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Population Externalities and Optimal Social Policy³

If fertility is not chosen in a socially optimal way, and if policies to directly target fertility are ineffective or politically infeasible, then public policies that affect fertility could have important welfare consequences through the fertility channel. We refer to these effects as population externalities, and in this paper we focus on one important variable that may have a causal impact on fertility: the education of potential parents. If increased education causes families to have fewer children, then a government would want to increase college tuition subsidies in the presence of environmental externalities such as climate change, to indirectly discourage families from having children who will generate future environmental costs. Alternatively, if fertility is inefficiently low, due to imperfect parental altruism for example, governments will want to lower tuition subsidies to encourage child-bearing. We present a simple model of the college enrollment decision and its fertility impacts, and show that such

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population externalities are quantitatively important: the optimal subsidy increases by about \$5000 per year with climate change, and decreases by over \$7000 per year with imperfect parental altruism. Our paper demonstrates how public economics can incorporate population externalities, and that such externalities can have significant impacts on optimal policy.

1. Introduction

A wide variety of variables can affect both a family's decision to have children and their desired number of children. Clearly, there is considerable heterogeneity in the fertility preferences of families, but a number of economic variables can enter the decision, including macroeconomic conditions, the employment status of the potential parents (Currie and Schwandt 2014), public monetary fertility incentives such as baby bonuses (Milligan 2005), family policies such as parental leave and benefits (Hyatt and Milne 1991; Phipps 2000; Cannonier 2014), and even programs that are not targetted at fertility, such as social assistance (Robins and Fronstin 1996).

In particular, one important variable that may have a causal impact on fertility is the education level of the parents: a number of papers have found significant negative effects of parental education on fertility in developed countries, including (León 2004), (Amin and Behrman 2014), (Lavy and Zablotsky 2015), and (Fort, Schneeweis, and Winter-Ebmer 2016).⁴ This is not a universal finding in the literature,⁵ but the weight of evidence of recent empirical studies for the U.S. generally supports the conclusion that education has a negative causal effect on fertility.

However, the fact that a number of policies and policy-related variables could impact fertility doesn't necessary imply a welfare impact. If fertility is chosen by households in a socially optimal way, then there are no population externalities from such policies, due to a standard envelope condition: even if subsidizing education (for example) leads to a more educated population that chooses a lower level of fertility, if households choose that fertility level optimally given their level of education, the derivative of welfare with respect to fertility is zero and there are no first-order welfare consequences of the resulting change in fertility.

But what if fertility is not chosen in a socially optimal way? It is not hard to imagine mechanisms that could lead to an inefficiently high or low level of fertility – and thus to an inefficiently large or small population – and we focus on two in this paper: environmental externalities such as climate change, and imperfect altruism on the part of parents who do not fully internalize the future utility of their potential children.

⁴ The latter paper finds a reduction in total fertility for the U.K. but not for continental Europe. Some other papers focus on timing of fertility and find that education causes a delay (Black, Devereux, and Salvanes 2008; Tropsf and Mandemakers 2017; James and Vujić 2019), though perhaps no overall effect on total fertility (Monstad, Propper, and Salvanes 2008).

⁵ Some papers find no effects at all, such as (Kan and Lee 2018) for Taiwan, and (Fort, Schneeweis, and Winter-Ebmer 2011) even finds a positive effect, but their study focusses on low-skill individuals affected by compulsory schooling laws for which positive income effects from increased education may outweigh negative substitution effects and lead to higher fertility.

Environmental externalities are relevant for questions of population because each person born adds to the amount of pollution produced, imposing social costs on the entire population. Several papers, including (Harford 1997), show that a standard pollution tax, or any other instrument that targets emissions per person, is ineffective in internalizing the population externality if the revenues from the policy are used in some way to benefit the population, include distributing them lump-sum to each individual. Indeed, (Harford 1997) shows that the optimal policy includes a “tax per child equal to the discounted present value of all pollution taxes that the child and her descendants would pay”; in the absence of such a child tax, fertility will tend to be inefficiently high, and the title of (Harford 1998) refers to this as “The Ultimate Externality.” We focus in this paper on the specific and profoundly important case of climate change.

In the opposite direction, the field of population ethics suggests a possible reason why fertility could be inefficiently low: parents may not fully account for the future utility of their potential children when making the decision to have children or not. That is, parents may be imperfectly altruistic towards their children, and if the social planner wants to maximize the discounted stream of present and future utility from all generations – rather than just maximizing the parents’ utility – the planner will place more weight on the utility of future children than their parents do, suggesting that fertility will be inefficiently low.⁶

If either (or both) of these arguments are correct, fertility and population levels may differ considerably from the socially efficient allocation. In such a case, the standard public economics response would be to attempt to correct the inefficiency at the source, such as with a subsidy or tax on childbearing, so as to ensure that parents face the efficient “price” of having a child. However, it is hard to alter fertility in rich countries; numerous studies summarized in (Gautier 2007) have found that while fertility subsidies may affect the timing of births, the overall effect on total completed fertility is considerably smaller, and (Blanchet and Ekert-Jaffé 1994) estimated that the total effect of fertility policies across various countries was about 0.2 children per woman. This implies that correcting any distortion to fertility using directly targetted policy could be quite difficult; in other words, if fertility responds slowly to subsidies and taxes, the optimal subsidy or tax could be infeasibly large.

