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On the infinity of the Dirac delta function

Open Mathematics Collaboration^{*†}

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Abstract

This is a microarticle on the infinity of the Dirac delta function [1].

keywords: Dirac delta function, infinity, fundamental property

The origin of Dirac delta's fundamental property

1. The formal definition of the Dirac delta is [2]

$$\delta(x) = \frac{1}{\pi} \lim_{\epsilon \rightarrow 0} \frac{\epsilon}{x^2 + \epsilon^2}$$

for $x \in \mathbb{R}$ and $\epsilon > 0$.

^{*}All authors with their affiliations appear at the end of this paper.

[†]Corresponding author: mplobo@uft.edu.br | Join the Open Mathematics Collaboration

2. It can also be defined as

$$\delta(x - a) = \begin{cases} \infty & x = a, \\ 0 & \text{otherwise.} \end{cases}$$

3. The delta function has the fundamental property that [3]

$$\int_{-\infty}^{+\infty} f(x)\delta(x - a)dx = f(a).$$

where $f(x)$ is a real-valued function [4].

4. A more detailed treatment of (3) can be found in the Appendix A of [5].

5. In (3), the **infinity** $\delta(a)$, in the limit $\epsilon \rightarrow 0$, “cancels out” the **infinitesimal** interval $(x - \epsilon, x + \epsilon)$, in which $dx = 2\epsilon$, resulting in $f(a)$.

6. The limit mentioned in (5) refers to the limit of the definition of integrals.

7. This was probably the *motivation* when Dirac created $\delta(x)$.

Final Remarks

8. We linked the fundamental property of $\delta(x)$, (3), with the result $f(a)$. In summary, we tried to elucidate an explanation for the creation of the Dirac delta function.

Open Invitation

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Ethical conduct of research

This original work was pre-registered under the OSF Preprints [6], please cite it accordingly [7]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

References

- [1] Dirac, Paul Adrien Maurice. *The principles of quantum mechanics*. Oxford university press, 1981.
- [2] Wolfram, Stephen. Official website. *Dirac delta function: Primary definition*. <http://functions.wolfram.com/PDF/DiracDelta.pdf>
- [3] Wolfram, Stephen. Official website. *Delta function*. <http://mathworld.wolfram.com/DeltaFunction.html>
- [4] Lobo, Matheus P. “The Metric Tensor Pullback.” *OSF Preprints*, 14 May 2019. <https://doi.org/10.31219/osf.io/puhzw>
- [5] Salasnich, Luca. *Quantum Physics of Light and Matter*. Springer International, 2017. http://cds.cern.ch/record/1707567/files/978-3-319-05179-6_BookBackMatter.pdf
- [6] OSF. *Open Science Framework*. <https://osf.io>
- [7] Lobo, Matheus P., and José C. de Oliveira Junior. “On the Infinity of the Dirac Delta Function.” *OSF Preprints*, 18 May 2019. <https://doi.org/10.31219/osf.io/6t98u>

The Open Mathematics Collaboration

Matheus Pereira Lobo (lead author, mplobo@uft.edu.br),¹

José Carlos de Oliveira Junior,¹ Tiago Sousa Moraes²

¹Federal University of Tocantins (Brazil); ²Colégio Estadual Rui Barbosa
(Tocantins, Brazil)