1 Reflections on the logic of living systems

2 Pietro Paolo Rosas*¹

3 ¹*Independent researcher. Via Ungaretti 38; 09039 Villacidro (SU); Italy; tel:*
4 *+39-3471014786.*

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6 **Abstract**

7 This article presents an abstract model of living systems and their evo-
8 lution. The Modern Synthesis describes the evolution of living systems
9 but the debate on the origin and evolution of these systems is not yet
10 finished. It starts from the assumption that a problem faced by any bio-
11 logical system is equivalent to search for a resolution of an algorithm. The
12 concept of algorithm treated in theoretical computer science allows to de-
13 fine an abstract machine able to program automaton or Turing machines.
14 These mathematical definitions were then analyzed from a physical point
15 of view. This leads to a definition of an abstract living system that allows
16 to describe the behavior of these systems based on the energy levels and
17 entropy to which they operate. In conclusion this abstraction allows to
18 have a mathematical approach to living systems.

19 **Keywords:** Evolution Life Computation Energy

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*pie.rosas@gmail.com
ORCID of the author: <https://orcid.org/0000-0003-1478-9164>

22 Introduction

23 The biological systems in their generality represent physical structures which
24 over time tend to preserve their functions. They evolve continuously and are
25 able to adapt to the most diverse environments. This is in contrast to what
26 is observed in nature for non-living processes, which are governed by paths to
27 decreasing energy, whereas entropy increases.

28 At present there is not yet a unambiguous accepted theory that can explain the
29 origin and why of the behavior of living systems compared to non-living systems,
30 whereas the evolution of such systems is largely proved by the observations (Dar-
31 win 1859; Ferraguti and Castellacci 2011). Furthermore, an abstract approach
32 to biological systems is lacking, but some authors have made contributions in
33 this sense (Solomonoff 1964; Wolfram 2002).

34 The Modern Synthesis represents the theory on the evolution of the most excit-
35 ing biological systems supported by experimental tests and certainly integrates a
36 large number of disciplines such as genetics, paleontology, comparative anatomy
37 etc. (Ferraguti and Castellacci 2011; Pigliucci and Muller 2010). However, there
38 are other hypotheses and theories that actually developing in this period but
39 the debate is certainly on and very lively (Gavrilets 2010). Some important
40 reflections concern the pattern of theory of punctuated equilibrium (Ayala and
41 Avise 2014; Ayala and Bendall 1983; Gould and Eldredge 1972) regarding the
42 discontinuity in evolution. In addition symbiosis (Margulis 1970; Margulis et al.
43 1967) and coevolution (Ferraguti and Castellacci 2011), certainly represent the
44 characteristics of living systems that make their description more complex and
45 interesting.

46 The concept of organism understood as an evolutionary unit seems to be linearly
47 describable with the concepts of genotype and phenotype, but in relation to the
48 evolutionary developmental biology this concept extends to the whole life cycle

of an organism up to embrace the environment in which he lives (Lewontin 2002). It is also interesting to note the approach to a molecular evolution as well as individual or population (Ferraguti and Castellacci 2011).

Likewise, the concept of species does not seem to be definable in an absolute sense, especially as regards unicellular organisms.

An interesting statement that begins to lay the foundations of a possible abstraction and synthesis of evolution concerns the fact that an organism is seen as a passive object embedded in the environment, which generates problems, and a genetic machine within the organisms that find solutions through random combinations, with recombination and mutations (Ferraguti and Castellacci 2011; Lewontin 2002). However, it is necessary to deepen the idea that the organism is seen as a passive object. In the present paper a different formulation of organism will be presented. Furthermore, it is observed that organisms modify the environment in which they live by influencing the selective pressures of successive generations (Lewontin 1983). A theory that combines these ideas and generalizes them is that living systems solve problems.

In this paper one presents an abstract, physical-mathematical description of these systems, regardless of the chemical and biological structures of which they are composed and of their organization. In short, every living system behaves like a thermal machine (Halliday et al. 2011; Mencuccini and Silvestrini 1987), uses energy, produces work and heat. Therefore a physical and mathematical description of such systems must start from a reflection on thermodynamics and statistical mechanics. Therefore, classical concepts of thermodynamics such as heat and work (Halliday et al. 2011; Mencuccini and Silvestrini 1987) will be re-defined in a different but equivalent way. This is to highlight a fundamental aspect of loss of information in the first case and information vehicle in the second one.

Thermodynamics and statistical mechanics arise from the inductive interpreta-

77 tion given by the interpretation and synthesis of the results of a numerous series
78 of experiments and then from the probabilistic study of the states of a system.
79 The present article one proposes an abstract approach to these theories.

80 Generally the alphabet is defined as a finite set of symbols (Church 1936;
81 Hopcroft et al. 2001; Turing 1937). This definition is modified in this article by
82 defining a symbol as the result of the application of a terminating algorithm. A
83 symbol is therefore not postulated but instead the algorithm or the procedure
84 necessary to obtain a symbol. For example, let a shepherd who has to count his
85 flock of sheep. He should force the sheep to cross a gate, recognizing them by a
86 sheep model, and link the recognition to a list of numbers. The recognition algo-
87 rithm works and the shepherd manages to count the sheep. On the other hand,
88 if the shepherd was, for example, short-sighted and without glasses he would
89 have great difficulty in recognizing a sheep and counting it. In this second case,
90 the symbols defined in a classical way do not take into account the fact that
91 the shepherd is short-sighted, while both the sheep and its biunivocal corre-
92 spondence with a number remains the same. Instead, considering the algorithm
93 that allows the shepherd to recognize a sheep and to assign this recognition to
94 a number, gives the most correct interpretation of the real process.

95 In the field of mathematics and theoretical computer science, most of the compu-
96 tational models studied are currently based on an evolution of a Turing machine
97 (Turing 1937). These models basically need a defined alphabet and a table in
98 which a program is stored and a memory or tape on which these symbols are
99 written and read.

100 This article presents a machine model able to create a program that can then
101 be executed on any theoretical machine that recognizes its language. This ma-
102 chine cannot be represented as a Turing machine, basically because it does not
103 need a starting point in the definition set and it does not need any table of
104 instructions. The model proposed by A. M. Turing was built to define the cal-

culability, while this machine represents the abstraction of a classical concept of thermodynamics, a thermal machine (Halliday et al. 2011; Mencuccini and Silvestrini 1987).

All biological systems are composed of molecules formed by chemical elements and each reaction within their metabolism corresponds to a chemical reaction. Regardless of the type of molecule they contain, it is fundamental to discuss the reaction dynamics. The most interesting models to start from are those concerning the potential energy surfaces (Atkins and De Paula 2010; Chang and Miller 1990) and the dynamics of electron transfer (Marcus 1956). On the basis of these models it is possible to construct a geometric generalization of the potential energy surfaces, that are usually defined as energy hypersurfaces.

Another important theoretical tool, used in this article is information theory (Hartley 1928; Shannon 1948). An important aspect of this theory is represented by a definition of entropy that starts from information rather than from the physical state of a system (Gray 2011). There is therefore a link between information theory and statistical mechanics (Jaynes 1957; Katz 1967).

A fundamental concept in physical systems and in particular in living systems concerns randomness. The definition of a random sequence of symbols is found in (Martin-Löf 1966), while for the randomness of a string, that is a set of groups of symbols, the definition is due to A. N. Kolmogorov (Li and Vitányi 2008; Zvonkin and Levin 1970). A still different approach to randomness is found in Ref. (Schnorr 1971).

Given an algorithm it is important to measure how complex its execution is from a computational point of view. To solve this problem there are two approaches, one given by the theory of computational complexity, which considers the resources in terms of time and memory that are necessary for the execution of an algorithm (Arora and Barak 2009) and another, given by the Theory of the Complexity of Solomonoff-Kolmogorov-Chaitin that instead takes into ac-

count the minimal information associated with a algorithm (Chaitin 1975; Li and Vitányi 2008). This theory is also called Algorithmic Information Theory, for a brief review see Ref. (Hutter 2007). Concerning the Theory of Complexity of Solomonoff-Kolmogorov-Chaitin (Li and Vitányi 2008), an interpretation of the Algorithmic Information Theory through statistical mechanics is interesting (Tadaki 2008). The complexity of an algorithm is essential for systems that seek a termination of computing as living systems do. As will be seen below, the non-termination of an algorithm implies a reduction in the survival of the living system. Regarding randomness many problems still remain unresolved (Solomonoff 2011; Zenil 2011)

There is a relationship between logical irreversibility and physical irreversibility, because the deletion of an information bit generates heat (Landauer 1961). This theoretical result was also demonstrated experimentally by Peterson et al. (Peterson et al. 2016).

In the biological field, the Modern Synthesis represents a fundamental step in the interpretation of living systems and their evolution. There is a vast literature concerning the description of living systems (Darwin 1859; Ferraguti and Castellacci 2011), but the debate is not concluded.

With regard to the origin of life, fundamental experiments have been carried out to try to demonstrate an abiotic origin of amino acids, nucleotides and cell membranes (Gould and Eldredge 1972; Hargreaves et al. 1977; Miller and Urey 1959; Parker et al. 2011). A very interesting aspect about this type of experiments concerns the fact that the chiral amonoacids that are synthesized are a racemic (Parker et al. 2014), while in biological systems this does not happen, because generally one enantiomer is observed (Bada 1985).

Finally, it is necessary to highlight the current uncertainty in the classification as living organisms of viruses and viroids, but there is also for them an evolution (Webster et al. 1992) like the one that happens for biological systems. They

161 interact with the cells (Agrios 1988).

162 A mathematical approach to biological systems concerns the study of natural
163 selection through the fitness of genes in which it is shown that the existing inher-
164 ited variance is fundamental for reproductive success (Ferraguti and Castellacci
165 2011; Fisher 1958).

166 In summary, in this text, a new definition of what is a symbol is presented.
167 For the mathematical part, a theoretical machine is described that can create
168 a program, while in the physical part a special definition of energy is given
169 and consequences are drawn on the observations. In the biological part an
170 evolutionary hypothesis is introduced based on the hypotheses made in the
171 mathematical and physical parts.

172 1 Mathematical part. Definitions and notations

173 In this part of the text the definitions necessary for the development of this
174 article will be introduced. In order to analyze the living systems from a math-
175 ematical point of view, concepts treated in theoretical computer science will be
176 introduced.

177 1.1 Turing machine

178 Formally one defines a *Turing machine*, hereinafter referred to as TM , a sextu-
179 ple, (Turing 1937), $\mathcal{M} = \langle Q, \Gamma, \mathfrak{b}, \delta, q_0, F \rangle$, where:

180 **Definition 1.** Q is the finite set of control states;

181 Γ is the alphabet of the symbols that can be written on the tape, besides
182 it defines $\Sigma \subseteq \Gamma$ which is the finite set of input symbols that the Turing
183 machine can read and write on the tape at the same time;

184 $\mathbf{b}$ is a special character called blank for which one has $\mathbf{b} \notin \Gamma$;

185 δ is a *transition function* $\delta : (Q - F) \times (\Gamma \cup \{\mathbf{b}\}) \rightarrow (Q \times (\Gamma \cup \{\mathbf{b}\}) \times D)$, which
 186 associates a pair $\{(q, x) \mid q \in Q, x \in (\Gamma \cup \{\mathbf{b}\})\}$, (where q is the state of the
 187 machine when it reads the symbol or sequence x), to a triple consisting
 188 of $\{(q', x', d) \mid q' \in Q, x' \in (\Gamma \cup \{\mathbf{b}\}), d \in D\}$ where set $D = \{r, l, S\}$ ¹, also
 189 places $S = \{s\}$;

190 q_0 is the initial state;

191 F is the set of final or accepting states.

192 In the following, instead of the set $(\Gamma \cup \{\mathbf{b}\})$ to simplify the notation, one will
 193 uses the set $\bar{\Gamma} = (\Gamma \cup \{\mathbf{b}\})$.

Definition 2. Given an alphabet $\bar{\Gamma}$, let k be the length of each word built upon $\bar{\Gamma}$. All words of length k belong to a set indicated with $\bar{\Gamma}^k$. The empty string is indicated with ε which is a null-length string, so one has

$$\text{for } k = 0, \quad \bar{\Gamma}^k = \{\varepsilon\}.$$

Furthermore all words or strings that can be formed with $\bar{\Gamma}$ are indicated with

$$\bar{\Gamma}^* = \bigcup_{k \geq 0} \bar{\Gamma}^k = \{\varepsilon\} \cup \bar{\Gamma}^1 \cup \bar{\Gamma}^2 \cup \dots \cup \bar{\Gamma}^k.$$

Definition 3. *Instantaneous configuration* of a MT is defined a string, (Ausiello et al. 2003):

$$c = xqy, \tag{1}$$

194 with:

195 (1) $x \in \bar{\Gamma}^* \cup \{\varepsilon\}$;

¹ r stands for *right*, l stands for *left* and S stands for *stationary*, this last character is a set, the reason is explained in Def. (8).

196 (2) $q \in Q$;

197 (3) $y \in \bar{\Gamma}\Gamma^* \cup \{\mathbf{b}\}$.

198 The TM is still an abstract model for the formal definition of an algorithm that
199 necessarily has an infinite memory. However, physical and biological systems
200 are assumed to be finite systems, so they must have finite memory.

201 In this article one will use the universal Turing machine definition $\mathcal{U}$, this
202 machine generalizes the behavior of a TM, and is able to simulate one or more
203 Turing machines $\mathcal{M}_0, \mathcal{M}_1, \dots, \mathcal{M}_n$, given the initial symbol on the tape and the
204 table of the transition functions of each $\mathcal{M}_n$, (Turing 1937).

205 **Turing machine and non-deterministic automaton** In this paragraph
206 one introduces the definition of *non-deterministic finite automaton*, as it could
207 model thermodynamic algorithms that contain one or more thermody-
208 namic operations with more than one degree of freedom, (Handey and Live
209 2014).

$$\lambda : Q \times \Gamma \rightarrow \mathcal{P}(Q \times \bar{\Gamma} \times D).$$

210 Unlike a TM, one defines *automaton* a TM that operates on a tape or a finite
211 memory. In the rest of this paragraph this definition is used.

In this case the non-deterministic automaton, (Safra 1988), can have more than one initial state, also since the function λ leads to the power set of Q . This corresponds to affirming that one can simultaneously have several possible states simultaneously, up to the whole set Q , including an empty state because one can have this: $\lambda \subseteq Q \times \Gamma \times Q$. This automaton has the same definitions given in (1) for the *TM*, except for the transition function λ , instead of a single value. This function is represented by a *transition probability matrix*, $\mathbf{P}(\Sigma)$, of dimension

$k \times k$, where $k = \mathbf{card}(Q)$. This matrix, for each $\Sigma \in \Gamma$ returns a probability that the automaton, finding itself in a generic state i , changes to a state j , for $i, j \in Q$. The transition probability matrix will be:

$$\mathbf{P}(\Sigma) = (p_{ij}(\Sigma)).$$

Given that $p(\Sigma)$ a probability, it must be:

$$p_{ij} \geq 0,$$

while the indices $(i, j = 1, \dots, k)$ are equal in number to the number of the possible states contained in the set Q . Given that for each state i different states j exist, each column of the matrix $\mathbf{P}(\Sigma)$ is precisely this condition, such a matrix must have on each column:

$$\sum_{j=1, \dots, k} p_{ij}(\sigma) = 1.$$

Finally, it is necessary to introduce a vector that represents the probability that the automaton is in the state i , this will be given by a probability vector associated with state i :

$$\mathbf{q}_i = (p(i_1), \dots, p(i_k)),$$

furthermore each element of the vector $\mathbf{q}_i$, being a probability, will be:

$$p(i_k) \geq 0,$$

and the sum of the probabilities of the i states in which the vector can be found must be :

$$\sum_{i=1, \dots, k} p(i_k) = 1.$$

2 On the Turing machine

The TM defined in the previous paragraphs operates on an alphabet. In general, all grammars operate on symbols whose set is called alphabet. For example, it is not necessary to show that the number one corresponds to a sign written on a sheet that corresponds to the abstraction of a measurable and unitary quantity, but it is universally accepted. Instead, assume that such a result is not given, but that the number one is the result of the execution of certain algorithm that returns the value one. There are infinite algorithms that will return this value, think of operations on $\mathbb{R}$.

Therefore, in general, a number can be obtained as a solution to an algorithm that uses other numbers. Instead to define an alphabet, the condition that must be postulated is that the solutions of this algorithm, that starts from the same point, must always give the same value, if performed by a computer or by a person. This example serves to show more generally that the symbol of an alphabet can be defined with these two axioms:

Definition 4. Consider any alphabet, within this alphabet one must verify that:

- (1) a symbol of the alphabet is defined by executing an algorithm,
- (2) for each symbol the algorithm must always return the same result.

If these two conditions occur, one can use this protocol to create an alphabet. Furthermore, the second condition implies that there is the possibility that an algorithm does not give the same result universally or does not give it at all (in the case of non-termination). If in this case there is a set that does not satisfy the second condition, not defining a symbol, it does not belong to an alphabet. For example the character b defined for a TM or the 0 for natural numbers can be understood as sets of all that does not belong to the respective alphabets.

239 A concept intrinsically linked to that of symbol is that any symbol must be
240 unique, so the result of a measure which returns a symbol must necessarily
241 be incompatible with that of another symbol belonging to the same alphabet
242 ².

243 With these conditions, one can theoretically nest this reasoning to infinity, since
244 the algorithm works with an alphabet, and the alphabet can be defined by
245 another set of algorithms, one can continue in this iteration by breaking down
246 any symbol of an alphabet in algorithms sequences ³.

247 So, if a symbol can be defined by an algorithm, this algorithm can be executed
248 by a TM.

249 3 Programming machine

250 A programming machine, in the following text will be called PM, is a machine
251 able to create a table of the transition functions of a TM.

252 The TM represents an abstract model that works on finite states and alphabet on
253 an infinite tape that contains a sequence of symbols of an alphabet. The symbol
254 sequence stored on the tape is defined as an ordered sequence of symbols, then
255 a tuple. In this Section one introduces a theoretical machine which, in place of
256 states and a given alphabet, contains and operates on an alphabet and states
257 given by stochastic processes. Then the representation of such processes is
258 possible through the use of sets of Markov matrices, (Gallager 2012; Häggström
259 2002), knowing the *initial probability distributions* of the different states. The
260 PM is able to create a table containing the transition functions that can then be
261 stored and executed on a MT. The introduction of stochastic processes in the

²If a measure could produce two symbols with compatible probability it would not be possible to distinguish them uniquely.

³This reasoning is valid at the theoretical level, as there may be a physical limit to this nesting.

PM will be explored later, however the idea is that each transition probability matrix represents the modeling of a measurement on reality.

The $\mathfrak{b}$ character used by the TM must be redefined for a PM. The $\mathfrak{b}$ character does not belong to the alphabet of a TM, but is recognized by it, so $\mathfrak{b}$ is unique to TM. This reasoning does not apply to PM, since the characters of its alphabet are represented in probabilistic way by Markov matrices. However, if a PM has to create a program (or table) that can be read by a TM, PM must also produce the $\mathfrak{b}$ character.