In such a second-best world, we need to consider the impacts of other policies on fertility: if a policy causes the privately-optimal level of fertility to increase, for example, this would generate added welfare gains in the case of imperfect altruism, and added welfare losses in the case of environmental externalities. In both cases,

⁶ An alternative mechanism leading to insufficient fertility could be market size effects on productivity and innovation: a larger population could generate more innovation in technologies that can benefit everyone, introducing a positive population externality.

various social policies could offer an indirect way to target fertility and population, and these welfare impacts should be considered when evaluating an optimal policy.

If education has a negative causal effect on fertility, as we will assume following the work of (Amin and Behrman 2014), then college tuition subsidies represent one such policy: if a higher tuition subsidy – or lower net costs of college – encourages more young individuals to become more educated, and if that in turn leads to reduced fertility, we may want a considerably larger subsidy than we otherwise would if we are concerned about climate change, whereas we may want a lower subsidy (and maybe even an education tax) if we are concerned about imperfect altruism.

We will consider this question in our paper, basing our analysis on (Lawson 2017), who considered optimal college tuition subsidies in the presence of fiscal externalities (positive effects of education on future tax revenues through higher incomes) and liquidity constraints. We show that we can slightly alter his analysis to incorporate the two inefficiencies discussed above, and we derive sufficient-statistics equations for the derivative of welfare with respect to the tuition subsidy, which we use to evaluate how climate change and imperfect altruism each impact the optimal subsidy. The estimated effects are large: in our baseline case, the optimal subsidy from (Lawson 2017) was about \$8000 per year, and we find that the optimal subsidy increases by about \$5000 per year in the presence of climate change, and decreases by over \$7000 per year in the presence of imperfect parental altruism. The significant magnitude of these impacts are consistent with the intuition from (Hendren and Sprung-Keyser 2020) that policies that impact very young children are likely to have particularly large welfare impacts; in our case, the children are impacted before they are even born.

It is important to note that the goal of this paper is not to calculate a precise estimate of the optimal college tuition subsidy. Rather, our goal is to demonstrate that public economists can incorporate population externalities in their analyses of public policy, and that the impact of such externalities can be very significant. Accordingly, a final section of our paper shows how our approach can be generalized to other policies: under certain assumptions, the population externality can be expressed as the product of the effect of the program on fertility (which is program-specific) and a term that captures the magnitude of the distortion to fertility.

In our analysis, we assume a unitary family model for simplicity, in which a college tuition subsidy offered to the one unit of parent impacts the number of units of child produced. However, this is isomorphic to a setting in which each family consists of a father and a mother, if we assume that the government is constrained to set the same subsidy for males and females and that the responsiveness of enrollment to subsidies is the same for male and female students. As we discuss in section 3, these are conservative assumptions, as the impact on optimal subsidies

would be larger (either for women alone, or for all students, depending on the situation) if the government could set different subsidies for men and women or if women responded more to subsidies than men.⁷

The rest of the paper proceeds as follows. Section 2 presents a baseline model of education and tuition subsidies based on (Lawson 2017), and solves for the optimal tuition subsidy in that setting, while section 3 shows how dramatically the optimal policy can be affected – in either direction – by population externalities. Section 4 then extends this idea to a more general setting, showing how population externalities can affect analyses of other social policies. Section 5 concludes the paper.

2. Optimal Policy in (Lawson 2017) Model of Education

To begin our analysis, we need to first present our baseline model of education and tuition subsidies in the absence of fertility considerations. What follows is a simplified version of the presentation from (Lawson 2017), using the same notation as in that paper for simplicity. We then summarize the optimal policy analysis and results from that paper, which serve as a baseline for our analysis of the impact of fertility subsidies on optimal policy.

2.1 (Lawson 2017) Model

The model consists of 12 periods of 4 years each, representing a working-age life of 48 years. In the first period, each individual i in a population of measure one chooses between attending college ($s_i = 1$) and working at wage w_{01} ($s_i = 0$); we abstract from gender and consider a unitary household model, but we will discuss issues of gender further in section 3. In periods $t = 2, \dots, 12$, the individual works at an exogenous wage w_{st} that depends on their first-period education choice s_i , where $w_{1t} > w_{0t} \forall t$. Individuals also choose labour supply l_{st} that is constant across time for simplicity, receiving an income of $Y_{sti} = w_{st}l_{si}$ per period. The real interest and discount rates are both equal to r , with the discount factor denoted as $\beta \equiv \frac{1}{1+r}$, and exogenous productivity growth causes wages to grow at a rate of g per period.

⁷ If the government could set different tuition subsidies for male and female students, then the estimated zero effect of husband's education on fertility in (Amin and Behrman 2014) would imply that the optimal subsidy for men would be that from section 2 of this paper, whereas the impact on optimal subsidies for women would be much larger than those estimated in section 3 since the effect of their education on fertility would be twice as large as we assume there. Meanwhile, if women responded more to tuition subsidies than men, as indicated by (Dynarski 2008) and (Card and Lemieux 2001), then fertility would respond even more to tuition subsidies than we estimate, imply larger effects of population externalities on optimal subsidies.

