mind, Eigen would have had some thirty years to contact Shannon and tell him that the quasispecies equation also applies to communication; and Shannon could have tested it. It all took longer...

**Eigen**

Thus, our present formula in which causes simply add up works well to explain how creativity flows. We predict and observe the emergence of creative groups that consist of things that are closely interrelated. Creativity flows back and forth in these groups, turning them into self-sustaining units of evolutionary selection.

**Modeling diversification**

Our next main goal is to model how diversity emerges. Diversity can be measured with Simpson's diversity index. This approach is taken for example in Figures 1-3 above.

**Defining diversity.**

The broad range of applications that has been found for Simpson's index by itself demonstrates how omnipresent diversity is, and how important it is to model it. Simpson's index was initially formulated in ecology and linguistics<sup>56</sup>, yet, it was independently formulated in economics<sup>57</sup>, and it later found applications in politics (there known as effective number of parties), in physics (there known as participation ratio), in virology<sup>58</sup>, in the humanities<sup>59</sup>, and in urbanism<sup>60</sup>.

**Simpson's index.**

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<sup>53</sup> Baciú 2019.

<sup>54</sup> Shannon 1948.

<sup>55</sup> Eigen 1971, Baciú 2019.

<sup>56</sup> Fisher 1943, Yule 1944, Simpson 1949.

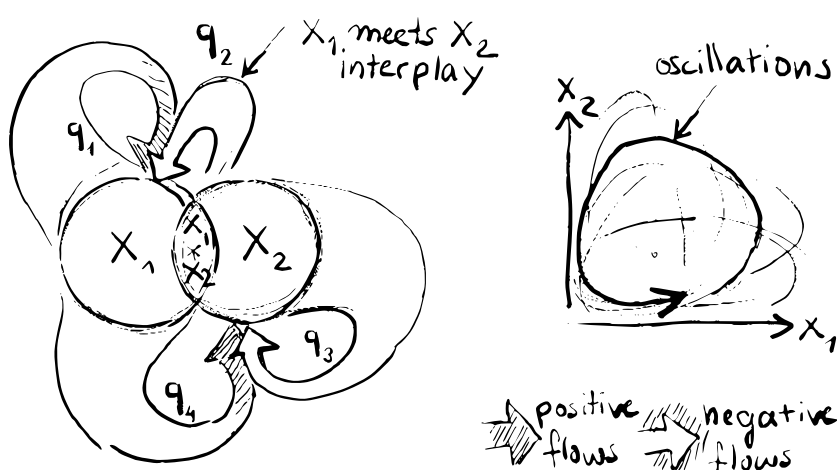
<sup>57</sup> Hirschmann 1945.

<sup>58</sup> Nowak et al. 1990.

<sup>59</sup> Baciú 2020.

As measured with Simpson's index, diversity is the probability that different things meet. The larger these things are, the likelier it is that they meet. Mathematically, the probability that they meet is obtained by multiplying their sizes. (For example speakers of English and speakers of Spanish are likely to meet because both of these languages are widely spoken.)

The creativity-equations just developed did not take into account meetings between  $X_1$  and  $X_2$ , but they can be modified to do so. Let us assume that meetings between  $X_1$  and  $X_2$  have an effect that flows back into  $X_1$ . This tiny change leads us from the basic model of creativity to a new model of interplay. I chose the term "interplay" to reflect that the model is designed to explain what happens when multiple parties meet or when multiple conditions are met at the same time. The equations together with a flow diagram are given in **Model 5**.



A first thing grows or shrinks  
 $\dot{x}_1 = q_1 x_1 x_2 + q_2 x_1$ ,  
 when it meets a second and by itself.  
 The second is created, grows, or shrinks due to  
 $\dot{x}_2 = q_3 x_1 + q_4 x_2$ .  
 the first, and it grows or shrinks by itself.

The model of interplay just created is not new. Mathematician and physical chemist Alfred Lotka formulated another version of it in the 1910s in an attempt to model an s-shaped growth curve<sup>61</sup>. Lotka's understanding was that s-shaped curves are a

<sup>60</sup> Baciu & Birchall 2021.

<sup>61</sup> Lotka 1910. See also Annex 2.

## Two parties meet

## Model of interplay

### Model 5. A small world of interplay.

Diversity is the probability that different things meet. This probability is calculated by multiplying the sizes of those things. Large, frequent things meet more often than small rare things. Once they meet, they can engage in interplay with each other. The nature of the interplay varies, but fact remains that they have met.

This small world of interplay is almost the same as the small world of creativity shown in Model 4. The difference is that one of the creative flows,  $q_2$ , was replaced with a causal flow that begins where  $X_1$  and  $X_2$  meet. Graphically, the meeting was interpreted as intersection between the circles of  $X_1$  and  $X_2$ .

## Lotka.

## S-curves

ubiquitous phenomenon of life. This has since been repeatedly reconfirmed<sup>62</sup>.

In essence, s-curves are a way to visualize that life grows first slow, then fast, and then slow again. Lotka's explanation went along the line that for life to grow, two conditions had to be met: first, life had to be successful in gaining access to resources needed for growth, and second, those resources had to be available in sufficient quantity. This led him to the conclusion that the s-shaped curve that he looked at could be explained very easily. First, life grows at a fixed rate, which initially results in exponential growth, but this growth leads to the depletion of resources, which again slows down the growth.

**Depletion of resources.**

Over time, it became evident that this idea of depletion of resources applied to many other settings. Lotka's prime examples became the interaction between prey and predators and between hosts and parasites. With only few changes, his model applied to these situations, too.

**Prey-predator interactions:**

In a way, all of the models that were later created in this line of thought deal with the same theme of "consumption": Fire consumes fuel; predators consume their prey; and parasites consume their hosts. Some do it from outside; others do it from within.

Eventually, when Lotka and his cohort of scientists experimented with these examples, they ended up with s-curve after s-curve, which turned the s-curves into waves. The prediction was that the sizes of the populations (prey and predators or parasites and hosts) wobble up and down in a wavelike manner: Populations of viruses come and go—and they can return, etc.

**oscillations,**

Thus, in this historical context, interplay turned out to have a very distinctive outcome. It led to s-shaped curves and to entire waves.

Along the way, one comes to realize that interplay often has negative impact, at least for one of the parties involved. This may have something to do with the fact that, if interplay were beneficial for all parties involved, these parties could try to find ways not to depend on the mere probability that they meet. They would stay together. Models have been created to account for such possibilities<sup>63</sup>.

**negative flows,**

The negative flows of interplay should not make it seem something undesirable. Although interplay may often be associated

**diversification,**

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<sup>62</sup> Bejan 2019, Bejan & Lorente 2012a.

<sup>63</sup> Pacheco et al. 2006.

with consumption and predation, the effects that it brings along can be beneficial and even crucial for ecosystems and cultures. Interplay can lead to diversification both in the short and in the long run: In ecosystems, prey-predator interactions are often responsible for keeping up the levels of diversity. Predators have an incentive to prey on the most abundant species, which gives rare species a chance to recover.