Definition 5 (Alphabet of a programming machine). It is the set of probabilities arrays of transition of the symbols of an alphabet Γ on itself, for Γ which belongs to the definition set of a Turing machine.

$$A = \{\mathbf{P}_\Gamma\} = \{\mathbf{P}_{\Gamma_1}, \mathbf{P}_{\Gamma_2}, \dots, \mathbf{P}_{\Gamma_l}\} \quad \text{with } l \in \mathbb{N}. \quad (2)$$

The alphabet A is recognized by a TM. So for a Turing machine one has $\Gamma \wedge \mathfrak{b} \subset A$.

Example 3.1. *Given the stochastic matrix:*

$$\begin{matrix} & s_1 & s_2 & \cdots & s_n \\ \begin{matrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{matrix} & \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{pmatrix} \end{matrix},$$

where $\{s_1, s_2, \dots, s_n\} \in \Gamma$.

One has that, the vector ${}^t\mathbf{x}_1 = (p_{11}, p_{12}, \dots, p_{1n})$ represents the probabilities of the transition of the symbol s_1 on itself and on all other symbols of the alphabet belonging to Γ . In other words, if any algorithm starts with s_1 , the vector ${}^t\mathbf{x}_1$ represents the probabilities of this algorithm ends with the symbol $s_1, s_2, \dots, s_n$. Finally, one must bears in mind that each algorithm must terminate so that the symbol can be read.

281 Thus any symbol is not considered as *a priori*, but represents the result of a
 282 processing.

283 **Proposition 1.** *Given an alphabet A , read by a PM, it is verified that the*
 284 *characters $\mathfrak{b}$, read by a TM that interprets this alphabet, are all the characters*
 285 *that in A are represented by Markov matrices with probability distributed on all*
 286 *the symbols of Γ . So a PM does not have a unique symbol called $\mathfrak{b}$ in accordance*
 287 *with Definition (1).*

Proof. Let A be the alphabet of a PM, it is read and interpreted by a TM, so
 it must contain both the $\bar{\Gamma}$ alphabet and all the transition probability matrices
 of A . So an A alphabet will consist of the symbols belonging to Γ and a subset
 that does not belong to Γ . Given that a TM recognizes only an alphabet with
 characters *almost certainly* equal to themselves, one has that for a TM:

$$\Gamma = A = \Gamma + \Gamma^{\mathfrak{C}}, \quad (3)$$

288 where $\Gamma^{\mathfrak{C}}$ represents a set of Markov matrices belonging to A that do not express
 289 a symbol in a univocal way.

In accordance with Definition (1) one has:

$$\bar{\Gamma} = \Gamma + \mathfrak{b},$$

as A must be an alphabet recognized by a TM for the Definition (2), one writes

$$A = \bar{\Gamma} = \Gamma + \mathfrak{b} \quad \text{and} \quad \mathfrak{b} = (\{\mathbf{P}_{\Gamma}\} - \Gamma) = \Gamma^{\mathfrak{C}}.$$

290 □

291 So the alphabet that it is reads by a TM corresponds to A if one accepts that the
 292 set of all the semi-definite or blurred characters given by Markov matrices⁴ are $\mathfrak{b}$.

⁴For example, imagine having a disk divided into three sectors, each colored with a funda-

293 Considering the Equation (3), to simplify the notation one can assume:

$$B = \Gamma^{\mathbb{C}},$$

$$A = \Gamma \cup B. \quad (4)$$

294 Obviously for a TM that reads the set A the relationship it is expected that
 295 $B = \mathfrak{b}$, i.e., the whole set B is recognized⁵ by a TM only with the symbol $\mathfrak{b}$.
 296 A PM therefore does not possess a $\mathfrak{b}$ character, but rather a set of transition
 297 probability matrices. If one indicates with $\mathbf{P}_b \in A$ a transient stochastic matrix,
 298 so given an initial symbol a terminal symbol is not uniquely determined, then
 299 one can write:

$$B = \{\mathbf{P}_{b_1}, \mathbf{P}_{b_2}, \dots, \mathbf{P}_{b_n}\} = \mathfrak{b} \quad \text{with } n \in \mathbb{N}. \quad (5)$$

300 The character concatenation operation is indicated with the symbol $\circ$. The
 301 concatenation of $\mathfrak{b}$ characters is equivalent to an empty string for a TM, then
 302 the following observation is derived.

Observation 3.1. If B is the set of Markov matrices that are recognized as $\mathfrak{b}$ from a TM, then an empty string, composed of matrices $\mathbf{P}_b$, for a TM corresponds to:

$$\varepsilon = \mathbf{P}_{b_1} \circ \mathbf{P}_{b_2} \circ \dots \circ \mathbf{P}_{b_n} \quad \text{with } \mathbf{P}_{b_1}, \mathbf{P}_{b_2}, \dots, \mathbf{P}_{b_n} \in B.$$

303 Therefore the alphabet of a PM is different from that of a TM that recognizes
 304 its language.

mental color, our eyes distinguish each of the three colors and one can create an alphabet with them. Now if the disk starts to rotate around the center the color appear there overlapped, everything appears white with different shades depending on the speed of the disc. Then a new alphabet can be formed from the three colors of the sectors plus the color given by the superposition of the three.

⁵The set A can be read both by a TM and by a PM, but they interpret it differently.

3.1 States of a PM

A PM must produce a program or an algorithm that a TM must then reproduce. Thus also the states of the set Q defined for a TM must belong also to the Q states of a PM. Therefore one gets the following definition:

Definition 6. The states of a PM represent the set of transition probability matrices that has as members the $Q = \{q_1, q_2, \dots, q_n\}$ states defined for a TM. So if this set is called K one has:

$$\begin{aligned} K &= \{\mathbf{P}_Q\} = \{\mathbf{P}_{Q_1}, \mathbf{P}_{Q_2}, \dots, \mathbf{P}_{Q_m}\} \quad \text{with } m \in \mathbb{N}, \\ K &= Q \cup \bar{Q}, \end{aligned} \tag{6}$$

where $\bar{Q}$ is the set of Markov matrices with distributed transition probabilities on all Q states, or the Markov matrices for which a state $q_n \in Q$ is not recognizable. From Eq. (6) one gets:

$$\bar{Q} = \{\mathbf{P}_Q\} - Q. \tag{7}$$

Moreover one has:

$$q_0 \in Q \quad \text{e} \quad F \in Q,$$

hence the initial state and also the set of final states, as defined for a TM, belong to the set Q .

The set $\bar{Q}$ is not defined for a TM⁶.

⁶To get an intuitive idea of the set $\bar{Q}$, take this example: imagine an observer who observes the void, without having any thought on which to reason, even for a moment. This situation can be represented as $\bar{Q}$.

3.2 Formal definition of programming machine

Taking into account the Definitions given in (1) and (4) and Equations (2) and (6), one defines a *programming machine*:

Definition 7. Formally a *PM* it is a 7-tuple:

$$\mathcal{M}_{\mathcal{P}} = \langle K, \bar{\mathbf{Q}}, A, B, \zeta, q_0, F \rangle \quad (8)$$

This machine operates on three potentially infinite⁷ and independent sets called *state spaces*:

$$\begin{aligned} K &= \{\bar{\mathbf{Q}}, q_0, q_1, \dots, q_n, F\}, \\ A &= \{B, s_1, s_2, \dots, s_q\} \\ \text{and } E &= \{S, r, l\}. \end{aligned} \quad (9)$$

K is the set of state transition probability matrices $Q = \{q_0, q_1, \dots, q_n, F\}$ defined in Eq. (6), where the stochastic matrix is:

$$\mathbf{P}_Q = \begin{pmatrix} p_{11} & p_{12} & \dots & p_{1n} \\ p_{21} & p_{22} & \dots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \dots & p_{nn} \end{pmatrix}; \quad (10)$$

F is a subset of Q , which represents the termination states defined in Eq.

(6) of a TM, $F = \{f_1, f_2, \dots, f_j\} \subset Q$. The corresponding matrix is:

$$\mathbf{P}_F = \begin{pmatrix} p_{11} & p_{12} & \dots & p_{1j} \\ p_{21} & p_{22} & \dots & p_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ p_{i1} & p_{i2} & \dots & p_{ij} \end{pmatrix}; \quad (11)$$

⁷In this article, these sets will be considered finite.

319

which the PM operates. The stochastic matrix defined in Eq. (2) is:

$$\mathbf{P}_\Gamma = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1r} \\ p_{21} & p_{22} & \cdots & p_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ p_{r1} & p_{r2} & \cdots & p_{rr} \end{pmatrix}; \quad (12)$$

320

ζ is the *transition function*:

$$\zeta : (K - F) \times A \rightarrow K \times A \times E; \quad (13)$$

321

E it is a finite set that contains the matrices of the transition probabilities

322

of the one-dimensional displacements within the n -tuple which represents

323

the code on which the PM operates. Let be $D' = \{r, l\}$ then it holds⁸:

$$E = \{\mathbf{P}_{D'}\},$$

$$\mathbf{P}_{D'} = \begin{matrix} & r & l \\ \begin{matrix} r \\ l \end{matrix} & \begin{pmatrix} p_{1,1} & p_{1,2} \\ p_{2,1} & p_{2,2} \end{pmatrix} \end{matrix}, \quad (14)$$

324

assuming that r and l are the shift of the head to the right and to the left,

325

respectively, while the head remains immobile whenever the $\mathbf{P}_{D'}$ matrix

326

does not express a unique result, i.e. the stochastic matrix is not absorbing

327

on a value of l or r .

328

In this case the whole set D is defined by following Definition.

⁸From a physical chemical point of view a so-called *displacement* should represent a measurable symmetry variation on hypersurfaces of energy distinguishable from that belonging to the symbol of initial measurement $\langle q_0, s \rangle$. Such energy hypersurfaces may have lower or higher levels of energy than the hypersurface where the variation occurred, but not equal, otherwise there would be no measurable displacement.

329 **Definition 8.** There is a set $\bar{D} \subset E$ where the matrices of the transition prob-
 330 abilities are transients on l and on r and therefore do not express a certain
 331 direction value.

A TM recognizes the elements of set $D = \{r, l, S\}$ as specified in Definition (1).
 One assumes for a TM, the equality $E = D$ and then one has

$$\bar{D} = S,$$

332 because the elements r and l also belong to E . So a TM recognizes the set $\bar{D}$
 333 as a single element $s \in S \subset D$.

334 The table of the combinations of the sets of ζ function is random (see Refs. (Li
 335 and Vitányi 2008; Zvonkin and Levin 1970) and (Martin-Löf 1966)) or intrinsi-
 336 cally or when the system of combination of elements is highly complex.

337 3.3 Transitions of a PM

338 For a TM an instant configuration is given if are known the content of the tape,
 339 the position of the read-write head, and the internal state. Whereas for a PM
 340 a configuration is not never determined uniquely.

Definition 9. Keeping in mind the definition given in Eq. (1), one defines as
 an instantaneous configuration of a PM a string composed of matrices:

$$\mathcal{C} = \mathbf{X} \circ \mathbf{P}_Q \circ \mathbf{Y}, \tag{15}$$

341 where:

342 (1) $\mathbf{X} \in A \circ A^* \cup B;$

343 (2) $\mathbf{P}_Q \in K;$

344 (3) $\mathbf{Y} \in A^* \circ A \cup B.$

345 It is noted, with respect to the definition given in Eq. (1), that in this definition
 346 does not appear the character $\mathfrak{b}$ in that it is contained in the set B . Furthermore
 347 the matrix $\mathbf{P}_B$ corresponds to a stochastic matrix that does not univocally define
 348 a symbol, see Eq.(5).

349 3.4 Matrix of the states Q

350 Consider a probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

Lemma 1. *Given the fifth axiom of probability theory (Kolmogorov 1956), one has*

$$p(A + B) = p(A) + p(B),$$

351 where $A, B \subset \Omega$.

352 Thus, the probability of an incompatible event can be added to each probability
 353 value or element of the matrix $\mathbf{P}_Q$. To make the transition probabilities vary,
 354 as mentioned, a null square matrix is defined,

Definition 10.

$$\mathbf{T} = \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1m} \\ q_{21} & q_{22} & \cdots & q_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ q_{m1} & q_{m2} & \cdots & q_{mm} \end{pmatrix}, \quad \text{where} \quad q_{11} = q_{1m} = \cdots = q_{mm} = 0. \quad (16)$$

355 The matrix $\mathbf{T}$ represents a matrix of the adjacency of a digraph whose nodes
 356 represent a transition code which are *not recognized neither by a TM nor a PM*.
 357 The elements of this code are part of the set $\mathfrak{Q} = \{\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3, \mathfrak{q}_4 \dots \mathfrak{q}_n\}$.

Furthermore, the following relationships are defined with respect to the sets

defined for a TM and a PM:

$$\Omega \notin Q \quad \text{and} \quad \Omega \notin K.$$

Then, from the direct sum

$$\mathbf{U} = \mathbf{P}_Q \oplus \mathbf{T},$$

one obtains a square matrix that is called matrix $\mathbf{U}$:

$$\mathbf{U} = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1r} & 0 & 0 & \cdots & 0 \\ p_{21} & p_{22} & \cdots & a_{2r} & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 & 0 & \cdots & 0 \\ p_{r1} & p_{r2} & \cdots & p_{rr} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & q_{11} & q_{12} & \cdots & q_{1m} \\ 0 & 0 & \cdots & 0 & q_{21} & q_{22} & \cdots & q_{2m} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & q_{m1} & q_{m2} & \cdots & q_{mm} \end{pmatrix}. \quad (17)$$

358 The newly defined matrix $\mathbf{U}$ can be divided into four blocks:

$$\mathbf{U} = \left(\begin{array}{c|c} \mathbf{P}_Q & \mathbf{L} \\ \hline \mathbf{M} & \mathbf{T} \end{array} \right) \quad \text{with} \quad \mathbf{L} = \mathbf{0} \text{ and } \mathbf{M} = \mathbf{0}. \quad (18)$$

359 The blocks $\mathbf{P}_Q$ and $\mathbf{T}$ have already been defined respectively in Def. (8) and in
360 Eq. (10). The $\mathbf{L}$ and $\mathbf{M}$ blocks, represented by null matrices, are modified by
361 the following definition.

Definition 11.

$$\mathbf{M} = \begin{pmatrix} b_{(r+1)1} & b_{(r+1)2} & \cdots & b_{(r+1)r} \\ b_{(r+2)1} & b_{(r+2)2} & \cdots & b_{(r+2)r} \\ \vdots & \vdots & \ddots & \vdots \\ b_{(r+m)1} & b_{(r+m)2} & \cdots & b_{(r+m)r} \end{pmatrix}. \quad (19)$$

Where for each index $i \in \mathbb{N}_{r+m}$ e $j \in \mathbb{N}_r$ has the elements:

$$(b_{ij}) = \{0, 1\} \quad \text{and} \quad \sum_{j=1}^r b_{ij} = 1. \quad (20)$$

362 Now let's consider the block $\mathbf{L}$ of the matrix $\mathbf{U}$. This block is replaced with a
 363 matrix that contains the probabilities of transition that link the symbols to a
 364 *transition code*.

Definition 12. The matrix is then defined as follows:

$$\mathbf{L} = \begin{pmatrix} l_{1(r+1)} & l_{1(r+2)} & \cdots & l_{1(r+m)} \\ l_{2(r+1)} & l_{2(r+2)} & \cdots & l_{2(r+m)} \\ \vdots & \vdots & \ddots & \vdots \\ l_{r(r+1)} & l_{r(r+2)} & \cdots & l_{r(r+m)} \end{pmatrix}. \quad (21)$$

There is the matrix $\mathbf{U}$ is, a stochastic matrix must be for each $i \in \mathbb{N}_r$ and $j \in \mathbb{N}_{r+m}$:

$$\sum_{j=1}^r p_{ij} + \sum_{j=r+1}^m l_{ij} = 1.$$

Then substituting with Eqs. 19 and 21 in Eq. 18, the null blocks of the stochastic matrix $\mathbf{U}$, one obtains the *matrix of states*:

$$\mathbf{U} = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1r} & l_{1(r+1)} & l_{1(r+2)} & \cdots & l_{1(r+m)} \\ p_{21} & p_{22} & \cdots & p_{2r} & l_{2(r+1)} & l_{2(r+2)} & \cdots & l_{2(r+m)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & & \vdots \\ p_{r1} & p_{r2} & \cdots & p_{rr} & l_{r(r+1)} & l_{r(r+2)} & \cdots & l_{r(r+m)} \\ b_{(r+1)1} & b_{(r+1)2} & \cdots & b_{(r+1)r} & 0 & 0 & \cdots & 0 \\ b_{(r+2)1} & b_{(r+2)2} & \cdots & b_{(r+2)r} & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{(r+m)1} & b_{(r+m)2} & \cdots & b_{(r+m)r} & 0 & 0 & \cdots & 0 \end{pmatrix}. \quad (22)$$

365 The matrix $\mathbf{U}$ is still a stochastic matrix, moreover this matrix represents the

Figure 1: Digraph of a matrix $\mathbf{U}$ of Example 3.2.

adjacency matrix of a digraph $\mathcal{D} = \langle V_u, E_u \rangle$ with $V_u = Q \cup \mathfrak{Q}$ and the set E_u represents the arcs of which the elements of the matrix $\mathbf{U}$ represent the probability that they are covered.

Example 3.2. The digraph of a matrix $\mathbf{U}$ of four states three of which, q_1, q_2, q_3 , belong to the set Q and one, q_1 , belongs to the set $\mathfrak{Q}$, is represented by:

$$\mathbf{U} = \begin{matrix} & \begin{matrix} q_0 & q_1 & q_2 & \mathfrak{q}_1 \end{matrix} \\ \begin{matrix} q_0 \\ q_1 \\ q_2 \\ \mathfrak{q}_1 \end{matrix} & \begin{pmatrix} p_{11} & p_{12} & p_{13} & l_{14} \\ p_{21} & p_{22} & p_{23} & l_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ b_{41} & b_{42} & b_{42} & 0 \end{pmatrix} \end{matrix}.$$

For $r = 3$ the digraph will be:

All nodes belonging to the set Q of the states are accessible and communicating with each other according to the values assumed by the inputs of the matrix $\mathbf{U}$. Each state can be determined based on the values assumed by the elements of

375 the matrix $\mathbf{U}$.

376 According to Def. (13) all the nodes belonging to the set $\mathfrak{Q}$, are neither accessible
377 nor communication between one another.

378 **Proposition 2.** *All states belonging to the set Q are **absorbent** if the state of*
379 *the node that precedes them belongs to $\mathfrak{Q}$.*

380 *Proof.* This follows from the Def. (19) and from Eq. 22. Moreover, the fact
381 that the matrix $\mathbf{U}$ is defined as stochastic, makes sure that each state belonging
382 to $\mathfrak{Q}$ has one and only one absorbing state in the set Q .

383 □

384 **Proposition 3.** *All the states belonging to the set $\mathfrak{Q}$ are **transitory** states.*

385 *Proof.* This follows from the Def. (21) and depends on the null block T from
386 Eq. 10. For this reason the elements of the matrix U , that lead from $\mathfrak{Q}$ to $\mathfrak{Q}$
387 are null.