Workers receive per-period utility $v^s(c_{vi}^s, l_{si})$ from consumption c_{vi}^s and labour supply, while students' utility $u(c_{ui})$ varies only with consumption, and utility functions satisfy the usual properties: $u', v_c^s > 0$, $v_l^s < 0$, and $u'', v_{cc}^s, v_{ll}^s < 0$, where subscripts denote derivatives. An individual who chooses not to attend college will select labour supply l_{0i} and consumption c_{vi}^0 per period, and receive lifetime utility $U_{0i} = \sum_{t=1}^{12} \beta^{t-1} v^0(c_{vi}^0, l_{0i})$. Individuals who choose to attend college will select post-schooling consumption c_{vi}^1 and labour supply l_{1i} , as well as consumption c_{ui} while in college, receiving lifetime utility $U_{1i} + \eta_i$, where $U_{1i} = u(c_{ui}) + \sum_{t=2}^{12} \beta^{t-1} v^1(c_{vi}^1, l_{1i})$ and where η_i represents idiosyncratic utility from schooling and is the only source of heterogeneity in this simplified form of the model.

We define $R_x \equiv \sum_{t=x}^{12} \left(\frac{1}{1+r}\right)^{t-1}$ and $\gamma_x \equiv \sum_{t=x}^{12} \left(\frac{1+g}{1+r}\right)^{t-1}$ to simplify notation, and then we can write the individual's budget constraints, for $s_i = 0$ and $s_i = 1$ respectively:

$$\begin{aligned} R_1 c_{vi}^0 &= (1 - \tau) \gamma_1 w_{01} l_{0i} \\ c_{ui} + R_2 c_{vi}^1 &= (b - e) + (1 - \tau) \gamma_2 w_{11} l_{1i} \end{aligned}$$

where e is the direct cost of college to the individual, τ is the marginal tax rate, and b is the government grant given to students, which incorporates all financial support provided to students by the government. Students may also face a liquidity constraint in the form of a debt limit A :

$$c_{ui} - (b - e) \leq A.$$

Therefore, the individual's maximization problem is to choose $\{s_i, c_{vi}^0, c_{vi}^1, c_{ui}, l_{0i}, l_{1i}\}$ to maximize $V_i = s_i(U_{1i} + \eta_i) + (1 - s_i)U_{0i}$:

$$\begin{aligned} V_i &= s_i[u(c_{ui}) + R_2 v^1(c_{vi}^1, l_{1i}) + \eta_i - \lambda_{1i}(c_{ui} + R_2 c_{vi}^1 - (b - e) \\ &\quad - (1 - \tau) \gamma_2 w_{11} l_{1i}) - \mu_i(c_{ui} - (b - e) - A)] + (1 - s_i)[R_1 v^0(c_{vi}^0, l_{0i}) \\ &\quad - \lambda_{0i}(R_1 c_{vi}^0 - (1 - \tau) \gamma_1 w_{01} l_{0i})]. \end{aligned} \quad (1)$$

The government chooses b and τ subject to a budget constraint:

$$Sb + G = \tau[S\gamma_2 E(Y_{11i}|s_i = 1) + (1 - S)\gamma_1 E(Y_{01i}|s_i = 0)] = \tau\bar{Y}$$

where $S = E(s_i)$ is the fraction of the population attending college, $\bar{Y}$ is mean total discounted lifetime income, and G is the exogenous discounted sum of other government spending over the 12 periods. Social welfare $V = E(V_i)$ is utilitarian with

equal weights, and an envelope theorem result then simplifies our welfare analysis: any changes in individual choices of labour supply, consumption, and college enrollment when b changes have no first-order welfare impact, because those variables were chosen by the individual to maximize their utility. As a result, welfare can be written as $V(b, \tau(b))$, and the welfare gain from increasing b is:

$$\frac{dV}{db} = \frac{\partial V}{\partial b} + \frac{\partial V}{\partial \tau} \frac{d\tau}{db} = E\left(\frac{\partial V_i}{\partial b}\right) + E\left(\frac{\partial V_i}{\partial \tau}\right) \frac{d\tau}{db}. \quad (2)$$

This welfare derivative combines the direct effect of raising b with the indirect effect of changing τ to balance the government budget. To evaluate it, we use the first-order conditions of the individual's maximization problem. Since η_i is the only source of heterogeneity, consumption and labour supply choices do not vary by individual conditional on the choice of s_i :

$$\begin{aligned} \frac{\partial V_i}{\partial b} &= s_i(\lambda_{1i} + \mu_i) = s_i u'(c_u) \\ \frac{\partial V_i}{\partial \tau} &= -s_i \lambda_{1i} \gamma_2 Y_{11i} - (1 - s_i) \lambda_{0i} \gamma_1 Y_{01i} \\ &= -s_i \gamma_2 Y_{11} v_c^1(c_v^1, l_1) - (1 - s_i) \gamma_1 Y_{01} v_c^0(c_v^0, l_0) \\ \frac{d\tau}{db} &= \frac{S}{\bar{Y}} \left[1 + \varepsilon_{sb} - \left(1 + \frac{G}{Sb} \right) \varepsilon_{\bar{Y}b} \right] \end{aligned} \quad (3)$$

where ε_{sb} is the elasticity of college enrollment with respect to student grants b , and $\varepsilon_{\bar{Y}b}$ is the elasticity of average income (accounting for foregone earnings) with respect to b .

