EVEN LENGTH PALINDROMES

THINH D. NGUYEN

1. PROBLEM STATEMENT

Definition 1. EVEN PALINDROME problem:

Input: Given a list of strings $[v_1, v_2, \dots, v_n]$ where $\Sigma|v_i|$ is even number.

Question: Is there a permutation σ such that $w = v_{\sigma(1)}v_{\sigma(2)}\dots v_{\sigma(n)} \in \{uu^R \in \Sigma^*\}$.

In other word, our problem is determining if a given set of strings can be ordered to form a palindrome (of even length as defined above). Moreover, we can restrict the cardinality of the alphabet $|\Sigma| = 2$?

We reduce 3-PARTITION problem to ours. 3-PARTITION is shown to be strongly hard by [1]. We claim that even with a binary alphabet, our problem is a very hard one.

Claim 2. *We have that 3-PARTITION $\leq_p$ EVEN PALINDROME*

2. REDUCING 3-PARTITION TO EVEN PALINDROME

The 3-partition problem is to decide whether a given multiset $S = \{a_1, a_2, \dots, a_n\}$ of $n = 3m$ positive integers, can S be partitioned into m triplets $S_1, S_2, \dots, S_m$ such that the sum of the numbers in each subset is equal?

Let $p = (\sum_i a_i)/m$

Note that the problem remains strongly NP-complete even if $p/4 < a_i < p/2$ (which forces each S_i to contain at exactly three elements).

We can build an equivalent EVEN PALINDROME instance in the following way:

We add the long string:

$$w = 1 0^p 111 0^p 11111 0^p \dots 0^p 1^{2m+1}$$

Then we add n more strings:

$$v_i = 0^{a_i}, i = 1 \dots n$$

and $1 + 3 + 5 + (2m + 1) = \sum_{j=1}^{m+1} (2j - 1)$ strings of length 1:

$$r_k = 1, k = 1 \dots \sum_{j=1}^{m+1} (2j - 1)$$

Note that $|w| = \sum_{i=1}^n |v_i| + \sum_k |r_k|$

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Proof. By construction the w contains substrings of odd increasing length $(1, 111, 11111, \dots)$, so it cannot be "placed" in the middle of uu^R ; but it must be placed entirely in u or entirely in u^R (without loss of generality suppose that it is placed in u).

Now the others v_i must be arranged in the opposite part u^R and in such a way that every p zeroes (that can be "totalized" using the v_i) an odd number of the r_k can be placed (to reflect the corresponding 1^{2j+1} substring of w). Furthermore all elements must be placed, so exactly 3 v_i must be picked to reflect each of the 0^p substring of w . So if an EVEN PALINDROME can be found it induces a valid 3-partition of the a_i of the original problem.

The opposite direction (a 3-partition exists $\Rightarrow$ there is an EVEN PALINDROME is immediate by construction).

Example:

3-Partition instance: $S = \{1, 1, 1, 2, 2, 3\}$, strongly NP-complete; so we can also represent the numbers in unary: $S' = \{1_1, 1_1, 1_1, 11_1, 11_1, 111_1\}$

EVEN PALINDROME reduction:

$w = 1000001110000011111$

$v_i = \{0, 0, 0, 00, 00, 000\}$

$r_k = \{1, 1, 1, 1, 1, 1, 1, 1\}$

Solution of EVEN PALINDROME:

$$uu^R = 1000001110000011111 \quad 11111 \ 0 \ 00 \ 00 \ 111 \ 000 \ 0 \ 0 \ 1\}$$

which corresponds to the 3-partition:

$$S_1 = \{1, 2, 2\}, S_2 = \{3, 1, 1\}$$

□

3. CONCLUSION

Garey and Johnson [1] shapes their theory based on previous primal works of Cook, Levin and Karp. Johnson [2] moves on with the guide to this theory. As long as we study a mathematical conjecture, we should encourage ourselves of having enough labouring hours on popular maths books like these. Then, reading some articles on theory of computing like [3] is a good practice. Only after that, could we think of the ultimate final for all mathematics sciences.

REFERENCES

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- Current address:* Department of Mathematics, Moscow State University
Email address: kosmofarmer@yandex.com