Jump-Diffusion Framework for Convertible Bonds

Tim Xiao

ABSTRACT

This paper presents a new framework for pricing convertible bond subject to credit risk. We show that the model prices fluctuate randomly around the market, indicating the model is quite accurate. Empirically, we do not find support for presence of a systematic underpricing as indicated in previous studies. We also show that the deltas increase with the underlying stock prices. At low market levels, the convertibles behave like their straight bonds with very small deltas. As the stock price increases, conversion becomes more likely. At certain market levels the convertibles are certain to be converted. In this case, the convertibles are similar to the underlying equities and the deltas are equal to the number of shares.

Key Words: jump diffusion, convertible bond, convertible underpricing, convertible arbitrage, default time approach, default probability approach, asset pricing and credit risk modeling.

1. Introduction

A company can raise capital in financial markets either by issuing equities, bonds, or hybrids (such as convertible bonds). From an investor's perspective, convertible bonds with embedded optionality offer certain benefits of both equities and bonds – like the former, they have the potential for capital appreciation and like the latter, they offer interest income and safety of principal. The convertible bond market is of primary global importance.

Tsiveriotis and Fernandes (1998) argue that in practice one is usually uncertain as to whether the bond will be converted, and thus propose dividing convertible bonds into two components: a bond part that is subject to credit risk and an equity part that is free of credit risk. A simple description of this model and an easy numerical example in the context of a binomial tree can be found in Hull (2003).

The jump-diffusion model was first introduced by Merton (1976) in the market risk context for modeling asset price behavior that incorporates small day-to-day diffusive movements together with larger randomly occurring jumps. Over the last decade, people attempt to propagate the model from the market risk domain to the credit risk arena.

Zhou (1997), Hilberink and Rogers (2002), Chen and Kou (2009), etc. introduce the jump-diffusion mechanism into the structural models, while Davis and Lischka (1999), Andersen and Buffum (2004), and Bloomberg (2009), etc. add a default jump to the stock price dynamics. We refer to the formers as the structural jump-diffusion models and the latters as the reduced-form jump-diffusion models.

In this paper, we mainly discuss the reduced-form jump-diffusion models. At the heart of the jump-diffusion models lies the assumption that the total expected rate of return to the stockholders is equal to the risk-free interest rate under a risk-neutral measure.

Because of their hybrid nature, convertible bonds attract different type of investors. Especially, convertible arbitrage hedge funds play a dominant role in primary issues of convertible debt. In fact, it is believed that hedge funds purchase 70% to 80% of the convertible debt offered in primary markets. A prevailing belief in the market is that convertible arbitrage is mainly due to convertible underpricing (i.e.,

the model prices are on average higher than the observed trading prices) (see Ammann, et al (2003), Choi, et al. (2009), Loncarski, et al. (2009), etc.). However, Agarwal, et al. (2007) and Batta, et al. (2007) argue that the excess returns from convertible arbitrage strategies are not mainly due to underpricing, but rather partly due to illiquid. Calamos (2011) believes that arbitrageurs in general take advantage of volatility. A higher volatility in the underlying equity translates into a higher value of the equity option and a lower conversion premium. Multiple views reveal the complexity of convertible arbitrage, involving taking positions in the convertible bond and the underlying asset that hedges certain risks but leaves managers exposed to other risks for which they reap a reward.

We model both equities and bonds as defaultable in a consistent way. When a firm goes bankrupt, the investors who take the least risk are paid first. Secured creditors have the best chances of seeing the value of their initial investments come back to them. Bondholders have a greater potential for recovering some their losses than stockholders who are last in line to be repaid and usually receive little, if anything. The default proceedings provide a justification for our modeling assumptions: Different classes of securities issued by the same company have the same default probability but different recovery rates. Given this model, we are able to back out the market prices.

Using the model proposed, we conduct an empirical study of convertible bonds. We obtain a data set from FinPricing (2015). The data set contains 164 convertible bonds and 2 years of daily market prices as well as associated interest rate curves, credit curves, stock prices, implied Black-Scholes volatilities and recovery rates.