388 □

389 3.5 Stochastic matrix of the alphabet Γ

390 Consider the Lemma (1).

391 Given a probability space $(\Omega_1, \mathcal{F}_1, \mathbb{P}_1)$. At each probability value of matrix
392 $\mathbf{P}_\Gamma$ the probability of an incompatible event can be added. In order to make
393 the transition probabilities vary, also in this case a null square matrix is de-
394 fined:

Definition 13.

$$\mathbf{S} = \begin{pmatrix} q_{11} & q_{12} & \dots & q_{1m} \\ q_{21} & q_{22} & \dots & q_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ q_{m1} & q_{m2} & \dots & q_{mm} \end{pmatrix} \quad \text{with} \quad q_{11} = q_{1m} = \dots = q_{mm} = 0. \quad (23)$$

395 The matrix $\mathbf{S}$ represents a matrix of the adjacency of a digraph whose nodes
 396 represent a transition code, *not recognized by either a TM or a PM*; the elements
 397 of this code are part of a set $\mathfrak{S} = \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3, \mathfrak{s}_4 \dots \mathfrak{s}_n\}$.

One obtains that the set $\mathfrak{S}$:

$$\mathfrak{S} \notin \Gamma \quad \text{and} \quad \mathfrak{S} \notin A.$$

Making the direct sum $\mathbf{V} = \mathbf{P}_\Gamma \oplus \mathbf{S}$ one obtains a square matrix:

$$\mathbf{V} = \begin{pmatrix} p_{11} & p_{12} & \dots & p_{1r} & 0 & 0 & \dots & 0 \\ p_{21} & p_{22} & \dots & p_{2r} & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 & 0 & \dots & 0 \\ p_{r1} & p_{r2} & \dots & p_{rr} & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & q_{11} & q_{12} & \dots & q_{1m} \\ 0 & 0 & \dots & 0 & q_{21} & q_{22} & \dots & q_{2m} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & q_{m1} & q_{m2} & \dots & q_{mm} \end{pmatrix}. \quad (24)$$

398 Also in this case the defined matrix can be divided into four blocks:

$$\mathbf{V} = \left(\begin{array}{c|c} \mathbf{P}_\Gamma & \mathbf{N} \\ \hline \mathbf{O} & \mathbf{S} \end{array} \right). \quad (25)$$

399 The submatrix $\mathbf{P}_\Gamma$ and the submatrix $\mathbf{S}$ have already been defined, while the

400 submatrices $\mathbf{O}$ and $\mathbf{N}$ are modified by the following

Definition 14.

$$\mathbf{O} = \begin{pmatrix} c_{(r+1)1} & c_{(r+1)2} & \cdots & c_{(r+1)r} \\ c_{(r+2)1} & c_{(r+2)2} & \cdots & c_{(r+2)r} \\ \vdots & \vdots & \ddots & \vdots \\ c_{(r+m)1} & c_{(r+m)2} & \cdots & c_{(r+m)r} \end{pmatrix}. \quad (26)$$

Where, for each $i \in \mathbb{N}_{r+m}$ and $j \in \mathbb{N}_r$ one has:

$$(c_{ij}) = \{0, 1\} \quad \text{and} \quad \sum_{j=1}^r c_{ij} = 1. \quad (27)$$

401 The $\mathbf{N}$ block is modified and becomes a matrix that contains the transition
402 probabilities that link the characters of the alphabet to the *transition code*.

403 The following definition is given

Definition 15.

$$\mathbf{N} = \begin{pmatrix} g_{1(r+1)} & g_{1(r+2)} & \cdots & g_{1(r+m)} \\ g_{2(r+1)} & g_{2(r+2)} & \cdots & g_{2(r+m)} \\ \vdots & \vdots & \ddots & \vdots \\ g_{r(r+1)} & g_{r(r+2)} & \cdots & g_{r(r+m)} \end{pmatrix}. \quad (28)$$

One set the $\mathbf{V}$ matrix is a stochastic matrix for each $i \in \mathbb{N}_r$ and $j \in \mathbb{N}_{r+m}$ must be:

$$\sum_{j=1}^r p_{ij} + \sum_{j=r+1}^m p_{ij} = 1. \quad (29)$$

Then substituting the submatrix $\mathbf{N}$ in the block the of stochastic matrix $\mathbf{V}$ one

obtains the *stochastic matrix of the alphabet*⁹:

$$\mathbf{V} = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1r} & g_{1(r+1)} & g_{1(r+2)} & \cdots & g_{1(r+m)} \\ p_{21} & p_{22} & \cdots & p_{2r} & g_{2(r+1)} & g_{2(r+2)} & \cdots & g_{2(r+m)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & & \vdots \\ p_{r1} & p_{r2} & \cdots & p_{rr} & g_{r(r+1)} & g_{r(r+2)} & \cdots & g_{r(r+m)} \\ c_{(r+1)1} & c_{(r+1)2} & \cdots & c_{(r+1)r} & 0 & 0 & \cdots & 0 \\ c_{(r+2)1} & c_{(r+2)2} & \cdots & c_{(r+2)r} & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{(r+m)1} & c_{(r+m)2} & \cdots & c_{(r+m)r} & 0 & 0 & \cdots & 0 \end{pmatrix}. \quad (30)$$

404 The modified $\mathbf{V}$ matrix is still a stochastic matrix. Moreover this matrix rep-
 405 represents the adjacency matrix of a digraph $\mathcal{D} = \langle V_v, E_v \rangle$ with $V_v = \Gamma \cup \mathfrak{S}$, where
 406 the set E_v includes the arcs whose probability to be covered is represented by
 407 the elements of the matrix $\mathbf{V}$.

Example 3.3. One considers the following matrix $\mathbf{V}_1$, with $s_1, s_2 \in \Gamma$ and $\mathfrak{s}_1 \in \mathfrak{S}$.

$$\mathbf{V}_1 = \begin{matrix} & s_1 & s_2 & \mathfrak{s}_1 \\ \begin{matrix} s_1 \\ s_2 \\ \mathfrak{s}_1 \end{matrix} & \begin{pmatrix} p_{11} & p_{12} & g_{13} \\ p_{21} & p_{22} & g_{23} \\ c_{31} & c_{32} & 0 \end{pmatrix} \end{matrix},$$

408 for $r = 2$ one has the digraph in Fig. 2:

409 3.6 Stochastic displacement matrix D

410 A TM reads and writes on a one-dimensional tape on which it is necessary to
 411 move the read-write head, so that the possible degrees of freedom are the three
 412 displacements $D = \{d, i, S\}$, and this set must be contained in a PM, according
 413 to Eq. (14). The set D , however, must be represented by a Markov matrix, as

⁹This stochastic matrix will be studied in the physical part of this text

Figure 2: Digraph of matrix $\mathbf{V}_1$ in Example 3.3.

it follows from the arguments used for U and V above.

Definition 16. Consider the lemma (1). Given a probability space $(\Omega_2, \mathcal{F}_2, \mathbb{P}_2)$, one defines a matrix:

$$D = \begin{pmatrix} p_{11} & p_{12} & h_{13} & h_{14} \\ p_{21} & p_{22} & h_{23} & h_{24} \\ d_{31} & d_{32} & 0 & 0 \\ d_{41} & d_{42} & 0 & 0 \end{pmatrix}. \quad (31)$$

A set $\mathfrak{D} \notin D$, which does not represent any real displacement, but a change in the direction of the head of the PM.

It should be noted that the Markov matrix $\mathbf{D}$ will give values that do not belong to the set $\{d, s\}$ i.e., values for which a true displacement can not be defined.

Then one has the following definition:

Definition 17. Given a Markov matrix $\mathbf{D}$, all the values such that the components of the matrix $\mathbf{D}$ do not yield a movement $\{d, s\}$ (that is for the set of matrices for which $\bar{\mathbf{D}} = \{P_D \mid i = j \mid p_{ij} \neq 1\}$) are not interpreted by a TM, and they hold the TM in a position $s \in S$.

Therefore, the fact that the head of a TM remains fixed in a position of the writing-reading tape is determined by an interpretation of the stochastic matrix $\mathbf{D}$. If the configurations r or l are clearly determinable the PM will memorize a

displacement, whereas if the reading is not clearly defined a TM interprets it as
immobility. Then, by considering the Definition (16) one explicitly the elements
of the stochastic matrix $\mathbf{D}$ to clarify how this matrix works:

$$\mathbf{D} = \begin{matrix} & \begin{matrix} r & l & \mathfrak{d}_1 & \mathfrak{d}_2 \end{matrix} \\ \begin{matrix} r \\ l \\ \mathfrak{d}_1 \\ \mathfrak{d}_2 \end{matrix} & \begin{pmatrix} p_{11} & p_{12} & h_{13} & h_{14} \\ p_{21} & p_{22} & h_{23} & h_{24} \\ d_{31} & d_{32} & 0 & 0 \\ d_{41} & d_{42} & 0 & 0 \end{pmatrix} \end{matrix},$$

with $\mathfrak{d}_1, \mathfrak{d}_2 \in \mathfrak{D}$.

3.7 Determination of a state, a symbol and a direction

A state, a symbol and a direction are determined by an initial probability distribution represented by three stochastic vectors: $\mathbf{p}_U^{(l)}$ for the states, $\mathbf{p}_V^{(m)}$ for the symbols and $\mathbf{p}_D^{(n)}$ for the directions. The components of $\mathbf{p}_U^{(l)}$ represent the probability distributions of the $q, \mathfrak{q}, f \in Q \cup \mathfrak{Q} \cup F$ states. The components of the stochastic vector $\mathbf{p}_V^{(m)}$ represent the probability distributions $s, \mathfrak{s} \in \Gamma \cup \mathfrak{S}$. The components of the stochastic vector $\mathbf{p}_D^{(n)}$ represent the probability distributions $d, \mathfrak{d} \in D \cup \mathfrak{D}$.

So the ordered tuple:

$$\langle \mathbf{p}_U^{(l)}, \mathbf{p}_V^{(m)}, \mathbf{p}_D^{(n)} \rangle, \quad \text{with } l, m, n \in \mathbb{N}, \quad (32)$$

represents the initial distribution of the system. The product of these stochastic vectors by the respective stochastic matrices determines the symbol, status and direction exiting a computation. There will be, for a given initial distribution of the three stochastic vectors defined by Eq. (32):

$$\mathbf{p}_U^{(0)} \cdot \mathbf{U} = \mathbf{p}_U^{(1)};$$

$$\mathbf{p}_V^{(0)} \cdot \mathbf{V} = \mathbf{p}_V^{(1)};$$

$$\mathbf{p}_D^{(1)} \cdot \mathbf{D} = \mathbf{p}_D^{(1)}.$$

445 So the tuple of the three stochastic vectors $\langle \mathbf{p}_U^{(1)}, \mathbf{p}_V^{(1)}, \mathbf{p}_D^{(1)} \rangle$ expresses respec-
 446 tively a state, a symbol and a direction usable by an PM.

Example 3.4. *One considers the following matrices:*

$$\mathbf{U} = \begin{matrix} & \begin{matrix} q_1 & q_2 & \mathfrak{q}_1 & \mathfrak{q}_2 \end{matrix} \\ \begin{matrix} q_1 \\ q_2 \\ \mathfrak{q}_1 \\ \mathfrak{q}_2 \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}; \quad \mathbf{V} = \begin{matrix} & \begin{matrix} s_1 & s_2 & \mathfrak{s}_1 & \mathfrak{s}_2 \end{matrix} \\ \begin{matrix} s_1 \\ s_2 \\ \mathfrak{s}_1 \\ \mathfrak{s}_2 \end{matrix} & \begin{pmatrix} 0 & 0.8 & 0.2 & 0 \\ 0.5 & 0.3 & 0.1 & 0.1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix};$$

$$\mathbf{D} = \begin{matrix} & \begin{matrix} r & l & \mathfrak{d}_1 & \mathfrak{d}_2 \end{matrix} \\ \begin{matrix} r \\ l \\ \mathfrak{d}_1 \\ \mathfrak{d}_2 \end{matrix} & \begin{pmatrix} 0.3 & 0.3 & 0.2 & 0.2 \\ 0.8 & 0.1 & 0.05 & 0.05 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}.$$

The tuple representing the initial probability distribution is:

$$\mathbf{p}_U^{(0)} = (0,1 \quad 0,2 \quad 0,4 \quad 0,3); \quad \mathbf{p}_V^{(0)} = (0,6 \quad 0,2 \quad 0,1 \quad 0,1);$$

$$\mathbf{p}_D^{(0)} = (1 \quad 0 \quad 0 \quad 0).$$

By multiplying for the respective matrices $\mathbf{U}, \mathbf{V}$ and $\mathbf{D}$, the following stochastic

vectors are obtained:

$$\begin{aligned}\mathbf{p}_U^{(1)} &= \begin{pmatrix} 0,6 & 0,4 & 0 & 0 \end{pmatrix} && \in K \cup \mathfrak{Q}; \\ \mathbf{p}_V^{(1)} &= \begin{pmatrix} 0,2 & 0,64 & 0,14 & 0,02 \end{pmatrix} && \in A \cup \mathfrak{S}; \\ \mathbf{p}_D^{(1)} &= \begin{pmatrix} 0,3 & 0,3 & 0,2 & 0,2 \end{pmatrix} && \in D \cup \mathfrak{D}.\end{aligned}$$

447 3.8 On the variation of the Markov stochastic process

448 The PM is defined by Markov stochastic processes, described by the matrices $\mathbf{U}$,
449 $\mathbf{V}$ and $\mathbf{D}$. If these matrices would remain the same at each step of computation,
450 one would obtain a *homogeneous stochastic processes*. However, in fact the prob-
451 abilities of the events may change at each step, and so should do the matrices
452 $\mathbf{U}$, $\mathbf{V}$ and $\mathbf{D}$. Therefore the following proposition is to be considered:

Proposition 4. *The difference between two square stochastic matrices of order n , generates a matrix $\mathbf{W}$ such that:*

$$\mathbf{W} = (w_{ij}), \quad i, j = 1, 2, \dots, n \in \mathbb{N};$$

with

$$-1 \leq w_{ij} \leq 1 \quad \forall \quad i, j = 1, 2, \dots, n \in \mathbb{N}.$$

Furthermore, for each line it must be verified that

$$\sum_{i=1}^n w_{ij} = 0.$$

Proof. Let $\mathbf{A} = a_{ij}$ and $\mathbf{B} = b_{ij}$ be two stochastic matrices. For the first condition one has that the elements a_{ij} of the stochastic matrix $\mathbf{A}$, vary within the interval $[0, 1]$ while, because of the difference

$$\mathbf{A} - \mathbf{B},$$

453 the elements of the matrix $-\mathbf{B}$ will vary within the interval $[0, -1]$. So the set
 454 given by $a_{ij} - b_{ij}$ must vary within the interval $[-1, 1]$.

Regarding the second condition, considering that for each row of each matrix one must have:

$$\sum_{i=1}^n a_{ij} = 1 \quad \text{and} \quad \sum_{i=1}^n b_{ij} = 1.$$

If one considers the first line of each of the two matrices, one has for $\mathbf{A}$:

$$\sum_{i=1}^n a_{i1} = a_{11} + a_{21} + a_{31} + \cdots + a_{i1} = 1.$$

Equally for $\mathbf{B}$:

$$\sum_{i=1}^n b_{i1} = b_{11} + b_{21} + b_{31} + \cdots + b_{i1} = 1.$$

Then the difference:

$$\sum_{i=1}^n a_{i1} - \sum_{i=1}^n b_{i1} = a_{11} - b_{11} + a_{21} - b_{21} + a_{31} - b_{31} + \cdots + a_{i1} - b_{i1} = 1 - 1 = 0.$$

455 The same reasoning can be repeated for each row of $\mathbf{A}$ and $\mathbf{B}$.

456 □

457 In order to let the content of one or more elements of the matrices $\mathbf{U}, \mathbf{V}$ and $\mathbf{D}$
 458 vary, the following definitions are given at each computation step.

Definition 18. One defines a matrix $\mathbf{W}_U$:

$$\mathbf{W}_U = \mathbf{U}_a - \mathbf{U}_b = \begin{pmatrix} w_{11} & w_{12} & \dots & w_{1n} \\ w_{21} & w_{22} & \dots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \dots & w_{nn} \end{pmatrix}. \quad (33)$$

459 Repeating the same reasoning for $\mathbf{V}$ and $\mathbf{D}$ one has:

Definition 19. One defines a matrix $\mathbf{W}_V$:

$$\mathbf{W}_V = \mathbf{V}_a - \mathbf{V}_b. \quad (34)$$

Definition 20. One defines a matrix $\mathbf{W}_D$:

$$\mathbf{W}_D = \mathbf{D}_a - \mathbf{D}_b. \quad (35)$$

460 According to Proposition (4) any $\mathbf{W}$ matrix can be obtained as the difference
461 between at least two stochastic matrices of the same order.

462 **Definition 21.** Adding a generic stochastic matrix $\mathbf{A} = (a_{ij})$ to a $\mathbf{W} = (w_{ij})$
463 matrix of the same order n one again obtains a stochastic matrix if and only if
464 the following conditions are met:

$$\begin{aligned} \sum_{j=1}^n a_{ij} + w_{ij} &= 1; \\ \sum_{j=1}^n w_{ij} &= 0; \\ a_{ij} + w_{ij} &\in [0, 1] \quad \forall a_{ij}, w_{ij} \in \mathbb{R}. \end{aligned} \quad (36)$$

465 The generic matrix $\mathbf{W}$ is able to modify the transition probabilities of the ma-
466 trices $\mathbf{U}, \mathbf{V}, \mathbf{D}$ at each computation step. Obviously the $\mathbf{W}$ matrix can also be
467 null. The justification for its construction is obtained by taking into account
468 the axioms of the probability theory (Kolmogorov 1956), discussed in Section
469 (1) below.