Examples for this phenomenon are well known from ecosystem management, for example in the United States. In the 19th century, when Yellowstone National Park was created, predators were often portrayed as a nuisance that had to be eliminated through hunting. However, the extinction of the wolves led to dramatic increase in a small number of species that were most successful at depleting the forests of resources. At a much later point, the re-introduction of wolves was a highly meaningful project that balanced out the ecosystem and helped it regain some of its original diversity. The wolves preyed on the most successful species, which provided other species a chance to recover.

Predation may also support diversification in through a more complex but equally important mechanism. Interplay can lead to diversification because species that are preyed upon find different techniques to escape predation, and those different techniques to escape may invite adaptations that are incompatible with each other. For example, some prey may acquire skills to hide in underground tunnels while other prey may get long hind legs that help them run away. However, the long legs don't fit into the tunnel. Thus, through this complex mechanism, predation chases the prey not only in physical space, but it also chases it into different evolutionary directions.

This latter conclusion can be supported with evidence obtained on a rather short time scale from the treatment of infections with antivirals or antibiotics. The antivirals and antibiotics are intended to kill viruses and bacteria. However, viruses mutate; bacteria acquire resistances<sup>64</sup>. If the treatment does not completely kill the viruses and bacteria, microbial diversity can increase as a consequence of the antiviral or antibiotic. Similarly, in the case of HIV, an infected person's immune system kills the virus, but the virus mutates, escapes, and diversifies, and virus diversity eventually overwhelms the capabilities of the immune system<sup>65</sup>.

The effects of interplay are maybe most vividly observable in human culture because there, we witness its effects on a daily

**formation of ecosystems,**

**escape in evolutionary space.**

**Antivirals as predators.**

**Immune system as predator.**

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<sup>64</sup> Nande & Hill 2021.

<sup>65</sup> Nowak et al. 1991.

basis. In a sense, humans consume news, culture, art, science the same way predators consume their prey. The predator species of human culture is the ability of humans to get bored of anything they encounter and consume. In technical terms, humans habituate against anything that they can sense. For example, the more ubiquitous an idea is, the easier it is for people to find it, to consume it, and to habituate against it. The idea becomes commonplace.

**Human habituation  
as predator.**

This interplay between ideas and habituation leads to fashion waves, and it also leads to diversification<sup>66</sup>. In human culture, diversity can form in the same two ways already observed in ecosystems: In ecosystems, diversity can form because predators have an incentive to prey on the most abundant species. Just the same way, habituation in human culture often occurs first and foremost in response to mainstreams because the mainstreams are ubiquitous, and so it is easy to get bored of them. They become commonplace most easily. This gives alternative cultures a chance to recover. Second, there is an incentive to actively search for novelty to escape the boredom. Mainstreams may evolve into different directions that surprise their audience.

**Diversification in  
human culture.**

Taken together, our basic model of interplay turns out to often be a model of consumption. The model explains s-shaped curves and entire waves. It is often a model in which predators consuming the largest prey population available. This recurrent pattern leads to diversification. (If predators consume the rarest prey available, diversity decays, as observed for example in illegal hunting of protected species in vulnerable habitats.)

From this vantage point of view, it is only another small logical step to understand why diversity is found most easily in the centers of density: Density makes it is easy to consume. If there is much of something, there's much to consume; and consumption leads to diversification. Thus, density indirectly leads to diversification.

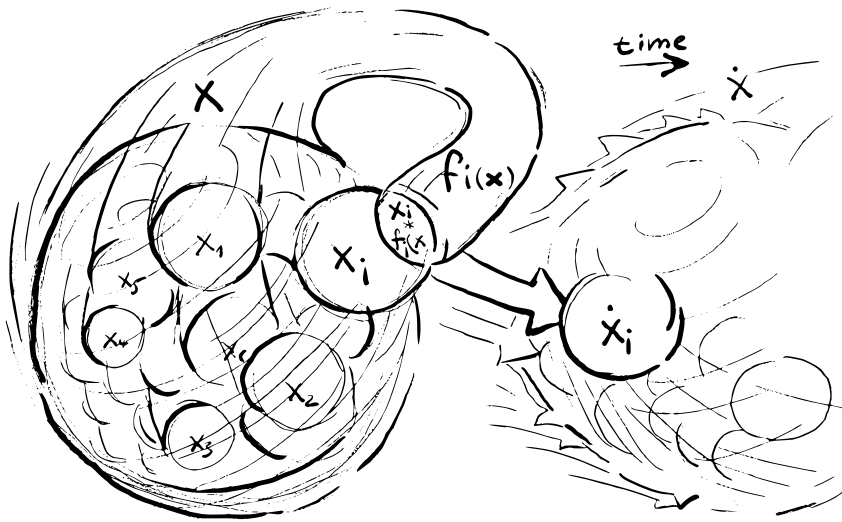
**Density brings along  
diversity.**

A general formula for interplay can be obtained by a sequence of trivial mathematical operations. If  $X_1$  finds itself in interplay, at the same time, with multiple other things such as  $X_2$  and  $X_3$ , we do not need to separately multiply  $X_1$  with  $f(X_2)$  and  $X_1$  with  $f(X_3)$ . We can simply add together  $f(X_2)$  and  $f(X_3)$ , and multiply this sum with  $X_1$  thereafter. This brings  $X_1$  out of the brackets and leads to the general model of interplay graphically rendered in **Model 6**.  $X_1$  is now in interplay with its entire environment. Anything that was creative in the previous model is now replaced by interplay.

**General model of  
interplay.**

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<sup>66</sup> Baciú 2018, 2019, 2020.



Any thing new is the product  
 $\dot{x}_i = x_i * f_i(x).$   
 of its own interplay with everything.

### Model 6. General model of interplay.

Generalizing model 5, a new model is obtained in which anything new emerges as product of its own interplay with the entire environment.

This model is the starting point for all compartmental models of epidemiology, ecology, and other sciences, and it is the most general model of game theory.

A first version of this model was developed by malaria scientist Ronald Ross in 1911.

This general formula is known to mathematical literature as "replicator equation"<sup>67</sup>. It serves as a unifying formula for all compartmental models in fields such as epidemiology<sup>68</sup>, ecology<sup>69</sup>, or humanities<sup>70</sup>. The same formula also is a heart-piece of evolutionary game theory<sup>71</sup>.

Historically, the origins of this formula go back to Lotka as well as to malaria scientist and Nobel laureate Ronald Ross, who, in 1911, formulated a theory of dependent happenings<sup>72</sup>. Ross believed that his theory and formulas applied to biology as well as culture. Later, Lotka jumped in and offered solutions to some of Ross's mathematical challenges<sup>73</sup>.