The empirical results show that the model prices fluctuate randomly around the market prices, indicating the model is quite accurate. Our empirical evidence does not support a systematic underpricing hypothesis. A similar conclusion is reached by Ammann and Wilde (2008) who use a Monte-Carlo simulation approach. Moreover, market participants almost always calibrate their models to the observed market prices using implied convertible volatilities. Therefore, underpricing may not be the main driver of profitability in convertible arbitrage.

We study the sensitivities of convertible bonds and find that convertible bonds have relatively large positive gammas, implying that convertible arbitrage can make a profit on a large upside or downside movement in the underlying stock price. Since convertible bonds are issued mainly by start-up or small companies (while more established firms rely on other means of financing), the chance of a large movement in either direction is very likely. Even for very small movements in the underlying stock price, profits can still be generated from the yield of the convertible bond and the interest rebate for the short position.

2 Model

Convertible bonds can be thought of as normal corporate bonds with embedded options, which enable the holder to exchange the bond asset for the issuer's stock. Despite their popularity and ubiquity, convertible bonds still pose difficult modeling challenges, given their hybrid nature of containing both debt and equity features. Further complications arise due to the frequent presence of complex contractual clauses, such as, put, hard call, soft call, and other path-dependent trigger provisions. Contracts of such complexity can only be solved by numerical methods, such as, Monte Carlo simulation, tree/lattice approaches, or PDE solutions.

Three sources of randomness exist in a convertible bond: the stock price, the interest rate, and the credit spread. As practitioners tend to eschew models with more than two factors, it is a legitimate question: How can we reduce the number of factors or which factors are most important? Grimwood and Hodges (2002) conduct a sensitivity study and find that accurately modeling the equity process appears crucial. This is why all convertible bond models in the market capture, at a minimum, the dynamics of the underlying equity price. Since convertible bonds are issued mainly by start-up or small companies (while more established firms rely on other means of financing), credit risk plays an important role in the valuation. Grimwood and Hodges (2002) further note that the interest rate process is of second order importance. Similarly, Brennan and Schwartz (1980) conclude that the effect of a stochastic interest rate on convertible bond prices is so small that it can be neglected. Furthermore, Ammann, et al. (2008) notice that the overall

pricing benefit of incorporating stochastic interest rates would be very limited and would not justify the additional computational costs. For these reasons, most practical convertible models in the market do not take stochastic interest rate into account.

The risk-free stock price process can be described as

$$dS(t) = r(t)S(t)dt + \sigma S(t)dW(t) \quad (1)$$

where $S(t)$ denotes the stock price, $r(t)$ denotes the risk-free interest rate, σ denotes the volatility, $W(t)$ denotes a Wiener process.

The expectation of equation (1) is

$$E(dS(t)|\mathcal{F}_t) = r(t)S(t)dt \quad (2)$$

where $E(\bullet|\mathcal{F}_t)$ is the expectation conditional on the $\mathcal{F}_t$.

Next, we turn to a defaultable stock. The defaultable stock process proposed by Davis and Lischka (1999), Andersen and Buffum (2004), and Bloomberg (2009), etc., is given by

$$dS(t) = (r(t) + h(t))S(t_-)dt + \hat{\sigma}S(t_-)dW(t) - S(t_-)dU(t) \quad (3)$$

where $U(t)$ is an independent Poisson process with $dU(t) = 1$ with probability $h(t)dt$ and 0 otherwise, $h(t)$ is the hazard rate or the default intensity, $S(t_-)$ is the stock price immediately before any jump at time t . The expectation of $dU(t)$ is $E(dU(t)|\mathcal{F}_t) = h(t)dt$.

The expectation of equation (3) is given by

$$E(dS(t)|\mathcal{F}_t) = (r(t) + h(t))S(t)dt - S(t)h(t)dt = r(t)S(t)dt \quad (4)$$

It is shown in equation (4) that the expected return of a defaultable stock under a jump-diffusion model also grows at the risk-free interest rate. Equation (3) is a simpler version of the Merton's Jump-diffusion model where the number of jumps is 1.

However, we wonder whether it is appropriate to propagate the jump-diffusion model directly from the market risk domain to the credit risk domain, as credit risk actually impacts the valuation of assets. This

is why financial institutions are required by regulators to report CVA. In fact, we will show in the following derivation that the expected return of a defaultable asset under a risk-neutral measure is actually equal to a risky rate instead of the risk-free rate. This conclusion is very important for risky valuation.