Denote by $\mathbf{A}^{(t)}$ and $\mathbf{W}^{(t)}$ respectively a stochastic matrix and a matrix $\mathbf{W}$ with
an index $t \geq 0$. Moreover, let $\mathbf{p}^{(0)}$ be the initial distribution of a family of ran-
dom variable X_t belonging to a set of finite S states, defined on a probability
space $(\Omega, \mathcal{F}, \mathbb{P})$. Then a homogeneous Markov chain, given the initial distribu-
tion $\mathbf{p}^{(0)}$, for t steps will be given by the following of Chapman-Kolmogorov

equation:

$$\mathbf{p}^{(t)} = \mathbf{p}^{(0)}(\mathbf{A}^{(t)} + \mathbf{W}^{(t)})^t, \quad (37)$$

with the conditions

$$\sum_{j=1}^n a_{ij}^{(t)} + w_{ij}^{(t)} = 1;$$

$$a_{ij} + w_{ij} \in [0, 1] \quad \forall a_{ij}, w_{ij} \in \mathbb{R}.$$

By considering in particular the matrices $\mathbf{U}, \mathbf{V}, \mathbf{D}$ e and the respective matrices $\mathbf{W}_U, \mathbf{W}_V, \mathbf{W}_D$, one defines three initial distributions on the sets $Q \cup \Omega, \Gamma \cup \mathfrak{S}, D \cup \mathfrak{D}$. The initial distributions defined in Eq. (32), are the vectors $\mathbf{p}_U^{(0)}, \mathbf{p}_V^{(0)}, \mathbf{p}_D^{(0)}$. one has then that:

$$\begin{cases} \mathbf{p}_U^{(l)} = \mathbf{p}_U^{(0)} \cdot (\mathbf{U}^{(0)} + \mathbf{W}_U^{(0)}) \cdot (\mathbf{U}^{(1)} + \mathbf{W}_U^{(1)}) \cdot \dots \cdot (\mathbf{U}^{(l)} + \mathbf{W}_U^{(l)}) \\ \mathbf{p}_V^{(m)} = \mathbf{p}_V^{(0)} \cdot (\mathbf{V}^{(0)} + \mathbf{W}_V^{(0)}) \cdot (\mathbf{V}^{(1)} + \mathbf{W}_V^{(1)}) \cdot \dots \cdot (\mathbf{V}^{(m)} + \mathbf{W}_V^{(m)}) \\ \mathbf{p}_D^{(n)} = \mathbf{p}_D^{(0)} \cdot (\mathbf{D}^{(0)} + \mathbf{W}_D^{(0)}) \cdot (\mathbf{D}^{(1)} + \mathbf{W}_D^{(1)}) \cdot \dots \cdot (\mathbf{D}^{(n)} + \mathbf{W}_D^{(n)}), \end{cases} \quad (38)$$

where:

$$\begin{aligned} \mathbf{U}^{(l)} &= \mathbf{U}^{(l-1)} + \mathbf{W}_U^{(l-1)} \\ \mathbf{V}^{(m)} &= \mathbf{V}^{(m-1)} + \mathbf{W}_V^{(m-1)} \\ \mathbf{D}^{(n)} &= \mathbf{D}^{(n-1)} + \mathbf{W}_D^{(n-1)}. \end{aligned}$$

470 In conclusion, three independent stochastic processes give rise to three stochastic
471 vectors. Furthermore the generated Markov processes are not homogeneous if
472 the matrices $\mathbf{W}_Q, \mathbf{W}_V, \mathbf{W}_D$ are not simultaneously null. To get an idea of this
473 procedure, consider the following example:

474 **Example 3.5.** *A library consisting of several floors contains a finite number*
475 *of books in each floor. A professor in the morning before opening to the public*
476 *enters the library and accesses a floor via an elevator, but the elevator is faulty,*
477 *the selected floor may not correspond to the floor where the elevator stops. The*
478 *teacher marks the plan, accesses it and chooses a book based on the title. It*

479 also marks the position by using the books to the left and right of the chosen
 480 text as a reference to resume it on the next morning. Immediately after the
 481 professor, a huge group of students entered the library to consult the books and
 482 at the end of the day put the books on the shelves in an order deliberately chosen
 483 by each student. In addition, the librarian often changes the titles of the books
 484 that are not known to all users. At the end of the evening, just before closing,
 485 the professor will return the book taken in the morning to the librarian.

486 The floor of the library to which the elevator will stop is represented by the
 487 vector $\mathbf{p}_D$, the position of the books inside the floor is specified by the vector
 488 $\mathbf{p}_U$, while the titles of the books are represented by the vector $\mathbf{p}_V$. The behavior
 489 of the elevator is described by a stochastic matrix given by $\mathbf{D}^{(n)} + \mathbf{W}_D^{(n)}$, the
 490 behavior of the students by $\mathbf{U}^{(l)} + \mathbf{W}_U^{(l)}$ and finally the behavior of the librarian
 491 by $\mathbf{V}^{(m)} + \mathbf{W}_V^{(m)}$.

492 Given an initial distribution of the three vectors, $(\mathbf{p}_U^{(0)}, \mathbf{p}_V^{(0)}, \mathbf{p}_D^{(0)})$ represents an
 493 initial probability distribution of: the arrangement of books within the library,
 494 the titles of the books that are present, the floor to which the elevator stops
 495 by pushing a button, respectively. Therefore, with this initial distribution it is
 496 possible to know the probability distributions of the three stochastic vectors at
 497 each successive access of the professor to the library.

498 As the example underlines, the state of the system is described by three stochas-
 499 tic processes, an initial distribution and three successive vectors $(\mathbf{p}_U^{(l)}, \mathbf{p}_V^{(m)}, \mathbf{p}_D^{(n)})$
 500 after l, m, n steps of each stochastic process.

501 The probability of finding the book chosen by the professor and returned to the
 502 librarian will be discussed abstractly in Section 6.

503 3.9 Determination of the configuration of the transitions 504 of a PM

505 To be able to determine the configuration of a PM, by recalling the definition
506 given in Section 3.1, it is necessary to make some observations:

507 **Observation 3.2.** The matrices $\mathbf{U}$ and $\mathbf{V}$ represent all the probability of tran-
508 sition of respectively states belonging to the set Q and the whole alphabet
509 belonging to the set Γ .

510 **Observation 3.3.** The sets $\mathfrak{Q}$ and $\mathfrak{S}$ and $\mathfrak{D}$ do not belong and are external
511 to the definition sets of the MT and the PM. They are transition sets that
512 determine the variation of the components of the matrices $\mathbf{U}$, $\mathbf{V}$ and $\mathbf{D}$.

513 Then according to the definition (9) the following equivalence relation is deter-
514 mined:

515 **Proposition 5.** *Given an instantaneous configuration $\mathcal{C}$ of a MP, it is equiva-*
516 *lent if shall be substituted the sets Q and A with the sets of the matrices of the*
517 *transition probabilities of the matrices U and V .*

Proof. For $n, m \in \mathbb{N}$ one has:

$$P_U = \{\mathbf{U}_1, \mathbf{U}_2, \dots, \mathbf{U}_n\} \quad \text{and} \quad P_V = \{\mathbf{V}_1, \mathbf{V}_2, \dots, \mathbf{V}_m\}, \quad (39)$$

Moreover one has:

$$Q \subset K \subset P_U, \quad (40)$$

$$\bar{\Gamma} \subset A \subset P_V. \quad (41)$$

518 Then one assumes that a TM can indifferently work with one of the three sets
519 of the states Q, K, P_U and with one of the sets of the alphabet $\bar{\Gamma}, A, P_V$. On the
520 contrary, a PM is able to work only with the sets of K, P_U and one of the sets
521 of the alphabet A, P_V .

522

□

Therefore, for a PM the instant configuration $\mathcal{C}'$ is given:

$$\mathcal{C}' = X' \circ P_Q \circ Y', \quad (42)$$

523 with:

524
$$(1) \ X' \in P_V \circ P_V^*;$$

525
$$(2) \ P_Q \in P_U;$$

526
$$(3) \ Y' \in P_V^* \circ P_V.$$

The configuration $\mathcal{C}'$ represents a complete instant configuration of a PM. A PM which carries out any computation, which will be indicated by the sign $\vdash$, passes through a series of instantaneous configurations c'_i and is indicated:

$$\dots \vdash c'_i \vdash c'_1 \vdash c'_2 \vdash \dots \vdash c'_j \vdash \dots$$

527 **Observation 3.4.** A PM, in its definition set, possesses initial states q_0 and
528 final states F but they are recognized as such only by a TM. All internal states
529 of a PM are equivalent for a PM.

530 If a computation is performed again with the same steps there will be a proba-
531 bility that it will give the same result from c'_i to c'_j .

532 Less formally, the PM contains in itself all the elements recognized by a TM, but
533 they are scattered within a set of random elements that must be focused in order
534 to be recognized by a TM. A TM recognizes well-defined internal states and
535 alphabet, thus representing stochastic matrices whose components give almost
536 certain values.

537 The $\mathbf{U}$ and $\mathbf{V}$ matrices may contain *absorbent* elements, whose values are almost

538 certainly close to one. This means that these states are recurring. This also
539 means each symbol according to the Definition (4), recognized by a TM is
540 determined by the values assumed by the components of the matrix.

Example 3.6. *Given an alphabet of four symbols s_1, s_2, s_3, s_4 belonging to Γ and of four elements $\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3, \mathfrak{s}_4$ belonging to $\mathfrak{S}$. The matrix $\mathbf{V}$ representing a symbol may be the following:*

$$\mathbf{V} = \begin{pmatrix} 0.3 & 0.2 & 0.2 & 0.3 & 0 & 0 & 0 & 0 \\ 0.3 & 0.2 & 0.1 & 0.4 & 0 & 0 & 0 & 0 \\ 0.3 & 0.3 & 0.2 & 0.2 & 0 & 0 & 0 & 0 \\ 0.4 & 0.2 & 0.2 & 0.2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

If one adds to this matrix a $\mathbf{W}_V$ matrix $\mathbf{V}$ is modified and this can make a symbol readable:

$$\mathbf{W}_V = \begin{pmatrix} 0.7 & -0.2 & -0.2 & -0.3 & 0 & 0 & 0 & 0 \\ -0.3 & 0.8 & -0.1 & -0.4 & 0 & 0 & 0 & 0 \\ -0.3 & -0.3 & -0.2 & 0.8 & 0 & 0 & 0 & 0 \\ -0.4 & -0.2 & -0.2 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Then adding up according to the Eq. (19):

$$\mathbf{V} + \mathbf{W}_V = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Given an initial stochastic vector $\mathbf{p}_V^{(0)} = (0, 0, 1, 0, 0, 0, 0, 0)$, one has:

$$\mathbf{p}_V^{(0)} \cdot (\mathbf{V} + \mathbf{W}_V) = \mathbf{p}_V^{(1)},$$

so that $\mathbf{p}_V^{(1)}$ can be used to define the symbol s_3 .

4 Definition of symbol by a TM

The example reported above allows us to introduce a formal definition of a symbol of an alphabet. A further example is proposed. The symbol that represents the capital letter A is recognized by everyone both on printed texts and in texts written manually. In fact to obtain this symbol it is necessary to execute an algorithm, but this algorithm is executable in very different ways, (think about the handwriting of each one of us). Yet, in a hand-written text, in most cases one can recognize the letter A. With this example in mind, one introduces the following Definition:

Definition 22. One defines an elementary TM, $\mathcal{M}_0$, able to read a symbol of the alphabet and end the computation in a final state keeping the symbol unchanged. At the end of the computation $\mathcal{M}_0$ is able to start a new computation.

Figure 3: Example of State Transition Graph according to Definition 22 for different symbols.

Observation 4.1. According to Definition (4) if there exists a TM able to terminate a computation starting from a state q_0 and ending in a state q_f , so that the computation:

$$\mathcal{M}_1 : \quad c_1 \vdash c_2 \vdash \dots \vdash c_n,$$

554 starts always the computation from $\langle q_0, s_1 \rangle$ and ends the computation always
555 with $\langle q_F, s_n \rangle$, then such TM is able to generate a symbol of an alphabet.

For any initial probability distribution of the three vectors

$$(\mathbf{p}_U^{(0)}, \mathbf{p}_V^{(0)}, \mathbf{p}_D^{(0)}),$$

556 defined in Eq. (32), $\mathcal{M}_0$ will always yield in the same value.

557 Now one assumes that the result obtained by the machine $\mathcal{M}_1$ is used by a Uni-
558 versal Turing machine (Turing 1937) $\mathcal{U} = \langle Q', \Gamma, \mathfrak{b}, \delta', q'_0, F' \rangle$ for a computation.
559 Therefore, in conclusion, it is possible to obtain the alphabet of a TM if one
560 considers that each symbol can be obtained from an elementary TM and this
561 alphabet can be used by a universal TM.

562 **Proposition 6.** *For a given alphabet Γ , consisting of n symbols, there must be*
563 *at least n initial states $q_0^1, q_0^2, \dots, q_0^n$ and n final states $f_1, f_2, \dots, f_n$.*

564 *Proof.* According to Definition (4) a TM must execute an algorithm to obtain a
565 symbol, thus for n distinguishable symbols must exist n distinct algorithms and
566 therefore $q_0^1, q_0^2, \dots, q_0^n$ initial states. All symbols must also be recognized by a

567 TM, therefore a TM must also terminate the maximal computation, so there
 568 must also exist n final unique states.

569 □

570 5 Operation of a PM

Taking into account the Eqs. (30), (39), (40) and replacing the sets of transition probability matrices in Eqs. (8) and (13) one has:

$$\mathcal{M}_{\mathcal{P}} = \langle P_U, \bar{\mathbf{Q}}, P_V, B, \zeta, q_0, P_F \rangle. \quad (43)$$

$$\zeta : (P_U - P_F) \times P_V \rightarrow P_U \times P_V \times P_D. \quad (44)$$

571 In addition the probability distribution of a state, a symbol and a direction is
 572 given by the tuple defined in Eq. (32). The definitions of the sets are given in
 573 Section 2, while the set $P_F \subset P_U$, is defined as the set of matrices of the transition
 574 probabilities of the final states in which a TM ends the computation.

Given a matrix of Markov U_0 , and an initial state q_0 it must be verified that the path starting from q_0 through the matrix U_0 must lead to q_0 itself, i.e. the path that starts from q_0 , must be absorbing on q_0 . Otherwise the q_0 state can not be clearly read by a TM interpreting the code. The same holds for any symbol of the alphabet Γ . To sum up, involved sets are:

$$\bar{\mathbf{Q}} = \{\bar{q}_0, \bar{q}_1, \dots, \bar{q}_n\},$$

$$Q = \{q_0, q_1, \dots, q_n\},$$

$$F = \{f_1, f_2, \dots, f_n\},$$

Figure 4: Graph of the program to run from a TM in Tab. 1

and:

$$B = \{\bar{b}_1, \bar{b}_2, \dots, \bar{b}_n\},$$

$$\Gamma = \{s_1, s_2, \dots, s_n\},$$

$$\bar{\mathbf{D}} = \{\bar{d}_0, \bar{d}_1, \dots, \bar{d}_n\}.$$

575 The code generated by a PM is not always readable by a TM, but when this
 576 condition occurs, i.e., when a Markov matrix of states has no absorbing states,
 577 these values are substituted with the sign $\sqcup$ to indicate a symbol belonging to
 578 $\bar{\mathbf{Q}}$ not defined for a TM. Any undefined symbol on a TM can not generate any
 579 transition function for a TM, and will be indicated by the abbreviation *n. d.*
 580 (not determined).

581 In Table 1 is reported an example of how a PM works. On the left are the
 582 values assumed by the definition sets of a PM, given a tuple of initial proba-
 583 bility distribution defined in Eq. (32). Then the function ζ generated by the
 584 previous sets appears. Subsequently, the function δ is presented, which belongs
 585 to the n -tuple of definition of a TM. The last column of the table presents the
 586 interpretation by a TM of the function δ . The symbol *n. d.* indicates that a TM
 587 does not recognize any function δ belonging to the definition n -tuple.

588 Points indicate a sequence of instructions that are not defined for a TM.

589 The described PM executes the instructions using sets not defined for a TM.
 590 The first algorithm executable by TM allows it to start the computation on the
 591 symbol s_1 . One can be seen graphically in Fig. 4.

592 **Observation 5.1.** The initial configuration of a TM is defined by the sequence:

Table 1: Example of how a PM works.

P_U	P_V	P_U	P_V	P_D	ζ	δ	TM Resp.
...	...	...	...	...	...	...	n. d.
$\bar{q}_1$	s_2	q_1	s_2	$\bar{d}_1$	$\zeta(\bar{q}_1, s_2) = (q_1, s_2, \bar{d}_1)$	$\delta(\sqcup, s_2) = (q_1, s_2, s)$	n. d.
$\bar{q}_2$	$\bar{b}_2$	$\bar{q}_2$	$\bar{b}_3$	$\bar{d}_4$	$\zeta(\bar{q}_2, \bar{b}_2) = (\bar{q}_2, \bar{b}_3, \bar{d}_4)$	$\delta(\sqcup, \mathfrak{b}) = (\sqcup, \mathfrak{b}, s)$	n. d.
q_1	$\bar{b}_2$	q_1	$\bar{b}_1$	r	$\zeta(q_1, \bar{b}_2) = (q_1, \bar{b}_1, r)$	$\delta(q_1, \mathfrak{b}) = (q_1, \mathfrak{b}, r)$	n. d.
$\bar{q}_2$	$\bar{b}_3$	$\bar{q}_2$	$\bar{b}_1$	r	$\zeta(\bar{q}_2, \bar{b}_3) = (q_1, \bar{b}_5, l)$	$\delta(q_1, \mathfrak{b}) = (q_1, \mathfrak{b}, l)$	n. d.
q_0	s_1	f_1	s_2	r	$\zeta(q_0, s_1) = (f_1, s_2, r)$	$\delta(q_0, s_1) = (f_1, s_2, r)$	
$\bar{q}_2$	s_1	f_1	s_3	l	$\zeta(\bar{q}_2, s_1) = (f_1, s_3, l)$	$\delta(\sqcup, s_1) = (f_1, s_3, l)$	n. d.
$\bar{q}_2$	$\bar{b}_3$	q_1	s_3	l	$\zeta(\bar{q}_2, \bar{b}_3) = (\bar{q}_2, \bar{b}_3, \bar{d}_2)$	$\delta(\sqcup, \mathfrak{b}) = (\sqcup, \mathfrak{b}, i)$	n. d.
...	...	...	...	...	...	...	n. d.
$\bar{q}_2$	$\bar{b}_3$	q_1	s_3	l	$\zeta(\bar{q}_2, \bar{b}_3) = (\bar{q}_2, \bar{b}_3, \bar{d}_2)$	$\delta(\sqcup, \mathfrak{b}) = (\sqcup, \mathfrak{b}, s)$	n. d.
q_0	s_1	q_1	s_2	r	$\zeta(q_0, s_1) = (q_1, s_2, r)$	$\delta(q_0, s_1) = (q_1, s_2, r)$	
q_1	s_2	q_2	s_1	r	$\zeta(q_1, s_2) = (q_2, s_1, r)$	$\delta(q_1, s_2) = (q_2, s_1, r)$	
q_2	$\bar{b}_1$	f_1	s_1	l	$\zeta(q_2, \bar{b}_1) = (f_1, s_1, l)$	$\delta(q_2, \mathfrak{b}) = (f_1, s_1, r)$	
...	...	...	...	...	...	...	n. d.

593 $\varepsilon q_0 s_1 s_2 s_3$, where $\varepsilon = \mathfrak{b} \circ \mathfrak{b} \circ \mathfrak{b} \circ \dots \circ \mathfrak{b}$. The probability that the sets of the
594 matrices P_U, P_V, P_D allow to express values close to one are low compared to
595 all the values between 0 and 1 that could have, so one can assume that any
596 value q_0 expressed by the matrix $\mathbf{U}$ can be preceded by an empty string ε long
597 enough to be applied to the initial configuration definition for a TM.