A closer look at this generic formula of interplay reveals its connections to our initial model of causality. Evidently, interplay is still a form of causality. The focus on interplay does not make it something altogether different. This conclusion is arrived at in **Sketch 2**.

**Interplay is a form of causality**

<sup>67</sup> Hofbauer & Sigmund 1998.

<sup>68</sup> Brauer et al. 2019.

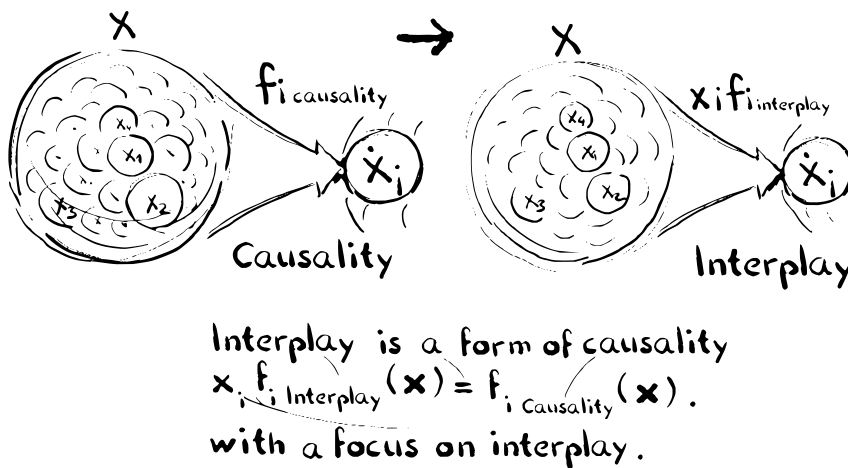
<sup>69</sup> Hastings & Gross 2012.

<sup>70</sup> Bejan & Lorente 2012b, Baciú 2018.

<sup>71</sup> Hofbauer & Sigmund 1998.

<sup>72</sup> Ross 1911, Smith 2012.

<sup>73</sup> Lotka 1912.



## Sketch 2. Causality and interplay.

Causality in general can be any function. Interplay is a more specific function. Thus, interplay is a special case. Causality is the general case.

Interplay is a mode of causality, but with a focus on parties that meet (or on preconditions that must be met for a certain outcome to be effectuated).

Nevertheless, the focus on interplay leads to one important limitation. Mathematically, the general formula of interplay (Model 6) is considered to be non-innovative. This means that, in an exclusive world in which there is only interplay, existing things can grow or shrink, but no new things can be created.

This conclusion can be reached intuitively. When we modeled interplay, we envisioned that its effects always flow back to one of the parties involved. This means that existing things are strengthened or weakened, but no new things are created along the way. Only parties that are already involved in interplay can benefit from it, according to this setup. In this narrowly defined form, interplay is as distinct as possible from creativity.

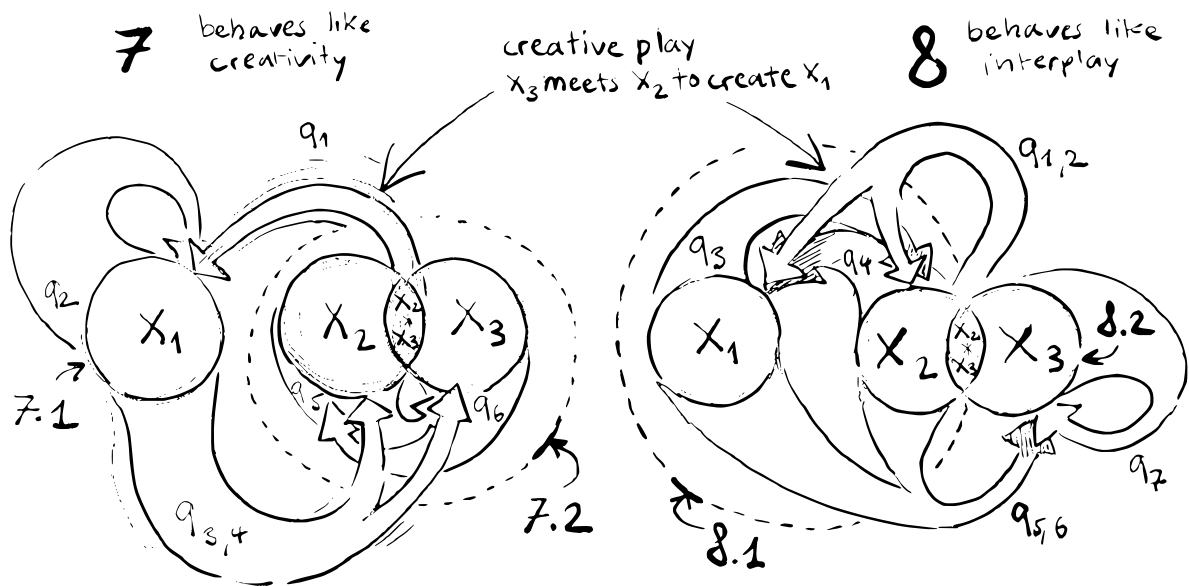
It is possible to change this setup in the general model of interplay. The effects of interplay can also be modeled to creatively flow away to a third party. This possibility opens up room for creativity and for creating new things. Parties that play can create new things. Let us call this causal flow "creative play".

## Creative play

While this type of causal flow can lead to the development of new causal models, these models behave in ways that we are already familiar with. The models tend to fall back into behaving like creativity or like interplay.

Consider **Models 7-8**. They represent two cases of creative play that are nearly the same. Both are small worlds. In each of them, there are three parties involved. The difference is that the three parties group together differently: In Model 7, the groups have a creative flow between them, whereas in Model 8, the two groups engage in interplay. Thus, these sample models of creative play lead once to creativity, once to interplay.<sup>74</sup>

<sup>74</sup> See also Annex 3.



(7.1)  
 One new thing is created and grows because  
 $\dot{x}_1 = q_1 x_2 x_3 + q_2 x_1$ ,  
 other things meet, and it grows by itself.  
 The other things are created and grow because of  
 $\dot{x}_{2,3} = q_{3,4} x_1 + q_{5,6} x_{3,2}$ .  
 the first and because of each other. (7.2)

(8.1)  
 Two new things are created and grow because of  
 $\dot{x}_{1,2} = q_{3,4} x_{2,1} + q_{1,2} x_2 x_3$ ,  
 each other but shrink when the second meets a third.  
 The third is created and grows because of  
 $\dot{x}_3 = q_{5,6} x_{1,2} + q_7 x_3$ .  
 the first two but shrinks by itself. (8.2)