In the reduced-form models, the stopping (or default) time τ of a firm is modeled as a Cox arrival process (also known as a doubly stochastic Poisson process) whose first jump occurs at default and is defined as,

$$\tau = \inf \left\{ t : \int_0^t h(s, \Phi_s) ds \geq \Delta \right\} \quad (5)$$

where $h(t)$ or $h(t, \Phi_t)$ denotes the stochastic hazard rate or arrival intensity dependent on an exogenous common state Φ_t , and Δ is a unit exponential random variable independent of Φ_t .

It is well-known that the survival probability from time t to s in this framework is defined by

$$p(t, s) := P(\tau > s \mid \tau > t, Z) = \exp\left(-\int_t^s h(u) du\right) \quad (6)$$

The default probability for the period (t, s) in this framework is defined by

$$q(t, s) := P(\tau \leq s \mid \tau > t, Z) = 1 - p(t, s) = 1 - \exp\left(-\int_t^s h(u) du\right) \quad (7)$$

We consider a defaultable asset that pays nothing between dates t and T . Let $V(t)$ and $V(T)$ denote its values at t and T , respectively. Risky valuation can be generally classified into two categories: the *default time approach* (DTA) and the *default probability (intensity) approach* (DPA).

Under a risk-neutral measure, the value of this defaultable asset is the discounted expectation of all the payoffs and is given by

$$V(t) = E\left[\left(D(t, T)V(T)1_{\tau > T} + D(t, \tau)\phi V(\tau)1_{\tau \leq T}\right) \mid \mathcal{F}_t\right] \quad (8)$$

where 1_Y is an indicator function that is equal to one if Y is true and zero otherwise, and $D(t, \tau)$ denotes the stochastic risk-free discount factor at t for the maturity τ given by

$$D(t, \tau) = \exp\left[-\int_t^\tau r(u) du\right] \quad (9)$$

Although the DTA is very intuitive, it has the disadvantage that it explicitly involves the default time/jump. We are very unlikely to have complete information about a firm's default point, which is often inaccessible. Usually, valuation under the DTA is performed via Monte Carlo simulation.

The DPA relies on the probability distribution of the default time rather than the default time itself. We divide the time period (t, T) into n very small time intervals (Δt) and assume that a default may occur only at the end of each very small period. In our derivation, we use the approximation $\exp(y) \approx 1 + y$ for very small y . The survival and default probabilities for the period $(t, t + \Delta t)$ are given by

$$\hat{p}(t) := p(t, t + \Delta t) = \exp(-h(t)\Delta t) \approx 1 - h(t)\Delta t \quad (10)$$

$$\hat{q}(t) := q(t, t + \Delta t) = 1 - \exp(-h(t)\Delta t) \approx h(t)\Delta t \quad (11)$$

The binomial default rule considers only two possible states: default or survival. For the one-period $(t, t + \Delta t)$ economy, at time $t + \Delta t$ the asset either defaults with the default probability $q(t, t + \Delta t)$ or survives with the survival probability $p(t, t + \Delta t)$. The survival payoff is equal to the market value $V(t + \Delta t)$ and the default payoff is a fraction of the market value: $\phi(t + \Delta t)V(t + \Delta t)$. Under a risk-neutral measure, the value of the asset at t is the expectation of all the payoffs discounted at the risk-free rate and is given by

$$V(t) = E\{\exp(-r(t)\Delta t)[\hat{p}(t) + \phi(t)\hat{q}(t)]V(t + \Delta t) | \mathcal{F}_t\} \approx E\{\exp(-y(t)\Delta t)V(t + \Delta t) | \mathcal{F}_t\} \quad (12)$$

where $y(t) = r(t) + h(t)(1 - \phi(t)) = r(t) + c(t)$ denotes the risky rate and $c(t) = h(t)(1 - \phi(t))$ is called the (short) credit spread.