In the expression for $\frac{\partial V_i}{\partial \tau}$, we make the conservative assumption that $S\gamma_2 Y_{11} v_c^1(c_v^1, l_1) + (1 - S)\gamma_1 Y_{01} v_c^0(c_v^0, l_0) \simeq \bar{Y} v_c^0(c_v^0, l_0)$, and we also normalize the welfare gain into a dollar amount, by defining $\frac{dW}{db} \equiv \frac{\frac{dV}{db}}{v_c^0(c_v^0, l_0)}$ as the welfare gain in terms of an equivalent amount of consumption among non-graduates. The welfare derivative thus becomes:

$$\frac{dW}{db} \simeq S \left[\frac{u'(c_u) - v_c^0(c_v^0, l_0)}{v_c^0(c_v^0, l_0)} - \varepsilon_{sb} + \left(1 + \frac{G}{Sb} \right) \varepsilon_{\bar{Y}b} \right]. \quad (4)$$

Finally, we decompose the ratio of marginal utilities into two empirically observable quantities, which can be called liquidity and substitution effects. An individual attends college if the idiosyncratic utility from schooling exceeds a critical value η^* :

$$\eta^* = R_1 v^0(c_v^0, l_0) - u(c_u) - R_2 v^1(c_v^1, l_1).$$

We assume that η_i follows some continuously differentiable distribution $F(\eta)$ with a density given by $f(\eta)$. As a result, $S = 1 - F[R_1 v^0(c_v^0, l_0) - u(c_u) - R_2 v^1(c_v^1, l_1)]$, and therefore:

$$\begin{aligned} \frac{\partial S}{\partial b} &= f(\eta^*) u'(c_u) \\ \frac{\partial S}{\partial a_1} &= f(\eta^*) [u'(c_u) - v_c^0(c_v^0, l_0)] \end{aligned}$$

where a_1 is a lump-sum of cash in the first period, representing a change in initial assets. Thus, we can rewrite (4)(2) as:

$$\frac{dW}{db} \simeq S \left[L - \varepsilon_{sb} + \left(1 + \frac{G}{Sb} \right) \varepsilon_{\bar{y}b} \right] \quad (5)$$

where $L = \frac{\frac{\partial S}{\partial a_1}}{\frac{\partial S}{\partial b} - \frac{\partial S}{\partial a_1}}$. The $\frac{\partial S}{\partial a_1}$ in the numerator of L is the liquidity effect, whereas the $\frac{\partial S}{\partial b} - \frac{\partial S}{\partial a_1}$ in the denominator is the substitution effect, as it represents the effect on enrollment of changing relative prices without transferring income to students. A higher value of L thus indicates more severe liquidity constraints among students.

2.2 Optimal Tuition Subsidies

As in (Lawson 2017), we can now use estimates of each of the terms in (5) – the sufficient statistics for welfare analysis – to calculate an estimated value of $\frac{dW}{db}$, and we can also perform statistical extrapolations of these quantities, modelling how their values change as b changes to find the optimal tuition subsidy b . The list below gives a brief summary of the values chosen for the sufficient statistics in Table 1, which are chosen to represent the United States in 2007; hats represent baseline values, and further details can be found in (Lawson 2017).

- To estimate the baseline b , we use data on receipt of federal and state grants, loans, and work-study in 2007-08 from (Wei et al. 2009), and an adjustment formula from (Epple, Romano, and Sieg 2006); financial support was about \$1690 per student, and lacking data on other forms of government assistance, we round this up to \$2000. As we will denote monetary amounts in thousands of dollars per year, we have $\hat{b} = 2$.
- (Deming and Dynarski 2009) find a general consensus that a \$1000 increase in price of college leads to a 4 percentage point decline in attendance, which implies an elasticity of $\varepsilon_{sb} \simeq 0.2$. As a lower bound, we use the fact that (Dynarski 2008) estimates that \$2500 of financial aid leads to a 4 percentage point increase in degree completion from a base of 27% to motivate a value of 0.1.
- The estimated college enrollment rate of 18-24-year-olds in 2007 was $\hat{S} = 0.388$, according to. Assuming a constant elasticity of enrollment with respect to grants, this implies that $S = \phi b^{\varepsilon_{sb}}$, where $\phi = \frac{\hat{S}}{\hat{b}^{\varepsilon_{sb}}}$.
- Our preferred estimate of L is zero, as numerous papers argue that income has no causal effect on enrollment. Alternatively, results in (Acemoglu and Pischke 2001) imply that a \$1000 increase in family income increases enrollment by 0.21% points, so we also consider $\frac{\partial \hat{S}}{\partial a_1} = 0.0021$, which implies $\hat{L} = 0.057$ or 0.121 depending on the value of ε_{sb} . As in (Lawson 2017), we model a decline in L with S according to $L = \max\{\frac{\hat{L}(0.16 - (S - \hat{S}))}{0.16 + \hat{L}(S - \hat{S})}, 0\}$.
- We assume that the interest and discount rates are 3% per year, which implies $r = 0.12$ and $\beta = \frac{1}{1.12} \simeq 0.893$. We assume wage growth of $g = 0.04$, since the average real growth rate in the SSA average net compensation series was 1% over 1991–2008.
- We assume that each year of schooling increases earnings by 8%, and that the elasticity of taxable income is 0.4, as found by (Gruber and Saez 2002); see (Lawson 2017) for the resulting algebraic expression for $\varepsilon_{\bar{y}b}$, with baseline values of 0.0063 or 0.0142 depending on the value of ε_{sb} .
- The baseline tax rate is $\tau = 0.23$ (incorporating a 15% federal tax, a 5% state tax, and 3% for the Medicare tax), and the CPS 2008 Annual Social and Economic Supplement gives us $Y_{01} = 34$ for high school workers, meaning that $Y_{11} = 34(1.08)^4 = 46.26$. These estimates imply $G = 68.606$ and $\frac{\hat{G}}{\hat{sb}} = 88.410$, and thus $\frac{G}{sb} = 88.41 \frac{\hat{sb}}{sb}$.