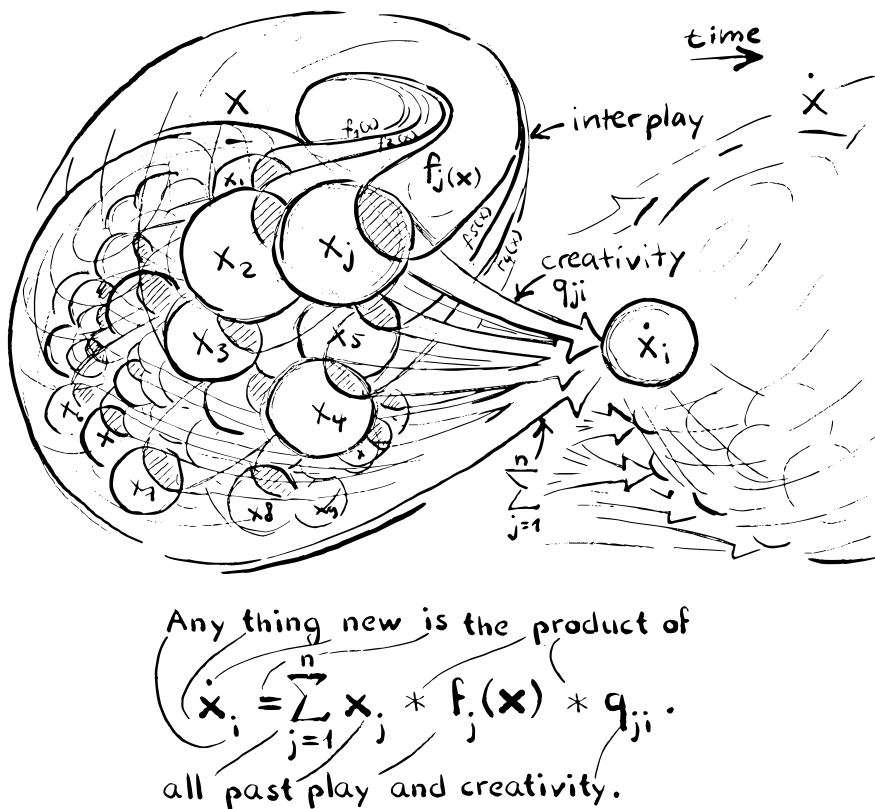
### Models 7-8. Creative play.

Interplay alone is not creative, but it can be turned into a creative flow if it is diverted from the place where it emerges.

Studying such flows in small worlds reveals that these worlds have properties of both creativity and of interplay: In Model 7, there are two overarching groups 7.1 and 7.2 with creativity between them. In Model 8, groups 8.1 and 8.2 engage in interplay.

A general model for creative play is obtained by uniting the general models of creativity and interplay. The resulting model can be taken to simply state, "Anything new is the product of all past creativity and play". The equation and the flow diagram are given in **Model 9**. A close inspection of this truly complex formula reveals that it is known to science under the name "replicator-mutator-equation".





### Model 9. General model of creative play.

Uniting the models of creativity and interplay, we receive the model of creative play. In this model, every party is in interplay with its entire environment, and this benefits (or has negative impact on) anything that exists or can be created.

The model has found applications in many different disciplines. The equation is also known to science as replicator-mutator-equation. It was developed by researchers who attempted to unite perturbation theory and game theory, and it is known to unify evolutionary dynamics.

Multiple researchers who searched for ways to unite perturbation theory with game theory have formulated the replicator-mutator-equation in the 1980s<sup>75</sup>, 1990s<sup>76</sup>, and early 2000s, eventually recognizing that their formula unifies evolutionary dynamics<sup>77</sup>. The equation found application across linguistics<sup>78</sup>, economics<sup>79</sup>, life science<sup>80</sup>, urbanism and humanities<sup>81</sup>, as well as other disciplines<sup>82</sup>.

The replicator-mutator-equation is very valuable even just because it puts together so many formulas into one, which results in a very complex formula, a "formula of formulas", maybe a mill of causality. In this one grand model, one can find at once creativity and play, and the model can fall back into behaving like creativity or like interplay as shown in **Sketch 3**.

<sup>75</sup> Haderl 1981.

<sup>76</sup> Bomze & Burger 1995.

<sup>77</sup> Page & Nowak 2002.

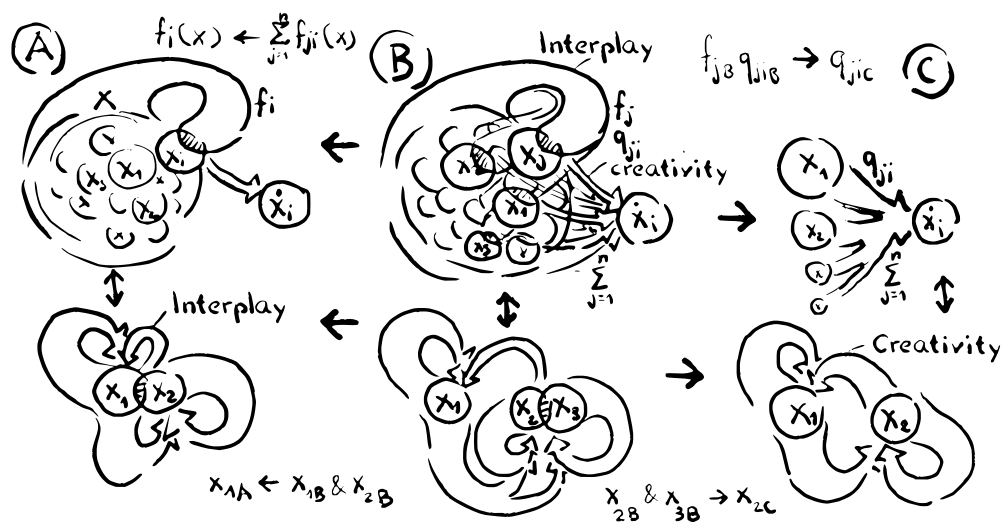
<sup>78</sup> Kauhanen 2020.

<sup>79</sup> Safarzynska & van den Bergh 2011.

<sup>80</sup> Garcia & Traulsen 2012.

<sup>81</sup> Baciú 2018, 2019, 2020, 2021; Baciú & Birchall 2021.

<sup>82</sup> Alfaro & Veruete 2020.



Creative play is a form of causality  

$$\sum_{j=1}^n q_{ji} x_j f_j(\mathbf{x}) \stackrel{\text{Creative Play}}{=} f_i(\mathbf{x}) \stackrel{\text{Causality}}{=} f_i(\mathbf{x}).$$
  
 with focus on creativity & interplay.

### Sketch 3. Model and models.

Creative play unites characteristics of interplay with those of creativity, and it can fall back in behaving like one or the other model.

Creative play becomes formally equivalent to creativity when  $f(\mathbf{x})$  is a linear function. Near equilibrium most functions look linear, which turns the model into a model of creativity.

On the other hand, when all  $x_i$  are distinct units of selection such that they are primarily in competition with each other, then there are no creative flows to be considered, and the model becomes a model of interplay.