Similarly, we have

$$V(t + \Delta t) = E\{\exp(-y(t + \Delta t)\Delta t)V(t + 2\Delta t) | \mathcal{F}_{t+\Delta t}\} \quad (13)$$

Note that $\exp(-y(t)\Delta t)$ is $\mathcal{F}_{t+\Delta t}$ -measurable. By definition, an $\mathcal{F}_{t+\Delta t}$ -measurable random variable is a random variable whose value is known at time $t + \Delta t$. Based on the *taking out what is known* and *tower* properties of conditional expectation, we have

$$\begin{aligned}
V(t) &= E\left\{\exp(-y(t)\Delta t)V(t+\Delta t)|\mathcal{F}_t\right\} \\
&= E\left\{\exp(-y(t)\Delta t)E\left[\exp(-y(t+\Delta t)\Delta t)V(t+2\Delta t)|\mathcal{F}_{t+\Delta t}\right]|\mathcal{F}_t\right\} \\
&= E\left\{\exp\left(-\sum_{i=0}^1 y(t+i\Delta t)\Delta t\right)V(t+2\Delta t)|\mathcal{F}_t\right\}
\end{aligned} \tag{14}$$

By recursively deriving from t forward over T and taking the limit as Δt approaches zero, the risky value of the asset can be expressed as

$$V(t) = E\left\{\exp\left[-\int_t^T y(u)du\right]V(T)|\mathcal{F}_t\right\} \tag{15}$$

Using the DPA, we obtain a closed-form solution for pricing an asset subject to credit risk. Another good example of the DPA is the CDS model proposed by J.P. Morgan (1999).

Under a risk-neutral measure the market price of risk and risk preferences are irrelevant to asset pricing (see Hull (2003)) and thereby the expectation of a risk-free asset grows at the risk-free interest rate. However, credit risk actually has a significant impact on asset prices. This is the reason that regulators, such as IASB and BCBS, require financial institutions to report a CVA in addition to the risk-free MTM value to reflect credit risk.

If a company files bankruptcy, both bonds and stocks go into a default status. In other words, the default probabilities for both of them are the same (i.e., equal to the firm's probability of default). But the recovery rates are different because the stockholders are the lowest priority in the list of the stakeholders in the company, whereas the bondholders have a higher priority to receive a higher percentage of invested funds. The default proceedings provide a justification for our modeling assumptions: Different classes of securities issued by the same company have the same default probability but different recovery rates.

According to equation (15), we propose a risky model that embeds the probability of the default jump rather than the default jump itself into the price dynamics of an asset. The stochastic differential equation (SDE) of a defaultable stock is defined as

$$dS(t) = (r(t) + h(t)(1 - \varphi_s(t)))S(t)dt + \sigma S(t)dW(t) = y(t)S(t)dt + \sigma S(t)dW(t) \tag{16}$$

where φ_s is the recovery rate of the stock and $y(t) = r(t) + h(t)(1 - \varphi_s(t))$ is the risky rate.

For most practical problems, zero recovery at default (or jump to zero) is unrealistic. For example, the stock of Lehman Brothers fell 94.3% on September 15, 2008 after the company filed for Chapter 11 bankruptcy. Similarly, the shares of General Motors (GM) plunged 32% on June 1, 2009 after the firm initiated Chapter 11 bankruptcy. A good framework should flexibly allow people to incorporate different recovery assumptions into risky valuation.

Equation (16) is the direct derivation of equation (15). The formula allows different assumptions concerning recovery on default. In particular, $\varphi_s = 0$ represents the situation where the stock price jumps to 0, and $\varphi_s = 1$ corresponds to the risk-free case. The expectation of equations (16) is

$$E(dS(t)|\mathcal{F}_t) = (r(t) + h(t)(1 - \varphi_s(t)))S(t)dt \quad (17)$$

Equation (17) says that the expected return of a stock subject to credit risk is equal to a risky rate rather than the risk-free rate. The risky rate reflects the compensation investors receive for bearing credit risk.

3. Empirical results

This section presents the empirical results. We use two years of daily data from September 10, 2010 to September 10, 2012, i.e., a total of 522 observation days. This proprietary data are obtained from an investment bank. They consist of convertible bond contracts, market observed convertible prices, interest rate curves, credit curves, stock prices, implied Black-Scholes volatilities, and recovery rates.

Figure 1. Histogram of convertible bonds by time to maturity

This histogram divides the convertible bonds in our sample, as of September 10, 2012, into different bins according to the time to maturity. The x-axis represents the maturity in years and the y-axis represents the number of convertibles in each bins. A maturity bin of n covers contracts with a time to maturity ranging from $n-1$ years to n years.