Table 1. Baseline Values of Sufficient Statistics

<i>Statistic</i>	<i>Definition</i>	<i>Value</i>
$\hat{S}$	enrollment rate	0.388
$\hat{b}$	per-year student grant	2
ε_{Sb}	elasticity of enrollment w.r.t. b	{0.1,0.2}
$\frac{\partial \hat{S}}{\partial a_1}$	effect of income on enrollment	{0,0.0021}
r	interest and discount rate per period	0.12
g	wage growth per period	0.04
$\hat{\varepsilon}_{\bar{Y}b}$	elasticity of mean income w.r.t. b	{0.0063,0.0142}
$\frac{\widehat{G}}{\widehat{Sb}}$	ratio of exogenous spending to grant spending	88.410

The optimal policy results, as estimated in (Lawson 2017), can be found in Table 2. Panel A presents an estimate of the welfare derivative at the baseline $b = 2$, in dollar terms relative to the annual amount within a period, so it is the present-value equivalent of the welfare derivative expressed per year over the 4 years of the first period. Panel B contains the estimated optimal grants, and panel C lists the estimated net welfare gains from moving to the optimum, which are calculated by numerically integrating $\frac{dW}{db}$ from $b = 2$ to the optimum; the welfare gain is then expressed as the dollar amount of an equivalent one-year per-person consumption increase, as well as (in brackets) a percentage of the initial size of the student grant program.

The results from the (Lawson 2017) analysis are dramatic: student grants should be increased by at least \$3800 per year, and by over \$6000 in our preferred case of $\varepsilon_{Sb} = 0.2$ and $\frac{\partial \hat{S}}{\partial a_1} = 0$. The welfare gains are also large, at about 41 cents per dollar spent at baseline in our preferred case, and with the optimal policy generating welfare improvements that are equivalent to as much as 0.17% of GDP at the economy-wide level. In our preferred case, enrollment increases from 38.8% to 51.3% at the optimal $b = 8.093$.

Table 2. Optimal Policy Results using (5)

ε_{sb}	$\frac{\widehat{\partial S}}{\partial a_1}$	
	0	0.0021
A. Estimate of $\frac{dW}{db}$ at $b = 2$		
0.1	0.1811	0.2282
0.2	0.4148	0.4370
B. Optimal Student Grants		
0.1	\$5843	\$8371
0.2	\$8093	\$8355
C. Welfare Gains from Moving to Optimum		
0.1	\$947 (30.5%)	\$1750 (56.4%)
0.2	\$3138 (101.1%)	\$3471 (111.8%)

Notes: Panel A presents the one-period per-year increase in welfare, expressed in dollars of consumption from a per-year one dollar increase in b . Panel C expresses welfare gains as a one-time lump-sum increase in consumption, and as a percentage of baseline spending on student grants. This format is used throughout all subsequent tables of this form.

3. Optimal Policy with Population Externalities

The model to this point has simply been a slightly simplified version of the baseline model from (Lawson 2017). We can now build upon this baseline by introducing a new population dimension to our model, in the form of a fertility decision by the model's individuals. Specifically, we assume that, in period 3 of the model, each individual i chooses how many children n_{si} to have, where the decision may depend on their s_i . The third period corresponds to ages 26–30, and (OECD Family Database 2019) indicates that a mother's average age at childbirth is about 29 in the United States; we avoid issues of discreteness by allowing n_{si} to be a continuous variable that represents expected children per individual.

We abstract from gender by considering a unitary family model in which the individual represents the mother-father unit, and we use the estimate from (Amin and Behrman 2014) that the causal impact of university education relative to high school is -0.678 children for women and roughly zero for men, which in a unitary-household model implies that the negative effect of university on fertility is -0.339 units of child per household. It is important to note that this is isomorphic to a

setting in which families consists of a separate father and mother, if we assume that the government must set the same subsidy for men and women and that the responsiveness of enrollment to subsidies is the same for men and women.