598 5.1 Programming machine with memory

599 A PM *with memory* is able to store the code it develops.

Definition 23. Given a PM that produces one or more codes, wich are executable by a TM, the set of such codes will be

$$T = \{\mathcal{M}_1, \mathcal{M}_2, \dots, \mathcal{M}_n\}. \quad (45)$$

600 Moreover, this set T of TMs can be simulated by a Universal Turing machine
601 $\mathcal{U}$.

602 In conclusion, once a PM produces a program that can be used by a TM, the

603 PM with memory is able to keep this code also for subsequent computations. A
 604 PM with memory is a theoretical machine more complex than a PM, because
 605 it has an additional set T .

606 6 Information Theory and stochastic matrices

607 A PM generates an algorithm that a TM can read and interpret. Any PM
 608 does not provide an algorithm in a unique and reliable way. Each step of an
 609 algorithm that is created is subject to an uncertainty given by the stochastic
 610 matrices of the transition probabilities $\mathbf{U}, \mathbf{V}, \mathbf{D}$. Since a PM combines these
 611 three matrices the result is still subject to uncertainty. However, it is possible
 612 to define a measure of the uncertainty of the result obtained from the Cartesian
 613 product of the definition sets of a PM.

614 **Definition 24.** The set on which the information is measured (see Def. 29), is
 615 represented by the transitions of the ζ function of a PM. The transitions of ζ
 616 function are completely random (see the last part of Sec. 3.1).

617 As one guesses that there are different algorithms, so one wonders how is it
 618 possible to distinguish them if performed by different TMs.

Definition 25. Let

$$\mathcal{M}_1 = \langle Q, \Gamma, \mathfrak{b}, \delta, q_0, F \rangle \quad \text{and} \quad \mathcal{M}_2 = \langle Q', \Gamma, \mathfrak{b}, \delta', q_0, F' \rangle$$

619 be two MTs. They perform two indistinguishable algorithms (Ausiello et al.
 620 2003) if:

- 621 (1) $\delta_0 = \langle q_0, a, Q, \bar{\Gamma}^*, D \rangle$ and $\delta'_0 = \langle q_0, a, Q', \bar{\Gamma}^*, D \rangle$ with $a \in \bar{\Gamma}^*$;
- 622 (2) $\forall y \in \bar{\Gamma}^* \circ \Gamma \quad \exists \quad c : \delta \langle q, y \rangle \in F \quad \text{and} \quad c' : \delta' \langle q', y \rangle \in F'$;
- 623 (3) $Q \equiv Q'$ and $F \equiv F'$.

624 From the first condition one can see that two algorithms are indistinguishable
625 if they start from the same initial transition function, for example one can not
626 distinguish two algorithms that both to compute from the number three, while
627 one can distinguish them if one starts from three and the other from two.

628 From the second condition it follows that there must exist a transition function
629 that ends with the same output for $\mathcal{M}_1$ and $\mathcal{M}_2$. The third condition implies
630 that there should not be a different state between $\mathcal{M}_1$ and $\mathcal{M}_2$, otherwise the
631 two algorithms would be distinguishable for that state.

632 Given a Markov matrix $\mathbf{U}$ or $\mathbf{V}$ and a distribution of starting point vectors,
633 these matrices can also be represented as matrices of the adjacency of a di-
634 graph. Whenever this digraph is traveled, it could define a symbol if it meets
635 the conditions involved in Definition (4). The production of a symbol by an
636 algorithm generates a *discrete source of information* that produces symbols be-
637 longing to an alphabet $\Gamma = \{s_0, s_1, \dots, s_n\}$. Thus a probability is also defined in
638 the space of the $Q = \{q_0, q_1, \dots, q_n\}$. The Q states are a set independent of Γ ¹⁰.
639 Each symbol or state is characterized by its probability, i.e. there is a proba-
640 bility that the algorithm that produced a symbol can repeat the computation
641 again and end with the same symbol as output.

642 A TM is able to perform a computation programmed by a PM if it starts with an
643 initial computation $c_0 = \varepsilon q_0 y$, moreover if a TM ends the computation it can set
644 itself indefinitely in the initial state and repeat the computation again.

645 **Definition 26.** Given the writing $c = xqy$, one indicates with $\mathcal{L}_\Gamma \in \Gamma \circ \bar{\Gamma}^* \cup$
646 $\{\varepsilon\}$ the language of the strings that can appear to the left of the state of a
647 configuration. One indicates with $\mathcal{R}_\Gamma \in \bar{\Gamma}^* \circ \Gamma \cup \{\mathbf{b}\}$ the language of the strings
648 that can appear to the right of the state of a configuration.

¹⁰Imagine a man and a rabbit observing the grass of a meadow, the man could think of mowing it, while the rabbit might think to eat it. The same object observed has different interpretations independent of the object.

A computation that starts at c_0 and leads to c_n is referred to as

$$c_0 \vdash c_1 \vdash c_2 \vdash \dots \vdash c_n,$$

where the transition from one configuration to another is described by a transition function δ . Imagine a TM operating within a PM. It executes the instructions that the PM produces gradually, recognizing its code and an initial state.

Definition 27. More computations are defined as symmetrical between themselves if, starting from the same initial configuration $c_0 = c'_0 = \varepsilon q_0 y$, for each configuration one has: $c_n \equiv c'_n$ with $\mathcal{L}_\Gamma = \mathcal{L}'_\Gamma$ and $\mathcal{R}_\Gamma = \mathcal{R}'_\Gamma$. Then for each step of computation on the tape, (defined for a TM), the same symbols appear .

Considering a PM that generates a code for a TM, defining a symmetric computation with respect to another makes sense only in probabilistic terms. Then in this case a TM can be defined as a probabilistic TM (indicated with PTM) (Arora and Barak 2009; Motwani and Raghavan 1995).

Definition 28. A PTM is defined as a six-tuple $\mathcal{M} = \langle Q, \Gamma, b, \delta, q_0, F \rangle$ with the transition function that instead of being deterministic for each computation step provides a probability distribution of the possible transitions.

One shrinks the observation to a limited set of PTM, those for which:

- (1) an initial instant configuration (elementary event) is almost certain $c_0 = \varepsilon q_0 y$ with probability $p_{c_0} = 1$;
- (2) there exists a terminal configuration $c_n = xfy$ with $f \in F$ and a probability distribution thereof $p_{c_n} \in [0, 1]$.

Then there will be a set of computations for which, given an initial state defined and almost certain, there is an uncertainty in the termination configuration:

$$c_0 \vdash^* p_{c_n}.$$

Where the $\vdash^*$ symbol indicates the reflexive and transitive closure of $\vdash$ to indicate that there is a finite number of transitions that lead from c_0 to c_n . This set is defined as:

$$G = \{c_0 \vdash^* p_{c_n} | p_{c_0} = 1 \text{ q.c.}, p_{c_n} = [0, 1]\}.$$

Thus, within the set E defined in Section 2, it will be interesting to calculate the probability of:

$$P(X_n = \langle f, s_n \rangle | X_0 = \langle q_0, s_1 \rangle) \quad \text{with} \quad X \in G, \quad (46)$$

669 where the stochastic variable X represents the expression of the transition func-
670 tion δ of a PTM.

671 From another point of view X also represents a symbol belonging to an alphabet,
672 emitted by a *stationary* and *discrete* source of information. Then it is possible
673 to measure how likely is it to observe this symbol (Shannon 1948):

Definition 29. One defines the information on the stochastic variable X ,

$$I(X) := -\log_b P(X). \quad (47)$$

Given an alphabet $\Gamma = \{s_1, s_2, \dots, s_n\}$, each symbol will have a probability

$$P(s_1), P(s_2), \dots, P(s_n),$$

674 to be expressed in a final configuration $\langle f, s_n, D \rangle$ of a PTM defined in the set G .
675 Moreover, for each symbol the information given by $I(s_n)$ is calculated.

Definition 30. One defines the information entropy (Shannon 1948):

$$H(X) := \sum_{n=1}^n P(s_n) I(s_n). \quad (48)$$

676 Information entropy provides a weighted average of the information associated
677 with each symbol. A set of algorithms able to end with an unambiguous sequence
678 of symbols can be used to create a *uniquely decipherable code*. One therefore
679 defines this code as:

$$\mathcal{A} = \{s_1, s_2, \dots, s_n\} \quad n \in \mathbb{N}. \quad (49)$$

680 7 Information of a source without memory and 681 with memory

682 So far one has considered the set G as a source without memory. However if one
683 considers a source with memory, that is, if one assume that a symbol emitted
684 by the source depends on what was emitted previously, then the information
685 relating to the emitted symbol changes and consequently decreases the entropy
686 value.

The probability of a memory source of a symbol $s_j \in \mathcal{A}$ of Eq. 49, conditioned
by the previous emission of a symbol $s_i \in \mathcal{A}$ will be:

$$P(s_j|s_i),$$

the entropy of a symbol s_i will therefore be:

$$H(\mathcal{A}|s_j) = \sum_i P(s_j|s_i) \log_2 \frac{1}{P(s_j|s_i)}. \quad (50)$$

It must also result:

$$H(\mathcal{A}|s_i) \leq H(\mathcal{A}), \quad (51)$$

687 (Shannon 1948).

688 8 Symbols of an alphabet

689 Once the matrices of the measurable symbols are obtained, and interpreted by
 690 a TM, it is possible to juxtapose the matrices V and obtain other symbols. So
 691 in general it is possible to obtain a new symbol starting from others:

692 **Definition 31.** One defines $\mathcal{V} = \{\mathbf{V}_1, \mathbf{V}_2, \dots, \mathbf{V}_n\}$ a set of stochastic matrices
 693 such that:

- 694 (1) Each matrix is able to describe an algorithm starting from a measurable
 695 initial symbol.
- 696 (2) At least one matrix $\mathbf{V}$ of the set $\mathcal{V}$ represents a computation that ends
 697 with a final measurable symbol.
- 698 (3) Any computation must always end with the same symbol.

699 Now let's consider an initial tuple consisting of $\langle q_0, \mathbf{s} \rangle$ with $\mathbf{s} = (s_1, s_2, \dots, s_n)$
 700 and $s_1, s_2, \dots, s_n \in \Gamma$. The $\langle q_0, \mathbf{s} \rangle$ pair represents the initial state of n TMs. This
 701 allows to have an initial state defined on n matrices $\mathbf{V}$. The matrices $\mathbf{V}$ each
 702 represent a probabilistic TM, hence the pair $\langle q_0, \mathbf{s} \rangle$ represents the initial state.
 703 Then one defines a symbol of an alphabet Γ' obtained by using the symbols of
 704 an alphabet Γ .

705 **Proposition 7.** *Given a pair $\langle q_0, \mathbf{s} \rangle$, defined as the initial state of $\mathcal{V}$, a symbol*
 706 *s' belonging to an alphabet Γ' is uniquely determined if for each initial state*
 707 *given by the pair $\langle q_0, \mathbf{s} \rangle$ there is at least one automaton or TM able to always*
 708 *end the computation with the pair $\langle f, \mathbf{s}' \rangle$.*

709 *Proof.* Let be the pair $\langle q_0, \mathbf{s} \rangle$ with $\mathbf{s} = (s_1, s_2, \dots, s_n)$ the initial state measured
 710 and applied to a set of stochastic matrices $\mathcal{V} = \{\mathbf{V}_1, \mathbf{V}_2, \dots, \mathbf{V}_n\}$. A symbol
 711 defined according to Definition (4) corresponds to a terminating computation.
 712 Then there must be at least one matrix belonging to $\mathcal{V}$ which ends the compu-
 713 tation with the symbol $s \in \mathbf{s}$. For each request given by $\langle q_0, \mathbf{s} \rangle$ to $\mathcal{V}$, there must

714 be a pair $\langle f, s' \rangle$ where $f \in F$.

715

□

716 In conclusion, since $\mathcal{V}$ represents a set of stochastic matrices, a compound symbol
717 must be probabilistic. Furthermore, the information relating to the outgoing
718 symbol must be zero or in any case very close to zero. This as a consequence of
719 the request to have uniqueness from the initial state to the final state. Obviously
720 through symbols derived from others it is possible to construct derived alphabets
721 and subsequently languages.

722 9 Physical part. Introduction

723 The second part of this paper aims to analyze and describe the physical con-
724 sequences of the definitions given in the first part. A study of living systems
725 must necessarily fit into an analysis of the physical system in which these are
726 observed.

727 A living system is necessarily also a thermodynamic system. Some examples
728 are proposed. When one looks at any object at a certain distance or when
729 one looks at a clock to know the time, one is actually comparing at least two
730 thermodynamic systems, one measuring and the other being measured. Such
731 systems will certainly have at least one level of energy in common at the time of
732 measurement. If two or more thermodynamic systems carry out a measurement,
733 there must be a common state that corresponds to the measurement itself. For
734 example, when an observer wants to establish the distance that separates him
735 from an object, he constructs an algorithm that divides this distance into parts
736 of a standard measure accepted by the observer, (meter, light year, etc.). The
737 fundamental characteristic of these measurement algorithms is that they must be
738 repeatable, that is, a second observer in the same conditions as the first one must

739 obtain the same result of measurement, or a change in the reference systems that
740 keeps unchanged all the energy levels on which they are performed the measures,
741 (i. e. not accelerating the system), must return the same measure. In the same
742 way, for the measurement of time, algorithms are applied that are able to divide
743 a time interval in relation to a cyclical system used as standard of measurement
744 and also in this case both the standard and the measurement must be repeatable
745 by different observers in the same conditions. These hypotheses can be true for
746 macroscopic systems, but the question is still debated for quantum systems. The
747 questions posed by J. S. Bell are interesting on this subject (Bell 1987).

748 These examples want to emphasize two fundamental aspects, the first is that
749 the observer making a measurement in space or time is applies an algorithm, an
750 exact sequence of instructions that will return a measurement value. The second
751 aspect concerns the fact that a second observer placed in the same conditions as
752 the first, applying the same sequence of operations, obtains the same result. The
753 measurement therefore requires that the algorithm necessary to carry it out has
754 a characteristic that will be called *symmetry* with respect to the algorithm of the
755 second observer and his reference system. So if a system remains unchanged
756 by repeating a number of times an algorithm that follows exactly the same
757 operations as the previous one, one gets the same measure, regardless of the
758 observer or the reference system.

759 Furthermore, the observers, the measured objects and the same standards used
760 for the measurement of space and time are actually thermal machines and
761 moreover these objects can be subsets of a more general thermodynamic sys-
762 tem.

763 The following discussion will be based on these two considerations:

- 764 (1) The observation of any physical system represents a measure made on that
765 system.

766 (2) The measurement obtained on a physical system is subject to an observer's
767 interpretation¹¹.

768 Finally, it is necessary to point out that in this article the observer makes a
769 measurement in two different steps:

770 (1) Initial instant called *positioning*, where he starts the measurement algo-
771 rithm.

772 (2) Final instant called *measurement*, where the measure actually takes place.

773 The fact that a measurement is carried out in two different steps implies that
774 there must exist an independent set of logical states, in which at least one initial
775 state and one final state must exist in order to make a measurement of a given
776 quantity. This procedure corresponds to the execution of a computation that
777 can be represented abstractly using a Turing machine.

778 In the quantum mechanics field, the measure theory is still debated (Bell 1987).

779 10 On thermodynamics and energy

780 A measurement process in practice always undergoes random errors. These
781 random errors may be due to the complexity of the system one wants to measure
782 or may be intrinsic to the system. Therefore it is assumed that each measure
783 can be represented as a stochastic matrix.

784 In the mathematical part above, the stochastic matrices were introduced to
785 define the symbol of a code. The observer in the act of observing an object begins
786 the measurement at some instant, but does not know how many initial symbols
787 are contained in the stochastic matrix to which the observation refers. That is,
788 the observer is assumed to know only some of the possible initial symbols. For

¹¹The observer must be understood in the most general sense possible, at the limit a thermodynamic system.

789 this reason the matrices considered are not bi-stochastic, because if they were,
790 it would also assume that the observer knows exactly all the initial symbols of
791 his observation, while this is not the case.

792 The stochastic matrix makes it possible to have a certainty in the outcome of
793 the observation, that is, given any initial symbol, the observer is certain that
794 there is an output measure.

Example 10.1. *One considers a stochastic matrix, and one assumes that the positioning stochastic vector is given by the pair $\langle q_0, \mathbf{p}_s = (1, 0, 0, 0) \rangle$. From the product of $\mathbf{p}_s$ by a stochastic matrix a vector is obtained:*

$$\mathbf{p}'_s = \mathbf{p}_s \cdot \begin{matrix} & s_1 & s_2 & s_3 & s_4 \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0.26 & 0.24 & 0.20 & 0.30 \\ 0.23 & 0.27 & 0.22 & 0.28 \\ 0.19 & 0.31 & 0.32 & 0.18 \end{pmatrix} \end{matrix}.$$

795 The measurement result returns the triple $\langle f, \mathbf{p}'_s = (0, 0, 0, 1), D \rangle$ in which the
796 vector $\mathbf{p}'_s$ will indicate the symbol observed with greater probability than the
797 others. The states q_0 , f and the set D represent respectively the initial state, a
798 final state the set of directions of a TM.

799 **Definition 32.** One assumes that a TM that operates through an alphabet
800 defined in Eq. (2) whose symbols are defined in Def. (22), a set of states
801 defined in Eq. (6) and a set defined in Def. (16) is a isomorphic model of
802 *measure* in the physical, chemical and biological fields.

803 This model operates on a space defined in Def. (53).

804 About the questions raised on measurement in physics (Bell 1987) one asserts
805 that, in general if it is possible to effect some measures on the Universe, then the
806 set that measures and the measured set they must interact. By Def. (32), this
807 interaction can be represented in an abstract way by assuming that the Measur-

808 able Universe can be represented with an ideal tape defined in Def. (1).

809 This tape is by definition discrete, while the Measurable Universe is not. This
810 problem is overcome by defining in a probabilistic way the symbols of the alpha-
811 bet through stochastic matrices defined in Eq. 30. In this way every measurable
812 and continuous variation on the Universe corresponds to a measurable and in
813 probabilistic way continuous variation on an ideal tape defined for a TM. Any
814 TM that makes a measurement on this tape must have an initial state for the
815 Def. (1). This assumption allows us to admit that the q_0 and f logical states, ac-
816 cording to Def. (1), can be attributed to a TM that performs the measurement,
817 while they may, not belong to the rest of the Measurable Universe.

818 **Example 10.2.** *Now let's assume to slide a ball on a α surface, perfectly*
819 *smooth, so smooth as to be able to imagine it as a surface formed by a sin-*
820 *gle isolated energy level. Now one substitutes the ball with any perturbation*
821 *localized on the surface. If one wanted to describe at which point of the surface*
822 *this perturbation is, such measurement would be impossible, since the probability*
823 *of the perturbation would be the same, instantaneously, for the whole surface.*
824 *So α surface represents a place where the probability of a perturbation becomes*
825 *the same everywhere.*

826 However, the presence of a ball or of a perturbation in a point of the surface
827 could be seen as a break of any symmetry of the α energy surface .