When life passes through such a model of models, it has some three options to persist. The first option is very basic; yet, it is relevant because it is the starting point of life. Life starts when some things are stable enough to remain unchanged. This is the option taken by nucleons, base pairs, and digits. They simply stay. At this most basic stage, our most complex model is as simple as it can be; it lets everything unchanged.

Once there are digits, a second option emerges. The second option is to combine and recombine the digits to form creative groups. Within each group, causal flows go back and forth in ways that reinforce the group. We have already encountered these groups by their various names: quasispecies, mutant swarms, creative clouds, fashions, styles, or whichever term you favor for them. They are large, creative units. These units are not stable; yet, they persist because they always tend to return towards some steady-state equilibrium (or else they disappear). Near steady-state equilibrium, our complex model of models suddenly begins to simplify in another direction; it becomes the general model for creativity (as shown in Sketch 3 on the upper right).

Next and finally, once there are creative units, a third option emerges. The creative units can start to engage in interplay. A first creative group can consume a second and force it to invent

new ways to escape in physical as well as evolutionary space. Thus, the creative groups begin to actively explore the limitless possibilities that life opens to them. With interplay, we have reached a stage at which the entire system is out of equilibrium. The sizes of the creative units begin to grow and shrink. They oscillate. Existing hierarchies are constantly overturned. As part of this process, the groups diversify, which helps them search new possibilities in as many directions as possible. At this large scale of diversity, our complex model of models mostly simplifies to become the general model of interplay. (Technically, the model simplifies to become a model of interplay when the units that are in interplay are independent creative units, i.e. when creativity is found only within them as shown in Sketch 3 on the upper left.) We are now at the scale of ecosystems, cities, and cultures.

Together, these three modes of action that life has at its disposition to persist give the overall architecture of the tree of life. At the bottom, there are stable units that go through life processes unchanged, at intermediary levels, life creates larger units that are in equilibrium, and at the highest levels, there is interplay and instability. All levels work together, and so, over time, the tree of life tends to become increasingly digitized by building more stability at the bottom, more creativity in-between, and, at the top, more diversity that allows life to search in more evolutionary directions. Through these adjustments, life must gain increasing access to the energy that flows through it.

I have now explained most of the key empirical observations mentioned in the previous section. The last empirical observation requires some additional modeling that has long fascinated me. The replicator-mutator equation and the multi-level tree of life bring in the idea that life evolves at multiple levels of sophistication.

Developing a multi-level model of life that features categories with subcategories nested in them (or causes and sub-causes) yields some final insights for our framework. Let us motivate this example with a topic that fits in this article. Let us model the history of scientific fields (Figure 3, previous section).

Life is studied in multiple scientific fields. Take the examples of physical science, natural science, human science, etc. Each has its own focus, and yet, they are nevertheless all part of science at large. Thus, our overarching category is science at large; the subcategories are the various scientific fields.

The larger category of science and the smaller subcategories of sciences interact in ways that individuals can often feel and experience. For example, it is customary to become bored of an

**Three main modes to pass the test of evolutionary dynamics;**

**three main types of level in the multi-level tree of life.**

**Multi-level models.**

**Science and scientific domains as example:**

experiment that one has run too many times. One will probably not want to see it again. However, when the same-old experiment shows up in a different scientific field, in any field, and when it opens up an entirely new path in that field, one might suddenly regain interest. If one is a scientist, one might even return to the files that one had once put aside. Thus, one can lose interest in the small scientific field that one is working in, but then, the safety ring comes science at large.

Let us now complete our causal model. In terms of neuroscience, we can say that people habituate against stimuli that come from any specific field of science that they hear about. Those who have habituated will tend to read and write less about the field in question. However, the people who have habituated to a field can nevertheless happen to encounter news about any other scientific field, which can, to some smaller extent, disrupt habituation and sensitize them again. They can re-gain interest.

**habituation and sensitization,**

To complete the model, let us also take into account that new scientific fields can emerge over time, and, finally, we shall also consider that somebody can habituate to all sciences whatsoever.

**creativity.**

The resulting model of interplay between categories and subcategories is graphically rendered in **Model 10**. On close inspection, this same model was used to simulate the interplay between immune system and viruses in persistent diseases<sup>83</sup>.

The insights obtained from this model are not completely new, and yet, they explain the last empirical observation on our list.

First, the model leads to short-range waves as customary for interplay. However, as soon as a causal flow is integrated that links categories and subcategories (labeled  $q_c$  in the figure), the short-range waves are overrun by longer-range waves. Intuitively, one can say that the small subcategories are responsible for the short-range waves, while the large categories are responsible for the long-range waves.

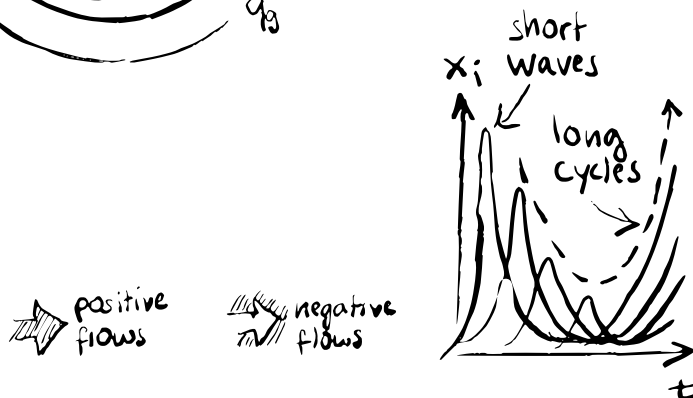
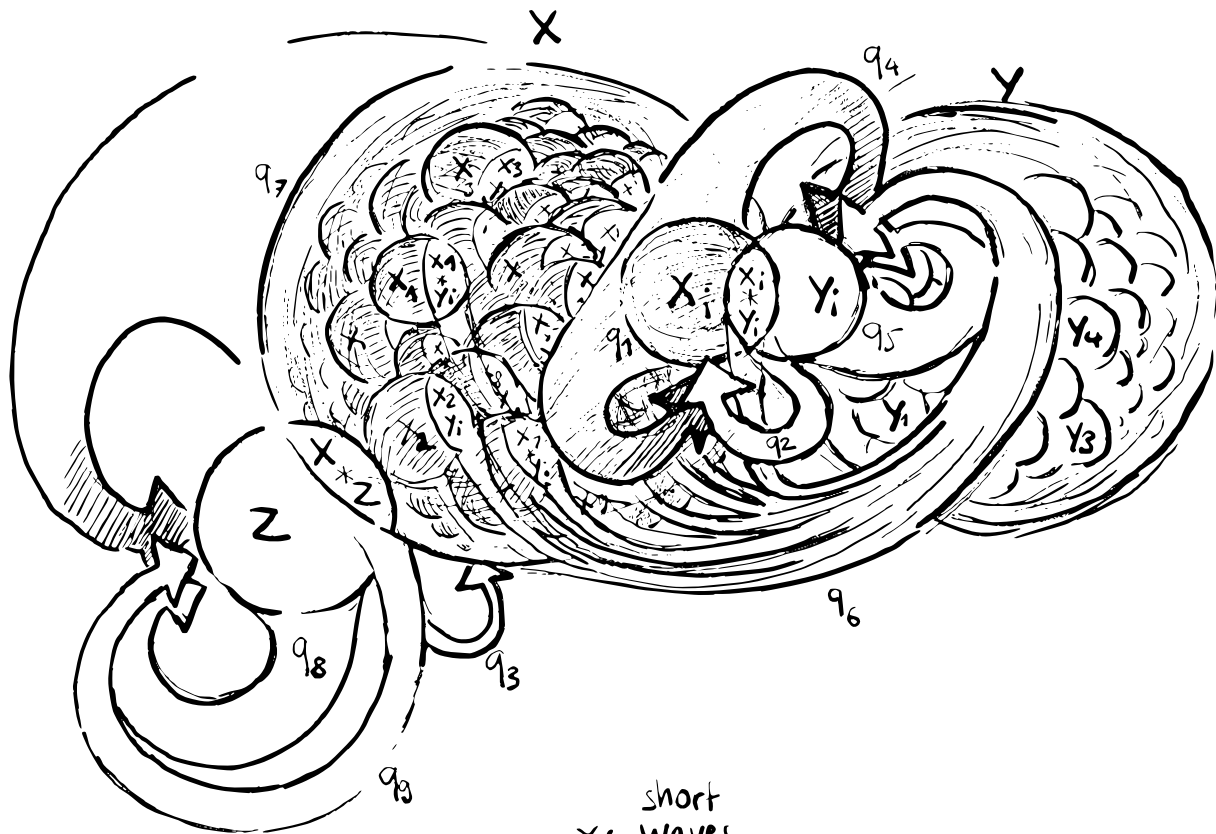
**Short and long waves**