It is also important to note that the latter are conservative assumptions that are likely to lead us to an underestimate of the impact of population externalities (either for all students or for women alone). If the government can set different tuition subsidies for men and women, then the finding in (Amin and Behrman 2014) that the effect of the husband's education on fertility is zero means that the father's optimal subsidy would be given by section 2 of this paper (with no fertility impacts), whereas the mother's optimal subsidy would be given by the analysis to come but with a doubled effect of education on fertility (-0.678 rather than -0.339). In that case, the effect of population externalities on optimal tuition subsidies, at least for women, would be even larger than estimated in this paper.

Alternatively, we could apply the finding in (Dynarski 2008) that the overall effect of financial aid on graduation is higher for women,⁸ which would imply that fertility responds even more to tuition subsidies, since those subsidies will impact women – whose education matters for fertility – more than the average. This would again imply even larger population externality effects than what we estimate in the current section of the paper.

For simplicity, we assume that the household's utility from children is additively separable from the rest of the utility function (1), taking the form of net private utility $q_s(n_{si})$, where the utility function varies according to whether an individual went to university or not, which will permit different fertility levels to be chosen by college graduates and non-graduates. Conditional on their choice of $s_i = \{0,1\}$, the individual will choose n_{si} such that $q'_s(n_{si}) = 0$; however, we will consider below the possibility that q_s is not equal to the social value of n_{si} children, so that the privately-optimal level of fertility may not be socially optimal.

In particular, we will now model – one at a time – the two possible population externalities that we consider in this paper: environmental externalities from greenhouse gases, and imperfect parental altruism.

3.1 Environmental Externalities from Population

We now introduce environmental externalities from population: we will assume that, while $q_s(n_{si})$ represents the private utility of individual i from their children,

⁸ (Card and Lemieux 2001) finds similar evidence of a greater responsiveness of female enrollment to tuition fees. On the other hand, while very few studies estimate effects of price on enrollment separately for men and for women, studies that consider only men – such as (Cameron and Heckman 2001) – find estimates of similar magnitude to the rest of the literature.

those children will also contribute to a public bad in the form of climate change. For simplicity, we assume this is the only effect of education on greenhouse gas emissions; that is, we assume that there is no direct effect of education on emissions (or that a carbon tax has been set optimally to offset this margin).⁹ It is possible that a direct effect of education on emissions could provide an alternative mechanism altering the optimal subsidy policy, but we abstract from this mechanism in our paper to focus on the fertility channel.

As a result of these assumptions, we can write a new social welfare function that extends (1) to this setting. Since the function q_s does not vary across individuals, there are only 2 possible levels of fertility in our model (n_0 and n_1), and so we can simplify our notation from n_{si} to n_s , giving us:

$$\hat{V}_i = V_i + s_i q_1(n_1) + (1 - s_i) q_0(n_0) - e(\bar{n}) \quad (6)$$

where V_i is identical to the original equation (1), and it is followed by 3 new terms. The first two represent the private utility from fertility (which is also valued by the social planner), whereas the final term represents the disutility experienced by everyone from the pollution generated by children, where $\bar{n} = S n_1 + (1 - S) n_0$ is the average number of children in the population.

Due to our simplifying assumption of additive separability of utility from children, the subsequent welfare analysis is unchanged from that in section 2 except for two things. First of all, the critical value η^* has to be replaced with a critical value of $\eta_i + q_1(n_1) - q_0(n_0)$, but this has no impact on the results, as it is simply a redefining of variables which are not among the sufficient statistics. Second, and more importantly, the welfare derivative in (2) has an extra term:

$$\frac{d\hat{V}}{db} = \frac{\partial V}{\partial b} + \frac{\partial V}{\partial \tau} \frac{d\tau}{db} + \frac{\partial \hat{V}}{\partial \bar{n}} \frac{d\bar{n}}{db}$$

because each n_{si} is the value that is privately optimal to the individual, but not the socially optimal value given the externality that operates through $\bar{n}$, which no individual is large enough to affect on their own.

⁹ (Bruderer Enzler and Diekmann 2015) provide a summary of the literature that studies the impact of individual characteristics on individual greenhouse gas emissions, and finds mixed results; a more recent paper by (Moser and Kleinhüchelkotten 2018) finds no significant effect of education on emissions. However, the consensus in the literature is that higher income raises emissions, so the overall effect of increased education is probably an increase in emissions.

Therefore, since $\bar{n}$ changes with b only because S changes with b , the welfare derivative in dollar terms can be written as:

$$\frac{d\widehat{W}}{db} = \frac{dW}{db} - \frac{e'(\bar{n})}{v_c^0(c_v^0, l_0)} \frac{d\bar{n}}{dS} \frac{dS}{db} \quad (7)$$

and to evaluate this derivative, we simply need to calculate the final term and subtract it from the $\frac{dW}{db}$ calculated in section 2. We already know that $\frac{d\bar{n}}{dS} = -0.339$, and that $\frac{dS}{db} = \frac{s}{b} \varepsilon_{Sb}$, so the main challenge is calculating $\frac{e'(\bar{n})}{v_c^0(c_v^0, l_0)}$, or the social environmental disutility caused by population in dollar terms.