We only consider the convertibles outstanding during the period and with sufficient pricing information. As a result, we obtain a final sample of 164 convertible bonds and a total of $164 \times 522 = 85,608$ observations. None of the convertibles in this sample actually defaulted during the time window.

Convertible bond prices observed in the market will be compared with theoretical prices under different volatility assumptions. The sample is segmented into two sets according to the time to maturity: a short-maturity class (0 ~ 8 years) and a long-maturity class (> 8 years). We first select a convertible bond from each group: a 7-year (or 5-year outstanding) contract and a 20-year (or 17-year outstanding) contract shown in Table 1.

Table 1. Convertible Bonds

We hide the issuer names according to the security policy of the investment bank, but everything else is authentic. In the market, either a conversion price or a conversion ratio is given for a convertible bond, where $\text{conversion ratio} = (\text{face value of the convertible bond}) / (\text{conversion price})$.

Convertible bond	Case 1 (a 7-year convertible)	Case 2 (a 20-year convertible)
Issuer	X company	Y company

Notional of bond	100	100
Annual coupon rate	2.625	5.5
Payment frequency	Semiannual	Semiannual
Issuing date	June 9, 2010	June 15, 2009
Maturity date	June 15, 2017	June 15, 2029
Conversion price	30.288	13.9387
Currency	USD	USD
Day count	30/360	30/360
Business day convention	Following	Following
Put price	-	100 at June 20, 2014

Let valuation date be September 10, 2012. An interest rate curve is the term structure of interest rates, derived from observed market instruments that represent the most liquid and dominant interest rate products for certain time horizons. Normally the curve is divided into three parts. The short end of the term structure is determined using the London Interbank Offered Rates (LIBOR). The middle part of the curve is constructed using Eurodollar futures that require convexity adjustments. The far end is derived using mid swap rates. The LIBOR-future-swap curve is presented in Table 2. We bootstrap the curve and get the continuously compounded zero rates.

Table 2: USD LIBOR-Future-Swap Curve

This table displays the closing prices as of September 10, 2012.

Instrument Name	Price
September 19, 2012 LIBOR	0.6049%
September 2012 Eurodollar 3 month	99.6125
December 2012 Eurodollar 3 month	99.6500
March 2013 Eurodollar 3 month	99.6500
June 2013 Eurodollar 3 month	99.6350
September 2013 Eurodollar 3 month	99.6200
December 2013 Eurodollar 3 month	99.5900
March 2014 Eurodollar 3 month	99.5650

2 year swap rate	0.3968%
3 year swap rate	0.4734%
4 year swap rate	0.6201%
5 year swap rate	0.8194%
6 year swap rate	1.0537%
7 year swap rate	1.2738%
8 year swap rate	1.4678%
9 year swap rate	1.6360%
10 year swap rate	1.7825%
12 year swap rate	2.0334%
15 year swap rate	2.2783%
20 year swap rate	2.4782%
25 year swap rate	2.5790%
30 year swap rate	2.6422%

The equity information and recovery rates are provided in Table 3. To determine hazard rates, we need to know the observed market prices of corporate bonds or CDS premia, as the market standard practice is to fit the implied risk-neutral default intensities to these credit sensitive instruments. The corporate bond prices are unfortunately not available for companies *X* and *Y*, but their CDS premia are observable as shown in Table 4. Usually the CDS market leads the bond market, in particular during crisis situation. Liquidity in the bond market is typically drying up during a financial crisis. Demand for insurance against default risk, on the other hand, increases if the issuer is experiencing financial stress. Consequently, prices and spreads derived from the CDS market tend to be more reliable. Said differently, CDSs on reference entities are

Table 3. Equity and recovery information

This table displays the closing stock prices and dividend yields on September 10, 2012, as well as the recovery rates

	Company X	Company Y
--	------------------	------------------

Stock price	34.63	23.38
Dividend yield	2.552%	3.95%
Bond recovery rate	40%	36.14%
Equity recovery rate	2%	1%

Table 4. CDS premia

This table displays the closing CDS premia as of September 10, 2012.