828 This introductory example is thus formalized.

829 Let α be a hypersurface of constant energy. Let any experiment be given on α
830 that, starting from a point A , causes a variation of symmetry on α . As a result
831 of the experiment, one expects to have a signal of value X with an associated
832 probability $p(X)$. Let us hypothesize that A_m *distinguishable* and independent
833 experiments on surface α are applicable. One experiment will be distinguishable
834 from another if $A_i \neq A_j$, $i, j \in m$. The results of each experiment will be
835 X_n with probability $p(X_n)$ representable as a stochastic vector of probability

836 distribution.

837 Let us consider the Def. (4) of symbol and assume that any random experiment
838 can be represented by an algorithm. If any symbol is definable by other random
839 experiments, then it is possible to have different symbols corresponding to the
840 results of such random generative experiments. One therefore assumes n dis-
841 tinct symbols. On these n symbols it is possible to construct n distinguishable
842 experiments for the initial symbol $s_1, s_2, \dots, s_n$. The pair $\langle q_0, s_n \rangle$, introduced in
843 Section (1), represents the initial state of each random experiment.

844 One can imagine that some results of random symbol-generating experiments
845 and some results of random experiments can not be represented as symbols but
846 they are observable because of the effects they have on symbols. For example,
847 an interaction between two bodies is not directly observable, but its effects on
848 bodies are observable. Therefore, it can occur that the results of a random
849 experiment can be modified by the results of other random experiments.

850 In this Section one wants to show how a Turing machine can represent the
851 abstraction of a work, while a programming machine is the abstraction of a
852 thermal machine¹².

853 Let us now consider one of the set of n random experiments and be Ω_1 the sample
854 space of all the results of this experiment. Given the probability space $(\Omega_1, \mathcal{A}, p)$
855 and repeating the same reasoning for each of the n random experiments it is
856 possible to put forward the following Definition:

Definition 33. Let α be a hypersurface of energy. If one performs n random experiments with $\Omega_1, \Omega_2, \dots, \Omega_n$ sample spaces, a probability space $(\Omega, \mathcal{A}, p)$ is defined for each random experiment. Let the matrix $\mathbf{V}_\alpha$ as defined in Eq. (30)

¹²Both definitions have been given in the mathematical part.

be associated with α :

$$\mathbf{V}_\alpha := \begin{pmatrix} p_{11} & p_{12} & \dots & p_{1r} & g_{1r+1} & g_{1r+2} & \dots & g_{1r+m} \\ p_{21} & p_{22} & \dots & p_{2r} & g_{2r+1} & g_{2r+1} & \dots & g_{2r+m} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & & \vdots \\ p_{r1} & p_{r2} & \dots & p_{rr} & g_{rr+1} & g_{rr+2} & \dots & g_{rr+m} \\ c_{(r+1)1} & c_{(r+1)2} & \dots & c_{(r+1)r} & 0 & 0 & \dots & 0 \\ c_{(r+2)1} & c_{(r+2)2} & \dots & c_{(r+2)r} & 0 & 0 & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{(r+m)1} & c_{(r+m)2} & \dots & c_{(r+m)r} & 0 & 0 & \dots & 0 \end{pmatrix}. \quad (52)$$

The matrix $\mathbf{V}_\alpha$ represents a probability distribution of a finite number of terminating random experiments¹³. An energy hypersurface is defined by $\mathcal{E} = (\Gamma, \mathfrak{S}, I, \mathbf{V}_\alpha)$ where:

Γ represents the alphabet or the set of symbols $s_1, s_2, \dots, s_n$ on which the random experiments are based, where the s_n symbols are the final and almost certain results of other random experiments generators.

$\mathfrak{S}$ represents the set of interactions between the symbols of the alphabet Γ . These interactions can not be measured directly, but their effects are measured on the symbols of Γ .

I Where is defined as $I := \langle q_0 \times \Gamma \rangle$, q_0 represents the initial state, (positioning), of each random experiment represented by a stochastic vector.

$\mathbf{V}_\alpha$ represents the stochastic matrix of transition probabilities.

¹³Terminating means of which it is possible to know the final outcome, as in Ref. (Li and Vitányi 2008; Turing 1937).

The matrix $\mathbf{V}_\alpha$ is divisible into four blocks, as reported in Eq. (25):

$$\mathbf{V}_\alpha = \left(\begin{array}{c|c} \mathbf{P}_\Gamma & \mathbf{N} \\ \hline \mathbf{O} & \mathbf{0} \end{array} \right),$$

While the elements of block $\mathbf{P}_\Gamma$ represents the probability of transition of measurable symbols belonging to an alphabet, the block $\mathbf{N}$ represents the distribution of probability of measurable interactions between bodies, the interaction between two bodies can be interpreted as a variation of probability with respect to an initial measure of measurable symbols.

Returning to Example (10.2), if one considers an energy hypersurface α , it is expected that all the symbols expressed by the metric associated with α , according by Def. (33), have the same average information value. That is, each symbol or interaction has a maximum information, due to the stochasticity of the matrix. Therefore, it is possible to assume that α has a random probability distribution. It is not possible to admit the existence of an associated matrix $\mathbf{V}_\alpha$ different from $\mathbf{V}_\alpha$. In other words, $\mathbf{V}_\alpha$ describes a hypersurface α if given an initial variation on α one has the same uncertainty in the measurement of the symbol or of the output value for all the measurable ones. If one represents α as an algorithm, this algorithm must be stochastic and minimal in the sense that it is not possible to imagine a simpler algorithm that, starting from initial variation on α terminates in the measurement on a symbol or interaction described by $\mathbf{V}_\alpha$. Incidentally, a proposition formulated by G.J. Chaitin on the randomness of a program, intended as a synonym of algorithm, would seem to connect intuitively energy and algorithms: *any minimal program is necessarily random* (Chaitin 1990).

Example 10.3. *Given a hypersurface α and the related stochastic matrix $\mathbf{V}_\alpha$ if the identifies a stochastic matrix. The first two columns of $\mathbf{V}_\alpha$ represent the coding of two symbols s_1 and s_2 belonging to an alphabet, for example two bodies, while the following two columns represent interactions i_1 and i_2 , they can not*

represent an alphabet but allow the transitions between the symbols.

$$\mathbf{V}_\alpha = \begin{matrix} & \begin{matrix} s_1 & s_2 & i_1 & i_2 \end{matrix} \\ \begin{matrix} s_1 \\ s_2 \\ i_1 \\ i_2 \end{matrix} & \begin{pmatrix} 0,5 & 0,5 & 0 & 0 \\ 0,5 & 0,5 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}.$$

Looking at the matrix $\mathbf{V}_\alpha$ one notices that it does not clearly express neither s_1 , nor s_2 . Suppose that an interaction intervenes on α precisely on the symbol s_2 with a probability equal to $p(i_1) = 0,5$, then $\mathbf{V}_\alpha$ must be modified, repeating the sum of the Example (3.6), as follows:

$$\mathbf{V}'_\alpha = \begin{matrix} & \begin{matrix} s_1 & s_2 & i_1 & i_2 \end{matrix} \\ \begin{matrix} s_1 \\ s_2 \\ i_1 \\ i_2 \end{matrix} & \begin{pmatrix} 0,5 & 0 & 0,5 & 0 \\ 0,5 & 0,5 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix}.$$

From $\mathbf{V}'_\alpha$ one notes that the symbol s_2 has zero probability, while i_1 contributes to give to the probability of the symbol s_1 the value one. For $\mathbf{V}_\alpha$ this is represented graphically in Fig. 5.

While for the $\mathbf{V}'_\alpha$ is represented graphically in Fig. 6.

One points out that the probability that the final state of the computation is s_1 is greater than that of s_2 .

An energy hypersurface can be defined as a probabilistic Turing machine where the matrix $\mathbf{V}$ represents all the transition functions for possible initial probability distribution given by the stochastic vector $\mathbf{p}_s$, while the interactions between energy hypersurfaces are represented by modifications of the respective matrices

Figure 5: Graph of matrix $\mathbf{V}_\alpha$ with start point on the symbol s_1 .

Figure 6: Graph of matrix $\mathbf{V}'_\alpha$ with start point on the symbol s_1 .

900 $\mathbf{V}$ according to the Eq. (38).

901 11 Symmetry variations and constraints to a ther- 902 modynamic operation

903 The matrix that represents a hypersurface of energy is not stationary, since
904 the interaction with other hypersurfaces of energy allows variations as described
905 by the Eq. (38). One gives the following

Definition 34. Let α be an energy hypersurface described by

$$\mathcal{E}_\alpha = (\Gamma, \mathfrak{S}, I, \mathbf{V}_\alpha).$$

906 Any perturbation that occurs on α due to the interaction with another hyper-
907 surface of energy must determine a variation of some entry of the stochastic
908 matrix associated with $\mathbf{V}_\alpha$. This perturbation is called variation of symmetry
909 of α .

910 The symmetry variation is observable and therefore measurable as a variation
911 of the state of the system. The experiments on the system will give at least a
912 statistically different result compared to the results before the variation. If two
913 or more hypersurfaces of energy interact the symmetry variations must occur
914 on each of these hypersurfaces. Moreover, changes in the income of matrices
915 associated with energy hypersurfaces can lead to the formation of transient
916 states or recurrent states, they can form communicating states or not.

917 Consider a thermodynamic system, containing a finite set of energy levels. Let
918 us assume that each energy level retains its symmetry if no external variations
919 occur. In other words, a system in a steady state tends statistically to leave its
920 symmetry unchanged. Therefore, in order to know the whole system it is suffi-
921 cient to know or measure only its symmetric or antisymmetric portion.

Definition 35 (Thermodynamic operation). Given a probability space

$$(\Omega, \mathcal{A}, \mu),$$

if a variation of symmetry on a hypersurface of energy is measurable on this probability space, then this variation of symmetry is defined as thermodynamic operation Ot .

A Ot represents the smallest measurable symmetry variation on a thermodynamic system. A single thermodynamic operation determines at least two symmetry variations on two interacting hypersurfaces of energy. In other words, if the measurable symmetry variation is a signal $S(t)$, it can be assumed that its source is an energy hypersurface α and that the emitted signal is received by the second hypersurface β . A variation of symmetry determines the transition between a previous state and a subsequent one of the surface. one can define a support $\mathcal{K} = \{\mathcal{O}, \overline{\mathcal{O}}\}$ of an algebra given by the Ot and its complementary $\overline{Ot}$. Furthermore, two fundamental operations can be defined: union and intersection of states. So the structure $(\mathcal{K}, \cap, \cup)$, represents the smallest σ -algebra for the set of the two energy hypersurfaces involved in the variation.

Proposition 8 (Termination of an Ot). *If an energy hypersurface changes its symmetry, this variation must have a start and end point. The graph corresponds to Def. (22).*

Proof. Let $\mathbf{V}_\alpha$ be the matrix associated with an energy hypersurface, if it varies its symmetry and this variation is measurable with a symbol s , the matrix $\mathbf{V}_\alpha$ must change at least the symmetry of another complementary s symbol so that the definition of stochastic matrix is maintained for the Eq. (29).

□

944 11.1 On thermodynamic systems

945 An Ot is defined between at least two energy hypersurfaces, in addition it must
 946 have an initial and a terminal points as shown in Graph (22). Then also an Ot
 947 is an algorithm, as according to Definition (4). It is assumed that which is as
 948 small possible algorithm. In fact it is allowed that an Ot has two states, one
 949 initial and one final, because only a starting and a final point.

950 It is possible to extend the concept of the interactions between bodies or particles
 951 have indeed Ot to probability space where random experiments act on abstract
 952 general states, which could not correspond to physical states¹⁴.

One consider the Definition (7), an initial stochastic vector $\mathbf{p}_U$, defined in Eq.
 (32) and a state spaces defined in Eqs. (9). Given a probability space of the
 logical states defined in Eq. (6) one has:

$$\Omega_1 = \mathbf{p}_U \cdot \mathbf{U}_{k \geq 0} = \{\bar{\mathbf{Q}}, \mathbf{q}_0, \mathbf{q}_1, \dots, \mathbf{q}_n, F, \mathfrak{Q}\}, \quad \forall \quad \mathbf{U}_k \in P_U.$$

The corresponding probability space is:

$$\mathcal{R} : (\Omega_1, \mathcal{F}_1, \mathbb{P}_1(\bar{\mathbf{Q}}, \mathbf{q}_0, \mathbf{q}_1, \dots, \mathbf{q}_n, F, \mathfrak{Q})),$$

where Ω_1 represents the outcome of all the possible experiments given the initial
 vector $\mathbf{p}_U$ and the whole set P_U . In addition for the alphabet defined in Eq.
 (30) set, one has:

$$\Omega_2 = \mathbf{p}_V \cdot \mathbf{V}_{h \geq 0} = \{\mathfrak{b}, \mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_n, \mathfrak{S}\}, \quad \forall \quad \mathbf{V}_h \in P_V,$$

¹⁴For example, one can think of a thermal machine, which is theorized as a MT, possessing
 logical states which can not be measured directly.

and the corresponding probability space is:

$$\mathcal{S} : (\Omega_2, \mathcal{F}_2, \mathbb{P}_2(\mathbf{b}, \mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_n, \mathfrak{S})).$$

953 One finally defines, by Eq. (16) for displacement set, a third probability space:

$$\Omega_3 = \mathbf{p}_D \cdot \mathbf{D}_{j \geq 0} = \{\bar{\mathbf{D}}, \mathbf{l}, \mathbf{r}, \mathfrak{D}\}, \quad \forall \quad \mathbf{D}_j \in P_D,$$

954 which is related to the probability space:

$$\mathcal{T} : (\Omega_3, \mathcal{F}_3, \mathbb{P}_3(\bar{\mathbf{D}}, \mathbf{l}, \mathbf{r}, \mathfrak{D})).$$

955 Now it is possible to define the product of these probability spaces.

956 These probability spaces are independent because the results of random exper-
957 iments belong to three different sets.

958 The sets denoted by $\mathfrak{Q}, \mathfrak{S}, \mathfrak{D}$, contain elements that do not directly produce
959 effects on the TM.

960 Each probability spaces generates a stochastic vector at each step of compu-
961 tation. Such vectors can be used as stochastic vectors that give rise to logical
962 states, alphabet and direction for the universal TM.

963 An O_t represents a measurement on an energy hypersurface. This measurement
964 must contain:

965 (1) An initial tuple of three stochastic vectors.

966 (2) Three stochastic matrices belonging to the sets P_U, P_V and P_D .

967 From the product of the stochastic vectors by the respective matrices a new
968 tuple is obtained, which represents the result of the measurement process. one

969 defines the product of the three probability spaces $\mathcal{R}, \mathcal{S}, \mathcal{T}$:

Definition 36. The *product space*

$$(\Omega, \mathcal{F}, \mathbb{P}), \tag{53}$$

970 is constituted by the following three sets:

- 971 • $(\Omega) = (\Omega_1) \times (\Omega_2) \times (\Omega_3)$, which is a triple constituted by experiments on
972 logical states, on the alphabet and on the direction.
- 973 • $\mathcal{F} = \mathcal{F}_1 \otimes \mathcal{F}_2 \otimes \mathcal{F}_3 := \sigma(\{Q \times \Gamma \times D | Q \in \mathcal{F}_1, \Gamma \in \mathcal{F}_2, D \in \mathcal{F}_3\})$. One therefore
974 obtains a σ -algebra produced from the three starting σ -algebras.
- 975 • $\mathbb{P}$ it is defined on the σ -algebras that generate $\mathcal{F}$. Given $q \in \mathcal{F}_1$, $a \in \mathcal{F}_2$,
976 $l \in \mathcal{F}_3$, one has $\mathbb{P}(q \times a \times l) = \mathbb{P}(q)\mathbb{P}(a)\mathbb{P}(l)$.

977 Furthermore one has that the null vector belongs to the set $\{\bar{\mathbf{Q}}, B, S\}$ of the PM
978 Definition given in (7).

979 To get an idea of the product space of the three probability spaces, refer to Ex-
980 ample (3.5). Using the value of $\mathbb{P}$ one can calculate a measure of the information
981 according to Definition (29).

982 The existence of two other probability spaces $\mathcal{R}$ and $\mathcal{T}$, in addition to $\mathcal{S}$, makes
983 it possible to tie together a number of *Ots* to constitute a third level of organi-
984 zation.

985 The third level of organization describes the *Ots* formed interact with each other.
986 Since it is impossible for a man to analyze the whole set of *Ot* only finite sets
987 will be considered, the following definition is put forward:

Definition 37 (Thermodynamic algorithm). Given a finite set

$$O = \{Ot_1, Ot_2, \dots, Ot_n\},$$

one defines a thermodynamic algorithm At a concatenation of $Ot \in O$ computable from a Universal Turing machine whose input, for each concatenated Ot , is composed by:

- (1) a coding of the transition table of each Ot ;
- (2) an initial state q_0 for the same Ot ;
- (3) a tape on which the Ots are stored .

With the first two elements of the list, for each Ot a Universal TM is able to simulate the computations of the Ots that belong to the concatenation. The Ots are arranged on a tape as if they were the code of a TM.

In addition being $\{s_1, s_2, \dots, s_n\} \in \Gamma$ and $\{\mathfrak{s}_1, \mathfrak{s}_2, \dots, \mathfrak{s}_n\} \in \mathfrak{S}$, an Ot represents the smallest possible thermodynamic algorithm on $\Gamma \cup \mathfrak{S}$.

Definition 38. Given a finite set of thermodynamic algorithms

$$W = \{At_1, At_2, \dots, At_n\},$$

one defines a *system of thermodynamic operations* $St \subseteq W$.

The following relation is then established:

$$Ot \subseteq St \subseteq W. \tag{54}$$

12 Constraints to an Ot

The values assumed by the various components of the $\mathbf{V}$ matrix of an Ot determine the possibility that this matrix may yield some state more likely than others or have absorbent or reflecting barriers. Thus the probabilities of $\mathbf{V}$ also determine constraints on the expression of a given code. As the transition probabilities of $\mathbf{V}$ vary, the symmetry of the associated energy hypersurface

1006 varies and the expressions relative to the measurable elements of this matrix
 1007 vary accordingly. A stochastic matrix with numerous absorbent states deter-
 1008 mines limits in the variability of the alphabet it expresses, so it is possible that
 1009 it always determines the same values within Γ . The information of a stochastic
 1010 matrix with many adsorbent state is low (see Def. 29 and Ref. (Shannon 1948)).
 1011 In general, a totally stochastic St is a thermodynamic system in which every Ot
 1012 that belongs to the system is able to vary its symmetry. In other words, every
 1013 element that belongs to the system has statistically access to all the energy
 1014 levels of this system.

1015 **12.1 On the stochastic state of a system of thermody-** 1016 **namic operations**

1017 An St composed of Ots with stochastic matrices whose components possess
 1018 similar probability measures determines Ots that can result in any symbol. This
 1019 type of Ots determines totally stochastic assemblies, i. e., within them it is very
 1020 unlikely to find a symmetry with respect to any symbol, all symbols tend to be
 1021 with equal probability. This situation obviously lasts as long as the system is not
 1022 measured because the measurement operation implies that there is a terminal
 1023 symbol. Such a St has a high information entropy of its symbols.