This model behavior that includes short and long waves explains our empirical observations in the history of science as well cities. In both of these cases, we found long waves that can take up to a century to pass, which makes them much longer than fashions. Characteristic for the long waves is also that they push the diversity index up and down: When the wave grows, the diversity index goes down, when the wave vanes, the diversity index returns to its initial level, and this is what has been observed while quantifying the public attention on science and scientific fields in Figure 3 of the previous section.

**Cycles of diversification**

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<sup>83</sup> Nowak & May 2000.



### Model 10. Multi-level interplay.

Human culture often comes in overarching categories and smaller subcategories. This hierarchy results in complex models that predict short fashions as well as longer cycles of growth and reform.

X represents the overarching category (science at large). Z is habituation or boredom against the overarching category. X1,2,3,...n are the subcategories (scientific fields). Y1,2,3,...n are habituation or boredom against the subcategories. Shaded are positive flows. White are negative flows.

Any variant in category X grows by itself, yet it shrinks

when it meets its matching pair in Y, or when it meets Z.

Matching pairs in Y grow due to the variants in X,

yet they shrink by themselves and when meeting any of X.

Category Z grows due to any of X,

yet shrinks by itself and when meeting any of X.

$$X = \sum_{i=1}^n x_i$$
  
X is the sum of all of its variants.

## **Conclusion**

Quantum physicists study the smallest detectable, first signs of life. Their observations lead seamlessly into physical chemistry, chemistry, genetics, and biology. All of these sciences study life's creativity and diversity.

Biology, by definition, is a "science of life." The Weldon committee, which awards one of the highest recognitions in biology, describes the field as "including zoology, botany, anthropology, sociology, psychology, and medical science." Yet, life does not stop with these sciences.

Life's creativity and diversity is also studied in the humanities and the sciences of the city. They, too, contribute their own bits and pieces to the study of the great causal tree of life that serves as starting point for this article.

Thus, life is a phenomenon that unites physics with chemistry and biology, as well as with the humanities and sciences of the city. All of these fields deal with life's creativity and diversity, and, in all of these fields, life poses scientific questions that are answered with causal models.

Along the way, causality becomes not only a great, unified theory of life, but also a way to classify the world into different causes, different effects, and different causal processes. The question that emerges at this point is how different models in different disciplines are related to each other.

In an attempt to answer this question, I developed a unified framework for causal modeling that unites key empirical insights as well as key modeling strategies that have come together for a very long time.

I began by stating that the present is caused by the past. Translating this statement into mathematics led me to a universally valid model of causality. Mathematically, causality is interpreted as a function or a relation that continuously maps the past onto the present.

Within this most general model, there were two main ways to handle causes. Some causes simply add up, while others are better multiplied.

Causes that add up are often related to each other. Such causes flow into each other and oftentimes form integrated units that can adapt to existing as well as entirely new conditions. I called this model a model of "creativity" because it can create new forms of life.

**Physical sciences**

**Biological sciences**

**Humanities and sciences of the city**

**Causal models are everywhere**

**Great, unified theory**

**Many models**

**Unified modeling framework**

**Causality as function or relation**

**Additions and multiplications**

**Model of creativity**

On the other hand, the physical meaning of multiplication is that of multiple parties that meet (or of multiple preconditions that must be met) for a certain causal mechanism to take effect. Such causes are in interplay with each other, and they can support the evolution of diversity. I called this model a model of "interplay". Interplay does not create by itself, but it can create new forms of life when it is supported by creativity.

These two sub-models—creativity and interplay—are less universal than the general model of causality, and yet they apply to all previously mentioned domains of science from physics to the humanities.

Creativity is the backbone of perturbation theory and quasispecies evolution, and already in the 19th century, architectural style was studied with a related model. On the other hand, interplay is used in evolutionary game theory as well as in compartmental models that are widespread in epidemiology, ecology, and in the humanities.

Re-uniting these models into a "model of models" made it possible to identify three ways in which life can persist:

First, life begins with basic building blocks that can remain stable over considerable evolutionary time. Obviously, as long as these building blocks do not change, they stay. Ancient philosophers theorized that such building blocks exist and called them atoms. In the various scientific domains, they are today known as nucleons, genetic base pairs, or digits.

Once such building blocks of life make their appearance, creativity can become active, and it combines and recombines them in various creative ways. Through this process, creativity leads to the evolution of larger units of selection. These units are not as stable as the basic building blocks, but they have the advantage that they are adaptable and creative. They can discover new possibilities that life offers them. Such creative units are known as quasispecies, mutant swarms, brain circuits, creative clouds, fashions, genres, or styles. So many names have been coined, that they are too many to list.

Finally, interplay sets in and opens the path to diversification. During interplay, multiple creative units interact, and they can chase each other through both physical and evolutionary space. They search in many, diverse directions; and they discover. The overall ensembles of such creative units that engage in interplay with each other are known to science as ecosystems, cities, cultures, or the like.

