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[microreview]

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An element of a finite monoid is right invertible if and only if it is left invertible

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Abstract

We prove the proposition addressed in the title of this paper.

keywords: finite monoid, left invertible, right invertible, abstract algebra

The most updated version of this paper is available at

https://osf.io/7vm92/download

Notation & Definition

1. $[1,2]$
2. S = finite monoid
3. $(x \in S,\ \exists x' \in S : xx' = 1) \to (x' = \text{right inverse of } x)$
4. $(x \in S,\ \exists x' \in S : x'x = 1) \to (x' = \text{left inverse of } x)$
5. $(x \in S,\ \exists k, \ell \in \mathbb{N},\ k < \ell,\ x^k = x^\ell) \leftrightarrow (x = \text{periodic})$

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Proposition

6. *An element of a finite monoid is right invertible if and only if it is left invertible.* [1]

Proof 1

7. Proposition: $(S = \text{finite monoid}) \to (S = \text{periodic})$.
8. $(\to)$
9. Suppose $x \in S$ is right invertible.
10. $\exists x' \in S : xx' = 1$
11. Since S is finite, from (7), S is periodic.
12. From (5) and (11), $x^k = x^{k+\ell} = x^{\ell+k}$ for some $k, \ell \in \mathbb{N}$.
13. $1 = xx' = x^k x'^k = x^{k+\ell} x'^k$
14. From (12) and (13), $x^{k+\ell} x'^k = x^{\ell+k} x'^k = x^\ell x^k x'^k = x^\ell$
15. Then $1 = x^\ell = x^{\ell-1} x$.
16. So $x^{\ell-1}$ is a left inverse of x.
17. $(\leftarrow)$
18. Similarly, if x is left invertible, it is right invertible.
19. Therefore,

$$(x \in S \text{ is right invertible}) \leftrightarrow (x \in S \text{ is left invertible}). \quad \square$$

Proof 2

20. $T(S)$ = transformation semigroup
21. From Cayley's theorem, $S \cong T(S)$.
22. So, the element $x \in S$ corresponds to a transformation $t_x : S \to S$.
23. If x is right invertible, then $xx' = 1$.
24. Thus $t_x t_{x'} = t_1$.
25. Since t_1 is a permutation and S is finite, t_x is also a permutation.
26. Then, for some n, we have $(t_x)^n = t_1$.
27. In other words, $(t_x)^{n-1}$ is the left-inverse of t_x, i.e., x^{n-1} is the left-inverse of x.

Final Remarks

28. S = finite monoid
29. ($x \in S$ is right invertible) $\leftrightarrow$ ($x \in S$ is left invertible)

Open Invitation

Review, *add* content, and **co-author** this paper [3, 4]. *Join* the **Open Mathematics Collaboration** (https://bit.ly/ojmp-slack). Send your contribution to mplobo@uft.edu.br.

Open Science

The **latex file** for this paper together with other *supplementary files* are available [5].

Ethical conduct of research

This original work was pre-registered under the OSF Preprints [6], please cite it accordingly [7]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

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References

[1] Cain, Alan J. *Nine Chapters on the Semigroup Art.* AJC Porto & Lisbon, 2020. http://www-groups.mcs.st-andrews.ac.uk/%7ealanc/teaching/m431

[2] Lobo, Matheus P. "Open Mathematics Knowledge Base." *OSF Preprints*, 13 May 2020. https://doi.org/10.31219/osf.io/evq8a

[3] Lobo, Matheus P. "Microarticles." *OSF Preprints*, 28 Oct. 2019. https://doi.org/10.31219/osf.io/ejrct

[4] Lobo, Matheus P. "Simple Guidelines for Authors: Open Journal of Mathematics and Physics." *OSF Preprints*, 15 Nov. 2019. https://doi.org/10.31219/osf.io/fk836

[5] Lobo, Matheus P. "Open Journal of Mathematics and Physics (OJMP)." *OSF*, 21 Apr. 2020. https://doi.org/10.17605/osf.io/6hzyp

[6] COS. *Open Science Framework*. https://osf.io

[7] Lobo, Matheus P. "An Element of a Finite Monoid Is Right Invertible If and Only If It Is Left Invertible." *OSF Preprints*, 28 June 2020. https://doi.org/10.31219/osf.io/7vm92

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