1024 In order to represent the stochastic Ots i.e. with high uncertainty on symbols
 1025 and on $\mathfrak{s} \in \mathfrak{S}$, (which represent interactions), one uses the symbol Ot_s , but this
 1026 notation does not represent any subset of Ot . Furthermore, a St composed of
 1027 Ot_s contains stochastic Ats , therefore with a high information entropy.

13 Thermodynamic computation

Given a St , and a pair $\langle q_0, s \rangle$, where s represents any symbol or interaction within St and q_0 is the initial state, $\langle q_0, s \rangle$ represents an initial variation of symmetry on St . By considering the definitions of TM one has that the pair $\langle q_0, s \rangle$ can be defined as initial computation c_0 , and the successive steps are indicated with c_i ($i = 1, 2, \dots, n$) up to a terminal computation, if it exists. So the execution of a particular At within a St starts from the pair $c_0 = \langle q_0, s \rangle$ if it exists:

$$At_1 : \quad c_0 \vdash c_1 \vdash c_2 \cdots \vdash c_n.$$

If an At defined according to Definition (1) then the execution of any At can be defined as a *thermodynamic computation*. This computation will end if the state of c_n will belong to a final state. Thus, a given terminating At is described by a table of transition functions, if the last step is of the type $c_f : \langle F, \Gamma, D \rangle$, where F is the set of final states. An At is considered terminated if the computation ends with a final state.

$$At : \quad c_0 \vdash c_1 \vdash c_2 \cdots \vdash c_f.$$

The termination of any At within a St allows the existence of a symmetry with respect to the computation, i.e. At can be executed again.

Example 13.1. Consider a shepherd, as evidenced in the following Example, counting the sheep of his flock. Every time he finishes the recognition and the count of the sheep, he frees himself and is able to recognize and count again, so he creates a symmetry in the execution of the recognition and counting algorithm.

In general, three different types of computation are created, two of which (i and ii) are called *maximal computations*:

- (1) computations ending in a final state (by convention it is called *accepting computation*);

1039 (2) computations that end because a further transition function is not defined
 1040 (by convention it is called *rejecting computation*);

1041 (3) non-terminating computation (no response).

1042 Within the first group of accepting computations (all ending with a final state)
 1043 two sets can be created again:

1044 (a) Set of repeatable algorithms indicated as A_r ;

1045 (b) Set of unrepeatable algorithms referred to as A_u ;

1046 A *accepting maximal computation* allows a TM to perform the same computation
 1047 again by positioning itself with the head on the first symbol indicated in the
 1048 table of the transition functions with the q_0 state. In the same way, the execution
 1049 of an At in a system of thermodynamic operations, if it ends with a final state
 1050 allows a new execution of the same algorithm. So the following Definition can
 1051 be given:

Definition 39 (Repeatable algorithm). Given a system of thermodynamic operations St and a thermodynamic algorithm At , if the execution of this algorithm ends with a final state, the At can be performed again. One indicates this by the symbol A_r . This occurs if the Ots that belong to it are stationary stochastic matrices with respect to the execution of the same At .

$$(At)_n : \{(At)_{n \geq 0} | (c_0)_1 = (c_0)_2 = \dots = (c_0)_n, (c_f)_n \in F\}. \quad (55)$$

1052 That is, the execution of an A_r , which ends with a final state, does not signif-
 1053 icantly modify the components of the matrices of Ots that constitute this At .
 1054 This is because small variations in the components of the stochastic matrices
 1055 can lead the algorithm to not be able to finish the computation since the final
 1056 state could become very unlikely to reach. So one has an A_r if there is a sym-
 1057 metry with respect to its execution, if it reaches the final state with the same

1058 number of computations.

1059 **Example 13.2.** *Imagine a clock, at each second a pointer indicates a variation*
 1060 *of time, the pointer executes a repeatable algorithm, because it is able to repeat*
 1061 *the same algorithm in succession.*

1062 Conversely, one can put forward the following Definition:

Definition 40 (Not-repeatable algorithm). Given a system of thermodynamic operations St and a thermodynamic algorithm At , is called *not-repeatable*, when her computation is finished, if a new accepting computation of it becomes improbable due to changes in the components of the stochastic matrices of its Ots present in the first computation. One indicates this by the symbol A_u . That is, if these stochastic matrices lose the bonds due to absorbing states.

$$(At)_m : \{(At)_{m \geq 0} | (c_0)_1 = (c_0)_2 = \dots = (c_0)_m, (c_f)_m \notin F\}. \quad (56)$$

1063 **Example 13.3.** *Suppose one has a sheet with a written text, this text can be*
 1064 *represented as an At executable by placing a reader on the first letter of the text,*
 1065 *with an initial state q_0 . This At represents a computation that ends with the*
 1066 *last letter of the text, thus it is also A_r , as it is executable again at the end of*
 1067 *the reading, a second reading and so on. Now it is assumed to execute a second*
 1068 *At which has as initial state q_0 a high energy Ot (e. g. fire), which determines*
 1069 *the combustion of the entire sheet. This second A_u makes it somewhat unlikely*
 1070 *that further execution of both of the previous algorithms will be improbable.*

1071 Let us assume to fill a cylinder with a movable wall with an ideal gas at a tem-
 1072 perature T_1 . Measurement takes place on the inner walls of the cylinder. The
 1073 first impact that each gas molecule has against the wall defines the initial state
 1074 of the At , $\langle q_0, s_1 \rangle, \dots, \langle q_0, s_n \rangle$. There will be a finite number of initial symmetry
 1075 variations and then of Ats . If the system is isolated and the impacts are con-
 1076 sidered elastic, it will remain indefinitely at the same temperature. However, if
 1077 the movable cylinder wall is displaced, a second measurement can be made on

1078 the system. Each collision with the movable wall will result in a terminal pair
 1079 $\langle f_1, s_1 \rangle, \dots, \langle f_m, s_m \rangle$ with $f_m \in F$ according to Definition (1). The gas collisions
 1080 with the displaced wall determine a measurable variation corresponding to the
 1081 termination of a number m of initial Ats . The system temperature drops to
 1082 T_2 , as the system has done work. Therefore T_1 must correspond to a number
 1083 n of A_u s, while T_2 corresponds to $n - m$ Ats . Then the work done by the gas
 1084 corresponds to the m A_r s. With regard to the efficiency of a thermal machine,
 1085 please refer to the Section 18.

1086 **14 On the thermal machine and on the PM**

1087 The statements given for a PM in Definition (7) can be applied to a thermal
 1088 machine. This last machine produces a work from the transfer of energy from a
 1089 hotter source to a colder source. The two hot and cold sources can be considered
 1090 as two St , one calls these systems St_h and St_c . The thermodynamic entropy of
 1091 the system must decrease during the transformation. A St_h contains a larger
 1092 number of Ots because it has a larger number of energy hypersurfaces accessible
 1093 to symmetry variations than a lower energy St_c system. Furthermore, St_h must
 1094 have an average information entropy lower than the Ots that compose it. This
 1095 lower information entropy corresponds to a lower total thermodynamic entropy
 1096 of St_h compared to St_c .

1097 One points out that an increase in energy of a St corresponds to an increase in
 1098 Ots accessible to changes in symmetry within St , but this does not determine
 1099 any change in the value in the components of the matrices that describe the
 1100 Ots ¹⁵. Thus the matrices associated with the Ots remain invariant during the
 1101 energy increase. Whereas, if the stochastic matrices associated with the hyper-
 1102 surfaces of energy are no longer stationary, then a variation of the information
 1103 entropy of these matrices is determined and consequently of the St .

¹⁵One considers an infinitesimal increase in system energy.

1104 A PM represents in part the abstraction of a thermodynamic system. Indeed,
1105 the internal energy of any thermodynamic system is represented by heat and
1106 work. A PM has a part of Ots with high information entropy and therefore
1107 models the effects of energy in the form of heat, and another part of Ot with
1108 low information entropy that models the effects of the work. While the part
1109 not not applicable to thermodynamic systems is represented by the set of logic
1110 states K of a PM defined in Eq. (6).

1111 15 Introduction to Biological part

1112 In this part the concepts introduced in the mathematical part and in the physical
1113 part for the modeling of biological systems will be used. In particular, one shall
1114 consider a broader concept, which is defined as a living system. The fundamental
1115 thought leading this study is that all living systems try to solve problems. If
1116 this statement is admissible and universal for living systems then it is necessary
1117 to define which the problem is, in which way one try to solve it and what it
1118 means to solve it.

1119 This paper aims to propose an alternative to the understanding of living sys-
1120 tems, starting from logical and mathematical bases instead of biochemical or
1121 biological ones. If this is correct, it means that the class of living systems may
1122 be larger than the one known and based on the DNA molecule. At the end of
1123 the discussion two concepts of freedom and happiness will be defined from a
1124 mathematical point of view.

1125 The following will be used a terminology not proper of biology. Instead, with a
1126 logical mathematical approach to biological systems it is preferred, so that it is
1127 easier to demonstrate and refute the different statements.

1128 16 On living systems

1129 A living system can be physically framed as a very complex thermodynamic sys-
1130 tem. The variety of forms and organizations of these living beings is very wide.
1131 It is possible to try to give a definition of a living system based on biochemical
1132 structures. This definition is based on the observable fact that all living systems
1133 possess DNA and RNA molecules in their structures and are able to reproduce.
1134 This definition clearly sets the boundaries of a biological system, but it does
1135 not clarify the evolution from non-biological systems to biological systems. Fur-
1136 thermore viruses and viroids have remained on the border of definition between
1137 living and non-living systems. Finally, the fact that there are only one type of
1138 chiral amino acids in biological systems is not explained.

1139 To address this topic, the definitions given in the mathematical part and in
1140 the physical part of this text will be used. Consider a PM in accordance with
1141 Definition (7) and let's operate this PM on an alphabet Γ formed on energy
1142 levels equal to those to which biological systems operate. One considers matter
1143 in standard conditions¹⁶, under which the atomic structures are stable. A sym-
1144 metry variation on any energy hypersurface at these conditions will represent a
1145 Ot as defined in Definition (35).

1146 Once the hypersurfaces of energy on which a PM operates are defined, the
1147 maximum and minimum limits of the energy of the thermodynamic system
1148 are also defined. A biological system operates and is defined within a finite
1149 thermodynamic system limited above and below by a maximum and minimum
1150 energy levels, and when they are exceeded these structures are not observable
1151 or otherwise do not work properly.

1152 The Definition (7) defined a PM without memory if it is not able to memorize
1153 the algorithms it creates.

¹⁶A PM is operated in standard chemical-physical conditions but this does not limit the possibility to apply a PM to different conditions and to operate in these different conditions.

1154 A more complex PM is a PM with memory, which is able to memorize the
1155 algorithms. So a living system must essentially be formed by two theoretical
1156 structures:

1157 **Definition 41** (Living system). This set must contain:

1158 (1) PM.

1159 (2) Memory, given by the set of algorithms contained in the system.

1160 Ultimately it is a set of PM with memory, defined in the mathematical part (see
1161 Section 5.1).

1162 In biological language, the term memory represents all that is preserved over
1163 time in a cell, for example, the cell membrane, its DNA, organelles and all
1164 internal structures. At a higher hierarchical scale the memory is represented
1165 by the typical structures of multicellular organisms, internal organs, dermis,
1166 exoskeleton, nervous system and so on. At a level of even greater complexity,
1167 for example for the population, the memory is represented by the individuals
1168 who survive, for example the fertile part of the population, the structures that
1169 are built and maintained for the following generations as nests, hives, dams
1170 in the case of beavers, but also territories occupied and maintained over time,
1171 in general ecological niches. One could still generalize by saying that memory
1172 represents all that is conserved from one generation to the next, considering also
1173 the learned techniques and behaviors. From our peculiar viewpoint, however, it
1174 is essential to underline that the memory contains all the biological constructs,
1175 ecological niche, behaviors and knowledge that are maintained over time in a
1176 specific biological system (organism, population, ecosystem, etc.) whose final
1177 goal is not the survival of the biological system in question, but rather to keep
1178 the ability to solve problems constant. This viewpoint should not contradict
1179 the current theories on the evolution of biological systems as survival is also
1180 a problem to be solved and therefore is consistent with the given definition.
1181 Moreover, this new definition of the purpose of a biological system allows us to

1182 explain also observable behaviors that do not seem to have reproduction as the
1183 last end.

1184 The stochastic part, fundamental for a PM, can be represented in a biological
1185 system as all the biochemical events that take place within the system and which
1186 possess an intrinsic uncertainty. For example, mutations of DNA, the motion
1187 of molecules and organelles within cells under the effect of heat. In addition in
1188 the context of populations, random variations in the position of individuals on
1189 a territory are stochastic.

1190 In a living system in general the concept of *environment* indicates what gen-
1191 erally surrounds a living being and what determines its phenotype. Moreover,
1192 also the genotype represents, together with the living being, something that re-
1193 mains constant to itself for at least an instant, something observable, therefore
1194 it can be seen as a whole as a memory. This memory can be divided into two
1195 parts:

1196 (1) External Memory given by the environment, on which the biological sys-
1197 tem can move remaining intact. This memory in an abstract way can be
1198 seen as the tape defined for a TM in Def. (1) as in (Turing 1937).

1199 (2) Internal memory of the living system, this memory represents all that a
1200 living system brings with it, of which it retains the symmetry, all that
1201 keeps the system equal to itself. The definition set is given in Def. (41).

1202 The system will comply with the energy limits of the thermodynamic system.
1203 With regard to a biological system, the energy range within which these struc-
1204 tures are observed must be defined. As a comparison, a crystal is of a substance
1205 is observed under certain conditions, in such conditions it is stable. In the same
1206 way, a biological system can be observed only in conditions where it is stable. A
1207 living system represents a more general whole compared to a biological system.
1208 Therefore, according to Definition (41), a biological system which is currently

1209 observed on Earth, can be represented by a set of PMs and by the memorization
1210 of a very large number of algorithms.

1211 In conclusion, above considerations, it emerges that the solution of a problem
1212 by a living system is possible through two different ways:

1213 (1) If it executes an algorithm already stored within the system itself.

1214 (2) If it builds a new algorithm starting from the structures stored in the
1215 system and from the stochastic part by Def. (7) (PM).

1216 The two paths could lead to a huge difference in computational complexity
1217 (Hopcroft et al. 2001; Li and Vitányi 2008).

1218 **Example 16.1.** *According to an example taken from Ref. (Lewontin 2002), if*
1219 *one has an electric razor built in the United States it will be suitable to operate*
1220 *with a 110 V main voltage, while in Europe the main voltage is 220 V, but*
1221 *usually the electric razors have a selector to set the correct voltage value. The*
1222 *razor then contains the instructions to operate at one of the two voltages and*
1223 *it is possible to immediately select one or the other voltage, thus there are then*
1224 *two options or switchable algorithms according to the ambient conditions (110*
1225 *V or 220 V). This corresponds to a solution of a problem of the type presented*
1226 *in the first point above.*

1227 *Suppose one finds ourselves with an electric razor in another country where the*
1228 *network voltage is 300 V. That electric razor does not provide that voltage and*
1229 *can not be used. In order to use it, one could build a transformer or look for a*
1230 *transformer store. There are many possible solutions, but all of them imply a*
1231 *greater time and complexity than in the previous case, and it could also happen*
1232 *that the shaver can not be used because a suitable transformer is not found.*
1233 *Therefore, using the razor with a 300 V voltage, corresponds to an application*
1234 *of the second point above.*

1235 17 Evolution and extinction of a living system

1236 The evolution of any biological system is generally understood as the survival
1237 of the most suitable or as an increase in the fitness of the most suitable genes to
1238 live in a certain environment and conversely a decrease in the frequencies of the
1239 genes less suitable for survival. In any case, the existence of the DNA molecule
1240 is fundamental for current theories.

1241 Each component of a cell, from the organelles to the cytoplasm to the mem-
1242 branes up to the DNA, preserves the memory of the system over time, while the
1243 variability present in the molecules and in the biological structures represents a
1244 PM as theorized in this article.

Any biological system evolves, as it is observed in Nature. In other words, the biological system remains in a probabilistic way equal to itself following an event that instead would tend to vary its symmetry with respect to its configuration. The evolution of a biological system can be represented as an adaptation to a given environmental situation. If such adaptation is possible, the individual or the population will be able to continue to survive. From a more formal point of view the environmental situation can be represented as a PM that produces an At that can modify the Ats stored in the set formed by an individual or by a population. Being able to adapt to an environmental situation means finding a solution to a problem. In other words, given a living system according to Definition (41) this system will contain a finite number of stored algorithms, and will have to solve a problem if at least one hypersurface of energy outside the living system prevents the termination of an At , contained in the living system. The living system survives, that is, remains the same if, as a result of environmental interference, the memory containing all the algorithms is not damaged. Formally, a living system St consisting of a set of a stochastic subset

and a subset of memory consisting of algorithms can be represented as:

$$\mathcal{S}_v = \{\mathcal{M}_p, \mathcal{M}_1, \mathcal{M}_2, \dots, \mathcal{M}_n\}.$$

1245 Like any thermodynamic system it must have a stochastic component¹⁷ repre-
1246 sented here $\mathcal{M}_p$ according to Def. 8 and the consequent Def. 43.

1247 A living system is a St that interacts with a limited environment¹⁸. The envi-
1248 ronment also represents a St . If two or more St interacting they have in common
1249 at least one variation on the same Ot according to definition (35). Since each
1250 Ot is part of an St , any symmetry variation that occurs on an Ot changes the
1251 whole St to which it belongs. In other words, if a living system interacts with
1252 another system that in this case is the environment, they will have at least one
1253 hypersurface of energy in common, which then will belong to both the At of the
1254 living system and an At of the environment. For a living system, the Ats con-
1255 tained in it are symmetric with respect to computation if they are terminal. A
1256 non-terminal algorithm prevents a TM that computes it to free itself and start
1257 a new computation. So if the environment produces a variation on an algorithm
1258 of a biological system it modifies its structure and could prevent or change its
1259 termination.

1260 **Definition 42.** A problem is solved or overcome if a terminal computation is
1261 found given the initial state provided by the problem, that is, if a $\mathcal{S}_v$ is able to
1262 formulate an At that ends in a final state.

1263 one has the death or extinction of the living system if:

- 1264 (1) it is not able to terminate the computations of the stored algorithms (see
1265 Sec. 13).

¹⁷The stochastic component of these systems is limited, i. e. it operates on a small set of Ats .

¹⁸Actually one could suppose that it interacts with the whole Universe, but these influences are ignored in this article.

1266 (2) it is not able to formulate new algorithms.

1267 If a living system dies out or dies it no longer has the capacity to find solutions
1268 to problems.

1269 **Example 17.1.** *Consider an algorithm for hunting a gazelle by a cheetah. The*
1270 *starting point of this algorithm is represented by the identification of the prey and*
1271 *then the algorithm ends with the killing the prey. This algorithm is reproducible*
1272 *a large number of times, but the cheetah dies if it is no longer able to terminate*
1273 *this algorithm, perhaps because it breaks a leg or maybe because it finds preys*
1274 *with much less frequency.*

1275 17.1 Evolution

1276 The evolution of a living system allows its adaptation to a thermodynamic
1277 system that contains it, which in general is called environment. Evolution is
1278 possible only and only if the living system is able to vary, that is to change
1279 some characteristics so as to be able to adapt. For example processes such
1280 as crossing over or sexual reproduction favor the increase of variability among
1281 individuals. Increasing the variability of a living system entails increasing the
1282 number of Ats present within that St that is, of the living system considered.
1283 In conclusion, based on the observations made so far, it can be stated that the
1284 increase in the variability of an St and in particular of a living system is possible
1285 in two ways:

1286 (1) Increasing the number of energy hypersurfaces accessible to the St .

