A short review on porous media convection

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Abstract

We briefly review the investigations of pattern formation and transport properties of Rayleigh-Darcy convection (or the Elder Problem), including laboratory experiments, theoretical analysis and numerical simulations. It is shown that the flow exhibits power-law-scaling characteristics at large Rayleigh-Darcy number Ra , a dimensionless parameter representing the ratio of the driving buoyancy forces to the diffusive forces. Namely, the mean spacing between neighboring interior plumes shrinks as $Ra^{-\alpha}$ with the scaling exponent $\alpha \leq 1/2$ and the convective flux increases linearly with Ra . However, more laboratory experiments are needed to validate these scalings. Additionally, many conditions, e.g. the inclination of the layer and hydrodynamic dispersion, etc., may lead to a large uncertainty in the flow pattern and transport efficiency.

Keywords: Porous media convection

1. Introduction

Buoyancy-driven convection in a fluid-saturated porous layer has been extensively studied due to its numerous applications in oil recovery [1], groundwater flow and geothermal energy extraction [2–7], transport in biological tissues [8], and carbon dioxide sequestration [9–23]. Moreover, porous media convection is also used as a classical example to study instabilities, bifurcations, pattern formation, and spatiotemporally chaotic dynamics [24–41].

In an infinitely-extended horizontal porous layer uniformly heated from the below and cooled from above, the system undergoes a series of bifurcations as the control parameter, the Rayleigh number Ra which represents the ratio of the driving buoyancy forces to the diffusive forces, increases. Above the critical Rayleigh number $Ra = 4\pi^2$, the basic conduction state becomes linearly unstable via a *stationary* bifurcation and the flow exhibits steady convective rolls [24, 25]. In a two-dimensional (2D) domain, the nonlinear steady rolls becomes unstable via a *supercritical Hopf* bifurcation at $Ra \approx 400$, and small-scale protoplumes, generated from thin upper and lower boundary layers near the walls, are periodically advected around the $O(1)$ aspect-ratio cells [28, 31–33]. For $400 \lesssim Ra \lesssim 1300$, the flow transitions between periodic and quasiperiodic convective roll motions [28–33]; and for $Ra \gtrsim 1300$, the large-scale convective rolls are completely broken down and the flow transitions to spatiotemporally *chaotic* motions [34, 36, 39].

At sufficiently large Rayleigh number, porous media convection exhibits a three-region columnar structure: adjacent to the upper and lower walls locate thin diffusive boundary layers; away from the walls the transport is controlled by nearly vertical columnar megaplumes; and between these two regions a series of protoplumes grow from the boundary layers and merge into the megaplumes [34, 36, 39]. As Ra is increased, the time-average inter-plume spacing δ decreases as a power of Ra , $\delta \sim Ra^{-\alpha}$, and the dimensionless transport flux F (i.e. the ratio of the transport in the presence of convective motion to the diffusive transport, which is also referred to as the Nusselt number or the Sherwood number in many studies) increases as a power law of the form $F \sim Ra^\beta$, where both α and β are positive.

One of the central challenges in studies of porous media convection is the determination of the scaling exponents α and β . In this manuscript, we summarize various investigations from laboratory experiments, theoretical analysis and numerical simulations for studying the pattern formation and transport properties of porous media convection. At the end, we discuss the effects of various conditions on the convective pattern and transport.

2. Experiments

The laboratory experiments by Elder [42] in both bead packs and a Hele-Shaw cell first confirm that convection sets in when $Ra \approx 4\pi^2$, consistent with the theoretical prediction of [24, 25]. The experiments by Lister [43] show that the lateral scale of the convective flow decreases with increasing Rayleigh number.

To explore the high- Ra scaling of convective transport in porous media, Neufeld *et al.* [14] perform laboratory experiments using solutions of methanol and ethylene-glycol mixing with water in a 2D chamber filled with glass beads. They find that the convective flux scales with the Rayleigh number to the $4/5$ power, i.e. $F \sim Ra^{0.8}$, for $5 \times 10^4 \leq Ra \leq 6 \times 10^5$. Similarly, the experiments by Backhaus *et al.* [44] using solutions of water mixing with propylene glycol in a Hele-Shaw cell reveal that $F = 0.045Ra^{0.76}$ for $6 \times 10^3 \leq Ra \leq 9 \times 10^4$.

3. Theoretical analysis

Linear stability analysis of the basic conduction solution indicates that the horizontal wavelength of the fastest growing disturbance decreases as $Ra^{-1/4}$, while that of the smallest unstable mode scales as $Ra^{-1/2}$ [24, 25]. Hence, to ensure the occurrence of convection, it requires $0 < \alpha \leq 1/2$.

The ‘marginally stable boundary layer’ hypothesis by Malkus [45] yields a Howard-Malkus-Kolmogorov-Spiegel scaling, $F \sim Ra$, for porous media convection [46]. Moreover, rigorous upper-bound analyses of the Darcy-Oberbeck-Boussinesq equations by Doering and Constantin [46] and Otero *et al.* [34], by maximizing the heat transport of piecewise linear ‘background’ profiles, show that $F \leq cRa$ with different prefactors c . Following this strategy, [47–49] compute the optimal upper bound on F under the Constantin-Doering-Hopf variational scheme. They find a lower prefactor c for the unit optimal scaling exponent in the F - Ra relationship. Moreover, these upper-bound variational analysis also yields a set of *a priori* eigenfunctions which can be used to construct low-/reduced-order models for porous media convection [50–52].

4. Numerical simulations

The dynamics and flow pattern of porous media convection for low and moderate Rayleigh numbers have been thoroughly investigated theoretically and numerically, as described in the introduction. In this section, we focus on the dynamics for the ‘turbulent’ high- Ra regime.