## **Model of interplay**

## **Apply to all sciences**

## **Applications**

## **Three ways to persist**

## **Digits**

## **Creative units**

## **Interplay**

These three ways that life has to persist are the main components for the tree of life: The tree begins with basic building blocks, continuing to creativity and to interplay.

**Three levels in tree of life**

Eventually, interplay and creativity together lead to the discovery of such sophisticated forms of life that these freshly discovered forms of life are able to invent and run their own tree of life. (Evolution leads to humans, and humans are smart enough to invent new digital systems entirely for their own purposes.) In consequence, the tree of life keeps growing. It keeps repeatedly proceeding from digits to diversity, and back to digits. A graphical version of this interpretation of the tree of life is rendered in **Sketch 4**.

**The way back to digits**

Many distinctions between creativity and interplay are already visible in the tree of life; yet, they can also be represented in multiple other ways.

**Distinctive outcomes of creativity and interplay**

In one such approach, causal models can be represented as vectors and matrices. A set of causes becomes a vector while the causal flow diagram becomes the matrix. The vector points in a direction that unites all causes. The matrix is simply another, less graphical version of the flow diagram. It can be imagined as large table filled with information that says how to map the past onto the present.

**Vector-matrix representation**

In the vector-matrix representation, creativity can behave in a very distinct way. It often pulls at the vector in one direction. The vector is stretched and begins pointing into the direction in which it is pulled. If the vector keeps growing, it also keeps pointing into the direction in which it grows. Thus, its tip draws an increasingly straight line. The physical meaning of this straight growth is that causal flows search equilibrium. Mathematically, such a matrix has eigenvalues representable as real numbers.

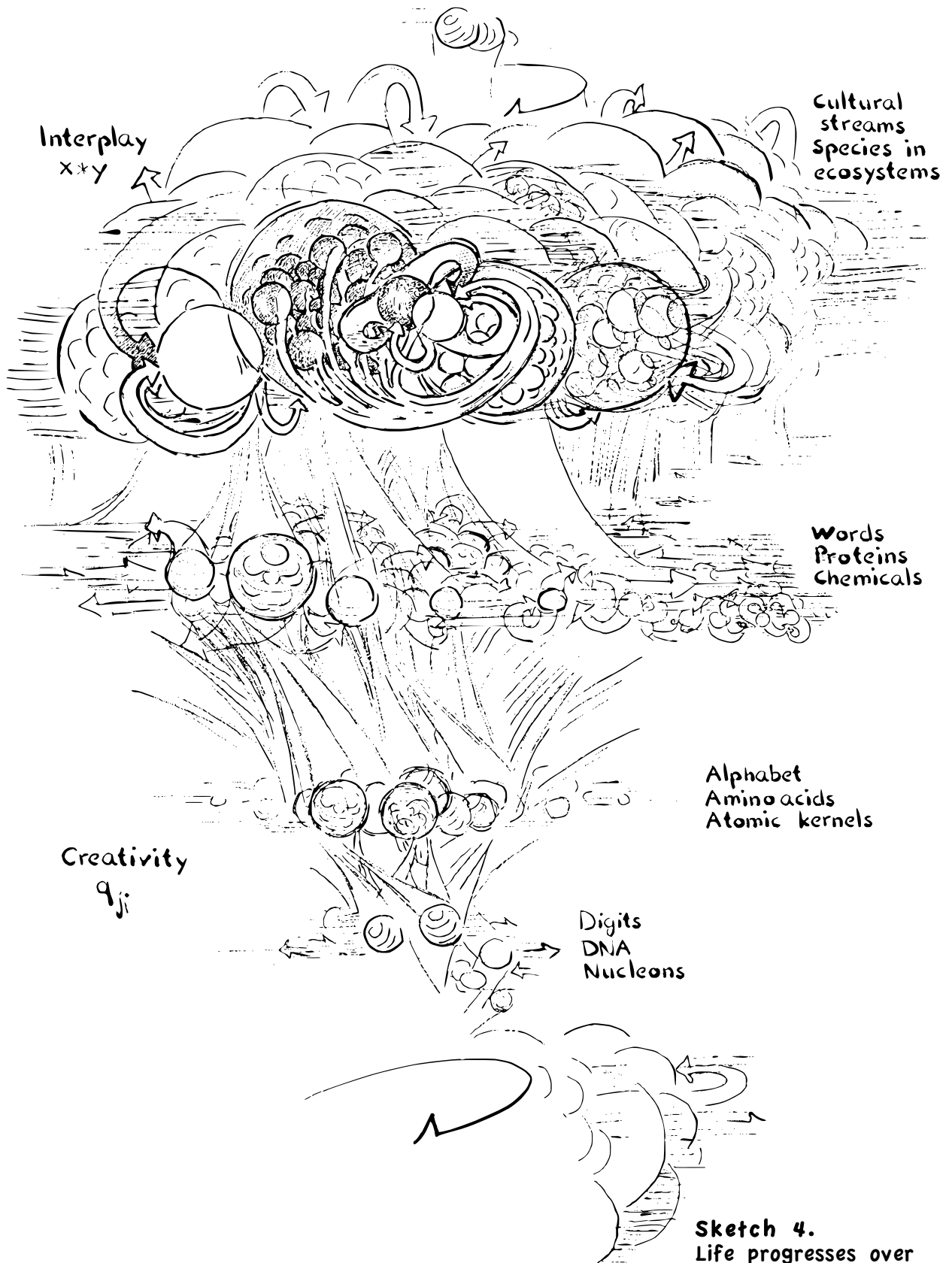
**Straight lines  
Real eigenvalues**

In contrast, curved lines result from matrices that have eigenvalues representable only as complex numbers. These matrices push and pull at the vectors in ways that make them spin. The physical meaning of this rotation is that existing hierarchies are constantly overthrown. The tips of the vectors draw curves, spirals, or waves. Such matrices are typically obtained from interplay models.

**Curved lines  
Complex eigenvalues**

Thus, one can say that creativity comes with equilibrium and with growth, straight lines, and real eigenvalues, whereas interplay comes with instability, re-orientation, curves, and complex eigenvalues.





**Sketch 4.**  
Life progresses over multiple levels of sophistication from digits over creativity to interplay and back to digits.

This mathematical perspective says something quite profound about causality and about the relationship between creativity and interplay. Straight lines are a special case of curved lines. Real numbers are a special case of complex numbers. No curves are more curved than curves. No numbers have yet been described that are more complex than the complex numbers.

## **Lines and numbers**

Real numbers and complex numbers are omnipresent. Straight lines and curved lines are omnipresent. Creativity and interplay are omnipresent. Nevertheless, scientists and humanists may observe forms of life that persist and prevail, and yet, they cannot be modeled either as creativity or as interplay.

## **Persisting in different ways**

Such observation could still fit in the basic model of causality that we began with. Yet, they would somehow not fit into any more specialized model of creativity or play.