To calculate this term, we need estimates of the emissions per person per year, the (discounted) number of years per person, and the social cost of each unit of emissions. For the first of these quantities, we use the estimate from (Climate Watch 2019) that carbon-dioxide-equivalent greenhouse gas emissions are about 20 tonnes per person in the United States; we assume that this average is also the marginal amount, as we are not aware of any estimates of the emissions impact of adding a marginal person to the population.

We assume that these 20 tonnes per year will be emitted by our marginal individual over a life expectancy of 20 periods, starting from period 4; however, we need to discount the costs of emissions occurring in the future. We continue to use a discount rate of 3% per year, but estimates from (EPA 2017) indicate that the social cost of carbon emissions also increases in real terms over time, at a rate of about 2% per year.¹⁰ With a resulting net discount rate of 1% per year or 4% per period, this implies that the 20 periods of life are the discounted present value equivalent of 12.565 periods.

Finally, we assume a social cost of carbon of \$100 per tonne. This value is a compromise between actual values used in the real world and recent estimates of the actual social cost from the scientific literature. For example, in 2015, the (EPA 2017) assumed a social cost of carbon of \$36 in 2007 dollars,¹¹ whereas (Ricke et al. 2018) estimate a global social cost of carbon of \$417 per tonne of carbon dioxide.

Therefore, a child born in our model will generate a social cost that is equivalent to $12.565 \times 20 \times 100 = \25130 per year over a 4-year period from their contribution

¹⁰ (EPA 2017) explains that “estimates of the social cost of these greenhouse gases increase over time because future emissions are expected to produce larger incremental damages as physical and economic systems become more stressed in response to greater climatic change, and because GDP is growing over time and many damage categories are modeled as proportional to gross GDP.”

¹¹ An exception to the rule that the values used in the real world are relatively low is given by Sweden, where the current carbon price is 110 Euros per tonne; see (Government Offices of Sweden 2020).

to climate change, and so $\frac{e'(\bar{n})}{v_c^0(c_p^0, l_0)} = 25.13$. This means that the term that we add to the welfare derivative $\frac{dW}{db}$ is $8.52 \frac{S}{b} \varepsilon_{Sb}$, and we can produce new estimates of the optimal policy results from Table 2 in the current model, which can be found in Table 3.

The welfare implications of the population externality from climate change are highly significant: the welfare derivatives nearly double, and the optimal grants increase by at least \$3500, with an increase of \$5000 per year in our preferred case of $\varepsilon_{Sb} = 0.2$ and $\frac{\partial \widehat{S}}{\partial a_1} = 0$. The overall welfare gains are also 2 to 3 times as large, and college enrollment S increases to 56.5% at the optimal policy in our preferred scenario. In additional results that are available upon request, we can produce estimates from simulations of the calibrated model instead of statistical extrapolation; as in (Lawson 2017), those results are similar to the ones in Table 3.¹²

We can also perform a sensitivity analysis with a social cost of carbon of \$417 as in (Ricke et al. 2018), and the impact of population externalities is extremely large in that case: in the preferred case of $\varepsilon_{Sb} = 0.2$ and $\frac{\partial \widehat{S}}{\partial a_1} = 0$, the welfare derivative increases to 1.7932, and the optimal grant is \$29106, which is approximately equal to the average worker's after tax income at baseline. The welfare gain becomes \$33279, equivalent to a 1.6% increase in GDP each year, and college enrollment S increases to 66.3% at the optimum. These results can most plausibly be interpreted as saying that, if the social cost of carbon is really \$417, we need to do a lot to reduce climate change, including a significant reduction to fertility – and we probably need to go well beyond education policy in doing so.

In any case, we can conclude that population externalities from climate change have a large impact on optimal tuition subsidy policy: if we are concerned about the contribution of population to climate change, but we lack efficient direct policy tools to affect fertility, very large subsidies to college tuition would be justified if that would lead to reductions in fertility.

¹² The welfare derivatives are slightly smaller in the simulations, but when $\varepsilon_{Sb} = 0.2$, the optimal grant is about \$1000 higher and the overall welfare gain is about 50% higher. The optimal b and the welfare gain are also significantly larger when $\varepsilon_{Sb} = 0.1$ and $\frac{\partial \widehat{S}}{\partial a_1} = 0$.

Table 3. Optimal Policy Results using (7)

ε_{sb}	$\frac{\partial \widehat{S}}{\partial a_1}$	
	0	0.0021
A. Estimate of $\frac{d\widehat{W}}{db}$ at $b = 2$		
0.1	0.3464	0.3935
0.2	0.7454	0.7676
B. Optimal Student Grants		
0.1	\$9380	\$12429
0.2	\$13072	\$12879
C. Welfare Gains from Moving to Optimum		
0.1	\$2826 (91.0%)	\$4115 (132.6%)
0.2	\$8338 (268.6%)	\$8694 (280.1%)

3.2 Imperfect Parental Altruism

An alternative potential reason for fertility deviating from the social optimum can be found in the field of population ethics: we assume that parents receive private utility $q_s(n_{si})$ from their children, and we now allow for the possibility that the real social utility $\tilde{q}_s(n_{si})$ generated by the children is larger than q_s . That is, the parent might not be perfectly altruistic and might underweight the utility of their children relative to what a utilitarian social planner would do if that planner wanted to maximize the discounted stream of all present and future utilities in the world.¹³

To model this possibility, we will explicitly specify both the parental utility function $q_s(n_{si})$ and the social utility function $\tilde{q}_s(n_{si})$. We assume that the utility function from children consists of two terms: a term that is linear in n_{si} , representing the utility per person that will be experienced by children, and a cost term that is increasing and convex in n_{si} , representing the monetary and/or effort cost for the parent to raise n_{si} children. We will then assume that the parent underweights the first term, underappreciating the utility that will be experienced by their children in the future.