Name	Company X	Company Y
6 month CDS spread	0.00324	0.01036
1 year CDS spread	0.00404	0.01168
2 year CDS spread	0.00612	0.01554
3 year CDS spread	0.00825	0.01924
4 year CDS spread	0.01027	0.02272
5 year CDS spread	0.01216	0.02586
7 year CDS spread	0.01388	0.02851
10 year CDS spread	0.01514	0.03003
15 year CDS spread	0.01544	0.03064
20 year CDS spread	0.01559	0.03101

The most important input parameter to be determined is the volatility for valuation. A common approach in the market is to use the at-the-money (ATM) implied Black-Scholes volatility to price convertible bonds. For the 5-year outstanding convertible bond (case 1 in Table 1), we find the ATM implied Black-Scholes volatility is 31.87%, and then price the convertible bond accordingly. The results are shown in Table 5. Our analysis actually indicates an overpricing of 0.42%.

Table 5. Model price vs. market price

This table shows the differences between the model prices and the market prices of the convertible bonds under different volatility assumptions, where $\text{Difference} = (\text{Model price}) / (\text{Market observed price}) - 1$. The convertible bonds are defined in Table 1.

	Case 1 (a 7-year convertible)	Case 2 (a 20-year convertible)
Type of volatility	ATM implied Black-Scholes volatility	Annualized historical volatility
Value of volatility	31.87%	18.07%
Model price	134.32	171.58
Market observed price	134.88	169.77
Difference	-0.42%	1.07%

We repeat this exercise for all contracts on all observation days. For any short-maturity convertible bond, we use the ATM implied Black-Scholes volatility for pricing, whereas for any long-maturity convertible bond, we perform valuation via the historical volatility. The results are presented in Tables 6.

Table 6. Statistics of underpricing for different maturity classes

An observation corresponds to a price snapshot of a convertible bond at a certain valuation date. Underpricing is referred to as the model price minus the market price.

Maturity	Observations	Underpricing			
		Mean (%)	Std (%)	Max (%)	Min (%)
≤ 8 years	82998	-0.13	1.37	0.79	-1.08
> 8 years	2610	1.67	2.03	2.24	0.58

Next, our sample is partitioned into subsamples according to the moneyness of convertibles. The moneyness is measured by the ratio of the conversion value to the equivalent straight bond value or the investment value. The underpricing of each daily observation with respect to the degree of moneyness is shown in Table 7, where moneyness between 0 and 0.9 corresponds to out-of-the-money; moneyness between 0.9 and 1.1 represents around-the-money; and moneyness higher than 1.1 is related to in-the-money.

Table 7. Statistics of underpricing for different moneyness classes

The moneyness is measured by dividing the conversion value through the associated straight bond value. An observation corresponds to a snapshot of the market used to price a convertible bond at a certain valuation date.

Moneyness	Observations	Underpricing	
		Mean (%)	Std (%)
< 0.5	5794	0.72	2.23
0.5 – 0.7	10595	-0.87	2.37
0.7 – 0.9	19850	0.51	1.64
0.9 – 1.1	14737	0.45	1.12
1.1 – 1.3	14379	-0.55	1.89
1.3 – 1.5	11631	-0.42	2.04
> 1.5	8622	-0.62	1.72

From Tables 7, it can be seen that the model prices fluctuate randomly around the market prices (sometimes overpriced and sometimes underpriced), indicating the model is quite accurate. Empirically, we do not find support for presence of a systematic underpricing as indicated in previous studies (see Carayannopoulos and Kalimipalli (2003), Ammann, et al. (2003), etc.). If there is no underpricing, how has the arbitrage strategy been successful in the past? Maybe convertible arbitrage is not solely based on underpricing

We calculate the delta and gamma values for the two deals described in table 1. The Greeks vs. spot equity prices are plotted in Figures 2~ 5. It can be seen that the deltas increase with the underlying stock prices in Figures 2 and 4. At low market levels, the convertibles behave like their straight bonds with very small deltas. As the stock price increases, conversion becomes more likely. At certain market levels the convertibles are certain to be converted. In this case, the convertibles are similar to the underlying equities and the deltas are equal to the number of shares (i.e., conversion ratios).