1287 (2) Increasing the entropy of the energy hypersurfaces belonging to the St ,
1288 without having access to higher energy levels.

1289 As above remarked, any living system operates in a finite range of energy levels,
1290 and the number of algorithms it contains are finite in number. Therefore, a
1291 living system observed at a given time is able to solve a finite number of prob-

1292 lems. Increasing the energy of a living system means giving the system new
1293 hypersurfaces of energy on which it can build new algorithms. This happens
1294 when a living system uses a new source of energy.

1295 Moreover, in order to increase the variability one must ensure that some of
1296 the algorithms it contains can be modified, but there is the risk that they can
1297 become non-terminal (see Sec. 13). For example, with sexual reproduction the
1298 variability of a population increases, but increases the risk that some individuals
1299 may fail to procreate, regardless of their adaptability to the environment.

1300 It is therefore deduced that living systems adopt two contrasting strategies to
1301 overcome the problems:

- 1302 (1) While a problem is being addressed by the system, this seeks solutions
1303 that minimize uncertainty and therefore tend to minimize information,
1304 according to Def. (29), about events that need to happen.
- 1305 (2) In order to overcome a problem and find a solution, they increase un-
1306 certainty and therefore intrinsic variability, this also corresponds to an
1307 increase in information on events.

1308 Finally, the consequences of the two systems to increase variability are not the
1309 same. If a living system uses higher energy levels to create new variability it
1310 must use additional energy sources, then all the algorithms built with these new
1311 energy levels will be executable only as long as these levels are accessible. So if
1312 it occurs that for some reason the higher energy levels are no longer accessible
1313 to the living system, it immediately loses the ability to perform all the *Ats*
1314 that need those energy levels, for example all the biochemical reactions with
1315 activation energies higher of the maximum energy available to the living system
1316 can not take place.

1317 On the other hand, if a living system increases variability by eliminating con-
1318 straints (see Sec. 11) on *Ots* or increasing degrees of freedom for some *Ots*, it

1319 is less likely that an external event will block the *Ats* found as a result of the
1320 increase in variability.

1321 **Example 17.2.** *Artificial selection in dairy cattle has led to an increase in*
1322 *the production of cow's milk. This increase in production, in order to be im-*
1323 *plemented, requires greater energy needs for the selected cows compared to the*
1324 *original population (Clark et al. 2001). Currently there is ample availability of*
1325 *raw materials for the feeding of selected cows, but if this availability drastically*
1326 *decreases the population of selected cows may not survive.*

1327 **Evolutionary mechanism** The fundamental mechanism that makes possi-
1328 ble the evolution of a living system consists, on one side, keeping a memory
1329 that contains the algorithms for solving outdated problems and on the other
1330 side possessing and increasing, when necessary, a stochastic part which allows
1331 the living system to look for new problem solving algorithms. Increasing the
1332 stochastic part allows to find more algorithms with less complex solutions. One
1333 could compare this mechanism to a rope that ends with a loop, and to a blind
1334 observer who holds the rope with his hands. The observer represents the mem-
1335 ory of the system, the rope the stochastic or variable part. The observer will
1336 begin to throw the rope without knowing the environment around, randomly,
1337 to look for an anchor point. Many attempts to look for an anchor will fail, if
1338 suddenly he finds a foothold, then he will be able to walk to the point found
1339 by following the rope. Furthermore, it is assumed that the observer is able to
1340 memorize the path he performs. After several attempts he will be able to make
1341 paths within that environment. Starting from these paths he will be able to
1342 explore more other environments by throwing the rope randomly. If the rope
1343 was too long, the observer may not be able to follow the rope to reach a found
1344 anchor because there are obstacles that should have been bypassed if the rope
1345 had been more short. The stochastic part must be proportional to the size of
1346 the memory or the observer.

17.2 Final considerations on the evolution of living systems

By thinking of the fact that a living system can evolve in two ways, some fundamental considerations can be drawn with respect to the main mechanisms for increasing its variability. If one increases the energy available for the living system, it will have a number of hypersurfaces of greater energy available and therefore greater availability of algorithms on which it is possible to construct an evolution. As a simple clarifying example, if an observer placed at the center of a territory to be explored, and he can only walk, the portion of the explored territory will be limited. If, however he will be given more energy through a car, he will be able to explore a larger portion, but if the car becomes unavailable he will soon find himself limited to exploring only the initial territory.

As already seen, it is possible to increase the variability of a living system and therefore the number of available algorithms by increasing the entropy within some *Ots*, i. e. ensuring that the algorithm that contains it may yield a larger variety of results. For example, if a population of bacteria is treated with β -lactam antibiotics it is known that at low doses eventually in the bacterial population can be isolated treatment-resistant individuals, which produce enzymes, the β -lactamases that catalyze the hydrolysis of the ring lactam present in the molecule of β -lactam antibiotics (Wilke et al. 2005). The production of β -lactamase does not imply that the bacterial population needs further sources of energy, but instead that it was made possible by modifying some portion of the bacterial DNA.

The second mechanism of increasing the variability of a living system is more durable, secure and sustainable than the first one. The discovery of a new source of energy can instead lead to the extinction of entire living systems if this energy source is no longer available.

1374 In conclusion, any living system to maintain its symmetry with respect to a
1375 variation due to the interaction with the environment implements two mecha-
1376 nisms:

- 1377 (1) tries to minimize the effects of this variation, ensuring that the variation
1378 does not modify its symmetry or its memory to such an extent that it can
1379 be extinguished. Therefore the external variation must not irreparably
1380 modify the memory of the living system by Def. (42);
- 1381 (2) increasing the internal variability of your system so as to cancel or reduce
1382 the effects of external variation.

1383 Examples of the first case are the shell of a mollusk or the carapace of the
1384 turtles which create an impediment to predator attacks or the current families
1385 of *Elephantidae*, which prevent predator attacks only increase the size of their
1386 individuals. Or even with polyploidy in many pteridodophytes that should
1387 increase resistance to lethal genetic mutations.

1388 The second case occurs when a living system increases the inherent uncertainty
1389 or entropy of information, for example by subdividing and distributing the ge-
1390 netic heritage or memory over a greater number of individuals. This happens in
1391 the spores of sporophyte plants that produce a large number of spores so that
1392 someone is more likely to sporulate.

1393 18 Freedom and Happiness

1394 The last topic addressed in this article concerns two concepts born and de-
1395 fined philosophically almost exclusively in the human sphere. It is a natural
1396 consequence of previous reflections.

1397 Let $\mathcal{S}$ be a stationary state of any St and therefore of a living system. A
1398 stationary state is a St without any variation on its hypersurfaces of energy.

1399 Such that this St remains symmetrically identical to itself. One indicates with
 1400 $\mathbf{card}(A)$ the cardinality of a generic set A .

Definition 43 (Freedom). *Freedom* is defined relative to a given St_x and is indicated with L_x , as the set of all the thermodynamic algorithms At_n that the St_x system is able to execute given the state $\mathcal{S}$ in which it is found.

$$L_x = \bigcup_{n=1}^k At_n \quad \text{with} \quad n \in \mathbb{N}^+. \quad (57)$$

1401 Each At within an St is distinguishable from another At from the different
 1402 initial state.

Proposition 9. *Given two systems of thermodynamic operations St_a and St_b , with $St_b \subset St_a$ it occurs that:*

$$L_b \leq L_a. \quad (58)$$

1403 *Proof.* The elements of the set St_b are contained within the set St_a therefore the
 1404 Ats contained in St_b must also belong to the set St_a . It follows that according
 1405 to Definition (43), also $L_b \leq L_a$.

1406 □

1407 According to Def. (43) and the previous statement, the following measure of
 1408 the elements of L_x is defined:

Definition 44. Let κ_1 be a positive constant different from zero, and each element of the set L_x let be different from the other elements for each pair $\langle q_0, s_n \rangle$, then:

$$\mathcal{L} = \kappa_1 \cdot \mathbf{card}(L_x). \quad (59)$$

1409 From the previous Definition one has that $\mathcal{L} > 0$ and in a St must exist at least
 1410 an executable At .

1411 Then, given a St , in an initial state $\mathcal{S}$, it is possible that a part of the ther-

1412 modynamic algorithms that it is able to perform be of the A_r type, (terminal),
 1413 while the complement of this set will be A_u s, then the following Definition is
 1414 given:

Definition 45 (Happiness). Happiness is defined as the finite set of all A_r s of the given St_x .

$$F_x = \bigcup_{i=1}^k A_r \quad \text{with} \quad i \in \mathbb{N}^+. \quad (60)$$

From Eq. (43) it follows that:

$$F_x \subseteq L_x.$$

1415 According to the Def. (45) and to the distinguishability of the algorithms from
 1416 the initial symbol, by Def.25, it is possible to count the elements of the set F_x
 1417 and therefore one has:

Definition 46. Let κ_2 be is a positive constant different from zero, one has:

$$\mathcal{F} = \kappa_2 \cdot \text{card}(F_x) \quad \text{whit} \quad 0 \leq \mathcal{F} < \mathcal{L}. \quad (61)$$

1418 **Example 18.1.** *Imagine a mouse that lives in a prairie, which can walk, any-*
 1419 *where over this surface. The paths it covers on that surface are distinguishable*
 1420 *and each one represents the algorithms that the mouse can perform. Among*
 1421 *these paths there are some paths that the mouse can follow but are very danger-*
 1422 *ous, as those too far from its nest. The mouse can follow them but the probability*
 1423 *of being preyed increases. All the paths that the mouse can cover represent the*
 1424 *freedom of the mouse, while its happiness is represented only by the paths in*
 1425 *which the mouse is more likely not to be preyed.*

Definition 47. The efficiency of a St and therefore also of a living system is given by:

$$\eta = \iota \frac{\mathcal{F}}{\mathcal{L}} \quad \text{where} \quad \iota = \frac{\kappa_1}{\kappa_2}. \quad (62)$$

1426 **Proposition 10.** *Every living system works to maximize the value of η or at*
 1427 *most to keep it constant.*

1428 *Given an Sv according to Definition (41), if any energy hypersurface accessible*
 1429 *to this system undergoes a variation due to an Ot , the Sv reacts so that this*
 1430 *variation determines the maximum reduction of information, defined in (29),*
 1431 *on the At where the variation is present.*

1432 *Proof.* According to Definition (41) a living system consists of two fundamental
 1433 parts, a stochastic part with a high entropy and a stable part called memory.
 1434 Let St be a thermodynamic system external to a living system but interacting
 1435 with it by means of an energy hypersurface common to both called α . This
 1436 means that a variation of symmetry that occurs on α involves both the living
 1437 system and the thermodynamic system. One assumes that to represent the
 1438 interaction between the two systems there is an At that contains a variation
 1439 of symmetry on α and let Ot_α be this variation. One also assumes that the
 1440 thermodynamic algorithm At belongs to the thermodynamic system external
 1441 St because the initial Ot is within this system. Without lacking of generality
 1442 it is also possible to assume that algorithm belong to the living system. If the
 1443 thermodynamic system external St interacts on the living system with an At
 1444 that one assumes terminal, then it will be possible to measure an effect at the
 1445 end of the computation. On the side of the living system two situations can
 1446 occur:

- 1447 (1) The Ot_α operation belongs to the stochastic part of the living system or
 1448 to its PM.
- 1449 (2) The Ot_α operation belongs to a thermodynamic algorithm contained in
 1450 the memory of the living system.

1451 In the first instance, Ot_α has a low probability of making measurable changes
 1452 and therefore observable to the living system, as Ot_α is contained within a set
 1453 formed by Ot_s s according to Definition (12.1) , then there will exist a consid-
 1454 erable number of At which will contain Ot_α and in each time the observation
 1455 will be different. On the contrary, the memory of the living system remains

1456 unchanged.

1457 In the second instance, instead the variation Ot_α leads to a modification of an
1458 At stored in the living system and therefore to an observable change of this At .

1459 □

1460 **Example 18.2.** *Consider the following mental experiment. An isolated beaker,*
1461 *filled with water and sealed with a cap. Through the hole steel rod capable of*
1462 *falling over the water can be fitted and using a magnetic switch it is possible*
1463 *to release the rod, which hits the free surface of the water without damaging*
1464 *the beaker. Subsequently the rod is extracted from the water and the system*
1465 *is restored to the initial condition. In this first test, an external observer who*
1466 *observes the system after a sufficiently long time is not able to say whether the*
1467 *rod hit the water or not. In a second test the rod is dropped towards the water*
1468 *with greater speed and this bump causes the beaker that contained the water to*
1469 *break. Subsequently the the rod is put in its initial position. In this case, the*
1470 *observer is able to establish that the rod hit the water.*

1471 This experiment shows how a system in which a stochastic part exists can remain
1472 unchanged even in the presence of environmental variations.

1473 However, even small variations in the memory of a living system can deter-
1474 mine the impossibility of terminating thermodynamic algorithms present in-
1475 side.

1476 18.1 Final considerations on the origin of life

1477 If one assumes molecules such as DNA or RNA as the foundation of life, it is
1478 necessary to ask a question about the ordering of the bases in the molecule, for
1479 example about what the primordial order should have been. The experiments
1480 (Miller and Urey 1959), (Parker et al. 2011) showed how it was possible to create

1481 organic amino acid molecules from inorganic compounds. But this is not yet
1482 sufficient to explain why in the biological systems amino acids are present only
1483 in l-form and not in racemic form.

1484 Starting from general theoretical considerations the living systems and therefore
1485 the biological ones were defined as a set consisting of two distinct parts, a
1486 stochastic part and a deterministic part enclosed in the same set

1487 It could therefore be assumed that in the formation of a living system and
1488 in particular of a biological system the following steps would have occurred
1489 chronologically passing through these points:

1490 (1) formation of membranes (e. g. phospholipids in biological systems) that
1491 by wrapping themselves up isolate a liquid portion and various molecules
1492 from the external environment;

1493 (2) selection of metabolic processes within these membranes to withstand en-
1494 vironmental energy lowering;

1495 (3) accumulation of memory within these membranes.

1496 Cell duplication should be a process that happened later. This is because also
1497 the cellular duplication is a solution to a problem of memory conservation.

1498 **19 Final conclusions**

1499 This paper presents a logical mathematical approach to biological systems allows
1500 us to give a description external to them and to construct hypotheses, which
1501 that can be verified experimentally. In the mathematical part the concept of
1502 symbol of an alphabet has been redefined. The symbol is defined by executing an
1503 algorithm that uses another alphabet. It is not the symbol that is postulated,
1504 but the algorithm necessary to obtain it. In this way it is possible to apply

1505 the concept of Turing machine to more complex and real systems if actually,
1506 a universal alphabet is not known with which the observable Universe is built.
1507 Subsequently, a programming machine has been defined able to formulate a
1508 code that is executed by a Turing machine. In its definition, randomness is
1509 fundamental. Randomness is expressed at three levels, in the symbol, in the
1510 combination of the steps of the code and in the combination of the different
1511 algorithms.

1512 The introduction of a randomly based programming machine could open up new
1513 scenarios for artificial intelligence studies.

1514 In order to study the living systems, it appeared that first a Section devoted to
1515 some physics was needed. Living systems, indeed, can be described as physical
1516 systems performing measurements on the real world.

1517 An energy hypersurface is defined as a probabilistic Turing machine represented
1518 by a stochastic matrix. The initial state is represented by a stochastic vector. A
1519 measurable change, on the phase space, of an energy hypersurface corresponds
1520 to a variation of the elements of the stochastic matrix.

1521 The abstract concept of a programming machine is applied to thermodynamics
1522 as an abstraction of a thermal machine. Heat and work are rethought, respec-
1523 tively, as stochastic or almost certain symmetry variations of a thermodynamic
1524 system. Using this representation through algorithms, conceived in an abstract
1525 way as Turing machines, one can deduce that also the measurement of space and
1526 time can be represented as algorithms that are reiterated every time a measure-
1527 ment is made. Therefore, the physical world could be represented, in general,
1528 by using algorithms, as defined in this paper. This could open a different inter-
1529 pretation of the reality that has not been investigated in this work.

1530 In the biological part the definitions given above in the mathematical and phys-
1531 ical parts have been applied.

1532 The introduction of a measure of the uncertainty of an event (or of the result
1533 of an algorithm) is interesting to understand the behavior of biological systems,
1534 as they tend to reduce this uncertainty to a minimum. For example, in the
1535 human sphere, a measure of information (Shannon 1948) is approximated by
1536 the currency of exchange, the latter being an abstract means used as a measure
1537 of the uncertainty of trade.

1538 The greater the intrinsic uncertainty of getting a good for a purchaser, the lower
1539 will be its monetary value. For instance, no one would buy a ticket for a flight if
1540 the probability to arrive to the destination is lower than 20 percent. It might be
1541 interesting to look for other forms of approximation of information measurement
1542 in other living systems other than humans.

1543 Regarding to the evolution of living systems this work has the fundamental pur-
1544 pose of giving a completely abstract theoretical interpretation, so as to define
1545 a more extensive class of systems called living system that should include a
1546 subclass represented by biological systems . This should help to better under-
1547 stand how their origin and evolution was possible. Furthermore, the continuity
1548 between the non-living world and the living world is underlined.

1549 Obviously, these hypotheses can also be applied in the human sphere. In fact it
1550 is seen that living systems that use more energy than others are more vulnerable
1551 to extinction, because if some of the highest energy levels are no more accessible,
1552 all the algorithms in which they are involved can no longer be executed. For
1553 example, the use of high energy levels in industrialized countries would lead
1554 to a rapid extinction of their growth models if current energy levels were no
1555 longer sustainable. This could be a further and fundamental incentive for the
1556 reduction and sustainability of energy use in these countries. As a consequence
1557 there is a new argument in favor of reducing carbon dioxide emissions in the
1558 atmosphere as well as trying to slow down global warming.

1559 In the animal breeding field, for example, artificial selection leads animals to

1560 convert food more efficiently and to use more energy to produce more milk and
1561 meat. These artificially selected breeds are economically competitive compared
1562 to the old domestic animal breeds and they certainly represent an important
1563 part of the food sources in the industrialized countries. A drastic decrease in the
1564 available energy levels could lead to a rapid decline or even to the extinction
1565 of these breeds and this would then produce a food deficit in the industrial-
1566 ized countries. A similar reasoning is possible for the artificially selected plant
1567 varieties to adapt to high energy levels.

1568 Finally, structures such as freedom and happiness are defined that deserve fur-
1569 ther investigation.

1570 Unfortunately it has not been possible to extend the discussion to include a
1571 philosophical viewpoint, which would be extremely necessary.

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1579 The author declares that he has no conflict of interest.

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