If such observations are made, they could raise an interesting set of questions: How would we have to model such observations? How are the resulting models related to the basic model of causality? What types of lines and numbers would these models lead to?

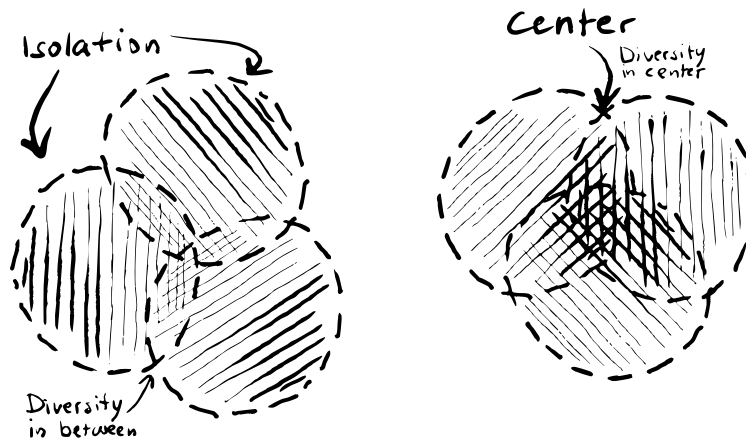
## **Outlook**

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## Evolutionary & physical space

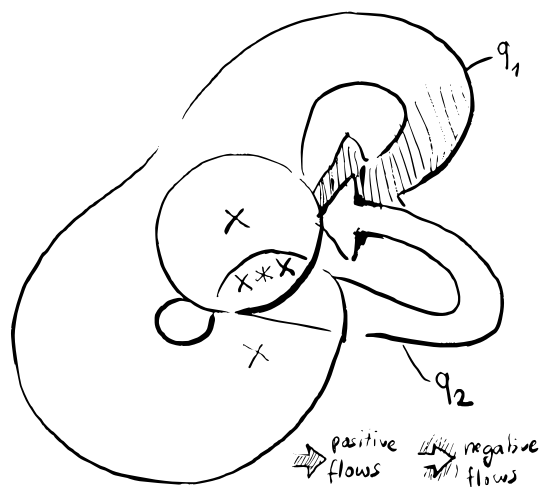


### Annex 1. Centers and isolation.

In evolutionary space, species often stand apart of each other. Members of different species can mostly not reproduce with each other (left).

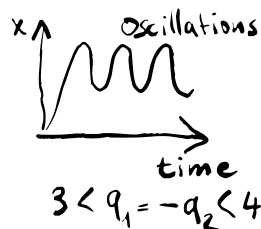
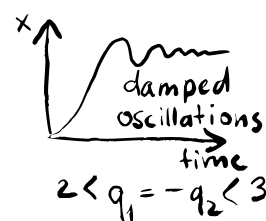
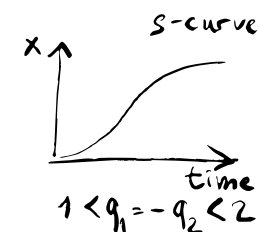
In contrast, in physical space, species diversity is often highest in the centers of density, and, most species are densest in joint centers of density (right).

## Something that consumes itself



Something grows by itself, but it  

$$x_{n+1} = q_1 x_n + q_2 x_n x_n$$
 shrinks in its own interplay with itself.



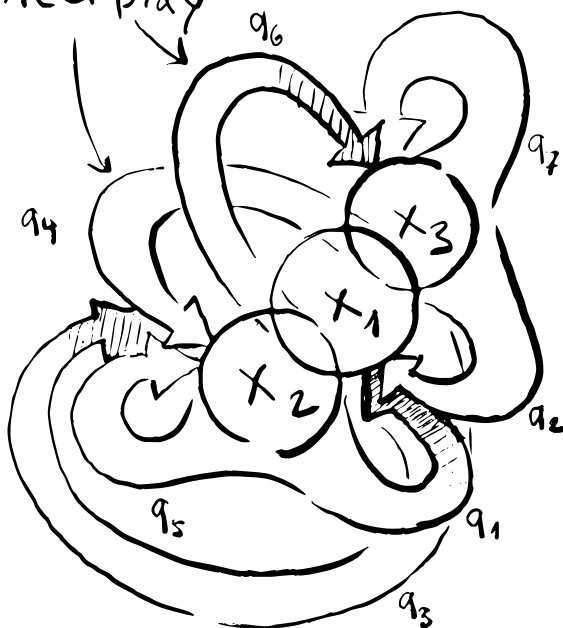
### Annex 2. Interplay in the logistic function.

Another equation similar to Lotka's is known as "logistic function." In this equation, X is in interplay with itself.

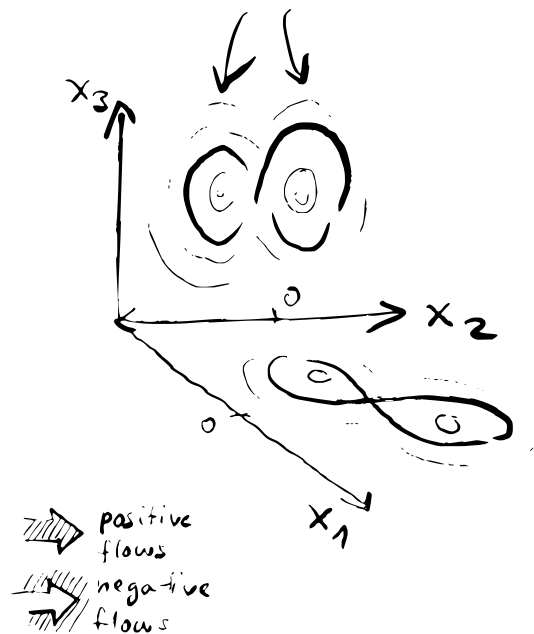
Similar to the equations that Lotka envisioned, this equation can lead to s-curves, damped oscillations, and aperiodic oscillations. (Equation published by Verhulst 1845. Behavior studied by May 1976.)

# Lorenz equations

creative  
interplay



oscillations



A first thing grows because of a second

$$\dot{x}_1 = q_1 x_2 + q_2 x_3,$$

but shrinks because of a third.

The second grows due to the first but shrinks due

$$\dot{x}_2 = q_3 x_1 + q_4 x_1 x_3 + q_5 x_2,$$

to interplay between the first and third and by itself.

The third grows due to

$$\dot{x}_3 = q_6 x_1 x_2 + q_7 x_3.$$

interplay between the first and second and by itself.

## Annex 3.

### Creative interplay in the Lorenz equations.

The Lorenz equations are a famous systems of equations. Like Models 7-8, they also model creative interplay between three parties.

The equations behave like interplay when they oscillate around one of two points. They behave like creativity when they occasionally change from oscillating around one point to oscillating around the other. (Equations published by Lorenz 1963.)

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