Figure 2. Delta vs. underlying price for a 7-year convertible bond

This graph shows how the delta of the 7-year convertible bond (described in Table 1) changes as the underlying stock price changes.

Figure 3. Gamma vs. underlying price for a 7-year convertible bond

This graph shows how the gamma of the 7-year convertible bond (described in Table 1) changes as the underlying stock price changes.

Figure 4. Delta vs. underlying price for a 20-year convertible bond

This graph shows how the delta of the 20-year convertible bond (described in Table 1) changes as the underlying stock price changes.

Figure 5. Gamma vs. underlying price for a 20-year convertible bond

This graph shows how the delta of the 20-year convertible bond (described in Table 1) changes as the underlying stock price changes.

The gamma diagrams in Figures 3 and 5 have a frown shape. The gammas are the highest when the convertibles are at-the-money. It is intuitive that when the stock prices rise or fall, profits increase because of favorably changing deltas. For this reason, convertible bonds are very good candidates for delta neutral hedging. Relatively large positive gammas of convertibles could be one of the main drivers of profitability in convertible arbitrage.

4. Conclusion

This paper aims to value hybrid financial instruments (e.g., convertible bonds) whose values may simultaneously depend on different assets subject to credit risk in a proper and consistent way. The motivation for our model is that if a company goes bankrupt, all the securities (including the equity) of the company default. The recovery is realized in accordance with the priority established by the Bankruptcy Code. In other words, different securities have the same probability of default, but different recovery rates.

We propose a hybrid framework to value risky equities and debts in a unified way. The model relies on the probability distribution of the default jump rather than the default jump itself. The model is quite accurate for pricing convertible bonds.

References

- Agarwal, V., W. Fung, Y. Loon and N. Naik, (2007), "Liquidity provision in the convertible bond market: analysis of convertible arbitrage hedge funds," CFR-working paper, Georgia State University.
- Ammann, M, Kind, A., and Wilde, C., (2008), "Simulation-based pricing of convertible bonds," *Journal of empirical finance*, Vol. 15, No. 2, pp. 310-331.
- Ayache, E., Forsyth, P. A., and Vetzal, K. R., (2003), "The valuation of convertible bonds with credit risk," *Journal of Derivatives*, Vol. 11, No. 1, pp. 9-30.
- Bloomberg, (2009), "OVCV model description," Quantitative research and development, Equities team.
- Calamos, Nick P., (2011), *Convertible arbitrage: Insights and techniques for successful hedging*, John Wiley & Sons.
- Carayannopoulos, P. and Kalimipalli, M., (2003), "Convertible bond prices and inherent biases," *Journal of Fixed Income*, Vol. 13, No. 3, pp. 64-73.
- Choi, D., Getmansky, M. and Tookes, H., (2009), "Convertible bond arbitrage, liquidity externalities, and stock prices," *Journal of Financial Economics*, Vol. 91, No. 2, 227-251.
- Davis, M. and Lischka, F. R., (1999), "Convertible bonds with market risk and credit risk," Working paper, Tokyo-Mitsubishi International plc.

Duffie, D., and Singleton, K. J., (1999), “Modeling term structure of defaultable bonds,” Review of Financial Studies, Vol. 12, No. 4, pp. 687-720.

FinPricing, 2015, Pricing Service, <https://finpricing.com/lib/FiZeroBond.html>

Goldman Sachs, (1994), “Valuing convertible bonds as derivatives,” Quantitative Strategies Research Notes, Goldman Sachs.

Hull, J., (2003), Options, Futures and Other Derivatives. 5th ed. Prentice Hall, Upper Saddle River, NJ.

Loncarski, I., Horst, J., and Veld C., (2009), “The rise and demise of the convertible arbitrage strategy,” Financial Analysts Journal, Vol. 65, No. 5, pp. 35-50.

J. P. Morgan, (1999), “The J. P. Morgan guide to credit derivatives,” Risk Publications.

Somanath, V.S., (2011), International financial management, I.K. International Publishing House Pvt. Ltd.

Xiao, T., (2013), “An accurate solution for credit value adjustment (cva) and wrong way risk,” Working paper, SSRN.

Zhou, C., (1997), “A jump-diffusion approach to modeling credit risk and valuing defaultable securities,” Technical Report, Federal Reserve Board, Washington DC.