At large values of Rayleigh number, convection in porous media appears in the form of three-region columnar flows, with a series of protoplumes growing from the thin upper and lower boundary layers and continuously merging with the interior megaplumes. The direct numerical simulations (DNS) of [36] show that the time-average inter-plume spacing in the 2D Rayleigh-Darcy system scales as $\delta \sim Ra^{-0.4}$ and the interior flow can be modeled using a single horizontal Fourier-mode ‘heat-exchanger’ solution. Subsequently, they perform three-dimensional (3D) DNS in [38] and find that the high- Ra convection still retains the columnar structure in the 3D system, but with a different scaling for the time-average inter-plume spacing, i.e. $\alpha \approx 0.5$. To explore the mechanism for this nonlinear scale selection, [36, 38–40] perform linear stability analysis of the heat-exchanger solution and the nonlinear steady solution. They find that the mean inter-plume spacing δ observed at large Ra results from an interplay between two types of instability: when δ is too small, a bulk mode controls the instability, causing plume merger and coarsening of the convective pattern; when δ is too large, a wall-mode instability dominates, causing small plumes generated from the walls to split the wider plumes into narrower ones. Moreover, all of these studies reveal that as $Ra \rightarrow \infty$, the convective flux increases linearly with Ra for both 2D and 3D systems, i.e. $\beta = 1$.

5. Conditions affecting the pattern and transport

In previous sections, we summarize various studies of the convective pattern and transport properties in the Rayleigh-Darcy domain. Most of these studies are performed in simplified/idealized geometries. In this section, we discuss several conditions which may affect the flow pattern and transport efficiency of porous media convection.

5.1. Domain aspect ratio

The small domain aspect ratio L , i.e. the ratio of the horizontal to the vertical dimension of the container, may significantly affect the convective pattern and transport. When L is smaller than the smallest unstable wavelength $4\pi/(\sqrt{Ra} + \sqrt{Ra - 4\pi^2})$ in the horizontal Rayleigh-Darcy domain, no convection occurs due to the stable conduction state [24, 25]. In the thermal convection problem, to obtain a reliable statistical results for heat transport flux, the domain should contain at least one sustained megaplume [48, 51].

5.2. 2D v.s. 3D

For simplification, most of the current studies on porous media convection are performed in the 2D systems. Although an extra freedom in 3D may lead to more complicated convective patterns and affect the transport efficiency, the studies by [38, 53, 54] show that

in 3D the flow still retains a similar columnar structure at large Ra in horizontal porous layers, and the convective flux increases linearly with Ra but with a slightly larger prefactor. Therefore, the analysis in 2D does, indeed, shed light on the real 3D system.

5.3. Inclination of layer

In a sloping 3D porous layer, the studies by [55–61] indicate that the conduction state is stable for $Ra \cos \phi \leq 4\pi^2$, where ϕ is the inclination angle above the horizontal, and four types of flow exist near the onset of convection for different ϕ : the basic unicellular flow (i.e. the conduction state); polyhedral cells; longitudinal helicoidal cells; and 2D transverse rolls. The experimental studies by [62, 63] reveal that a small amount of slope of layer enhances the convective transport compared to the horizontal case.

In the inclined 2D porous layer, the non-convective basic state at large Ra is linearly stable for $\phi \geq 31.3^\circ$ [64]. However, it is shown in [65] that the basic state is not energy stable so that convective states may be realizable. In fact, the DNS results in [65, 66] indicate that for $0 < \phi < 25^\circ$, the flow still exhibits the three-region wall-normal columnar structure as in the horizontal case; while for $\phi > 32^\circ$, the flow transitions to a large-scale traveling-wave convective roll state, and the heat transport is significantly reduced.

5.4. Hydrodynamic dispersion

As large Rayleigh number, the strong convection leads to fast small-scale roll motions. When the flow velocity is large enough, the mechanical dispersion may dominate the convection [67–69]. Recent studies by [70–80] indicate that at large Ra hydrodynamic dispersion significantly affects the flow pattern and transport of convection in porous media. Although most of the studies agree that hydrodynamic dispersion coarsens the convective pattern, [71, 72] show that the hydrodynamic dispersion enhances the convective mixing, while the laboratory experiments by [75, 77] reveal that it reduces the increase of convective flux with increasing permeability.

5.5. Anisotropy

Due to the inherent anisotropy of many sedimentary geological formations, it is necessary to explore the effect of anisotropic permeability on convection. [10, 11, 81, 82] investigate the onset of convection in anisotropic porous media with a vertical-to-horizontal permeability ratio $\gamma = k_v/k_h$, and find that the anisotropic system ($\gamma < 1$) becomes more unstable compared with the isotropic one. The DNS results from [83] for $0.25 \leq \gamma \leq 1$ up to $Ra = 4 \times 10^4$ show that the anisotropic permeability does not change the unit exponent in the F - Ra scaling relationship at large Ra but increases the prefactor.

6. Summary

We have briefly reviewed various investigations of flow pattern and transport characteristics of convection in porous media, including laboratory experiments, theoretical analysis and numerical simulations. The theoretical analysis and numerical simulations show that in a sufficiently extended porous layer and as $Ra \rightarrow \infty$, the inter-plume spacing shrinks

as a power-law scaling of Ra with the scaling exponent $\alpha \leq 1/2$ and the scaling exponent in the F - Ra relationship is unity. However, these scalings have not been verified due to the lack of laboratory data. Moreover, at large Rayleigh number many conditions, e.g. the inclination of the layer and hydrodynamic dispersion, etc., may lead to a large uncertainty in the pattern formation and transport efficiency.

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