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# Vacuously true sentences: Are they non-Boolean and undecidable?

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**Abstract**

We propose a discussion on the non-Boolean mathematical object embedded in vacuously true sentences.

**keywords**: logic, mathematics, vacuously true, Boolean, non-Boolean

# Introduction

1. In [1], you will find a very pedagogical approach to vacuously true sentences.

# Definitions

2. A **Boolean** sentence is *either* True **or** False.

3. A **non-Boolean** sentence is True **and** False *simultaneously*.

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# A non-Boolean object embedded in a Boolean sentence

4. Let $A = \varnothing$ be the empty set.
5. $\forall x \in A.P(x)$ is true, where $P(x)$ is any predicate about $x$.
6. (5) is vacuously true.
7. Suppose $Q(x) = \neg P(x)$.
8. $\forall x \in A.Q(x)$ is true.
9. (8) is vacuously true.
10. So, for each $x \in A$, $P(x)$ and $\neg P(x)$ are both true.
11. (10) means that for each $x \in A$, $P(x)$ is both true and false.
12. Therefore, due to (11), for each $x \in A$, $P(x)$ is non-Boolean.

# Decidability

13. Since there are no elements in the empty set, can vacuously true sentences be proved?

# In Summary

14. $A = \varnothing$
15. $B \equiv (\forall x \in A.P(x) \wedge \neg P(x))$
16. $B$ is vacuosly true.
17. A vacuously true sentence has a *non-Boolean* object embedded in it since for each $x \in A$, $P(x)$ is **both true and false**.

# Final Remarks

18. Vacuously true sentences are Boolean.

19. However, there are non-Boolean objects in vacuously true sentences.

20. (19) means that **a non-Boolean object is inside the Boolean logic**.

21. There are mathematical proofs that rely on vacuously true sentences.

22. **Are vacuously true sentences undecidable**?

# Note

23. There is an analogy between the **empty set** and the **quantum vacuum** in physics; while in the former, everything is possible, in the quantum vacuum there are **superposition** of different physical states.

# Open Invitation

*Review*, *add* content, and **co-author** this paper [2, 3].
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# Ethical conduct of research

This original work was pre-registered under the OSF Preprints [4], please cite it accordingly [5]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

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