¹³ Alternative mechanisms that could generate inefficiently low fertility might include market-size effects that cause larger populations to discover a larger number of positive innovations, though the functional forms might be different in that case.

Before presenting the utility functions, we have to address an important question: why do more-educated individuals have fewer children? In the previous subsection, we did not explicitly model the parental utility functions, as envelope conditions easily eliminated them from the welfare derivative, but if we are to model $q_s(n_{si})$, we have to model the mechanism that allows education to affect fertility. There are two possibilities: more-educated individuals' costs are higher, or their perceived benefits in terms of future child utility are lower. The second approach seems problematic, as it implies that more-educated individuals are less altruistic than they would be if they had less education (which could make individuals reluctant to attend college as that would reduce their altruistic utility). Instead, we choose the first approach and assume that more-educated individuals have fewer children because higher education raises their costs of child-bearing; this seems plausible, if more-educated individuals have more valuable uses for their time. Therefore, we assume that the net parental utility function is:

$$q_s(n_{si}) = \alpha \hat{u}_2 n_{si} - \frac{\theta_s}{\chi} n_{si}^\chi$$

whereas the social utility function from children is:

$$\tilde{q}_s(n_{si}) = \hat{u}_2 n_{si} - \frac{\theta_s}{\chi} n_{si}^\chi$$

where $\alpha < 1$ represents the degree of imperfect altruism and $\chi > 1$ represents the degree of convexity in the cost function. $\hat{u}_2 \equiv u_2 - \bar{u}_2$ represents the expected utility u_2 of the child relative to a critical value $\bar{u}_2$; ¹⁴ we assume that the child's expected utility does not vary with the education decision of the parent, to keep the algebra simple. θ_s is the parameter that determines the level of the cost function, and we assume that $\theta_1 > \theta_0$ as discussed above. As a result, the number of children chosen by individuals depends only on their education decision, with $n_s = \left(\frac{\alpha \hat{u}_2}{\theta_s}\right)^{\frac{1}{\chi-1}}$, and so $n_1 < n_0$ as expected.

At the private optimum, the utility experienced by the parent from their children is then given by:

$$q_s(n_s) = \left(\frac{\chi-1}{\chi}\right) \theta_s \left(\frac{\alpha \hat{u}_2}{\theta_s}\right)^{\frac{\chi}{\chi-1}}$$

¹⁴ The critical value is necessary because a value of zero in our utility functions does not necessarily have an obvious economic interpretation.

whereas the social utility function at that same value of n_s is:

$$\tilde{q}_s(n_s) = \left(\frac{\chi - \alpha}{\alpha \chi} \right) \theta_s \left(\frac{\alpha \hat{u}_2}{\theta_s} \right)^{\frac{\chi}{\chi-1}}$$

The planner has no way of affecting the value of n_s chosen by the parent conditional on s_i , but if they want to encourage fertility, they can try to discourage individuals from going to college. To be precise about this intuition, we can solve for the welfare derivative, starting from the welfare function:

$$\hat{V}_i = V_i + s_i \tilde{q}_1(n_1) + (1 - s_i) \tilde{q}_0(n_0) \quad (8)$$

where as before, the critical value η^* will need to be replaced with a critical value of $\eta_i + q_1(n_1) - q_0(n_0)$, which simply shifts the mean of the distribution of η .

It is now simplest to write the extra term of the welfare derivative in terms of S , because s_i is not chosen efficiently due to the difference between the social utility function $\tilde{q}_s$ and the private utility function q_s :

$$\frac{d\hat{V}}{db} = \frac{\partial V}{\partial b} + \frac{\partial V}{\partial \tau} \frac{d\tau}{db} + \frac{\partial \hat{V}}{\partial S} \frac{dS}{db}$$

and for the partial derivative with respect to S , we have:

$$\frac{\partial \hat{V}}{\partial S} = \frac{\partial V}{\partial S} + \tilde{q}_1(n_1) - \tilde{q}_0(n_0)$$

whereas the individual's optimization condition gives us:

$$\frac{\partial V}{\partial S} + q_1(n_1) - q_0(n_0) = 0.$$

Therefore our partial derivative with respect to S becomes:

$$\begin{aligned} \frac{\partial \hat{V}}{\partial S} &= (\tilde{q}_1(n_1) - q_1(n_1)) - (\tilde{q}_0(n_0) - q_0(n_0)) \\ &= \left(\frac{1 - \alpha}{\alpha} \right) (\alpha \hat{u}_2)^{\frac{\chi}{\chi-1}} \left[\theta_1^{\frac{-1}{\chi-1}} - \theta_0^{\frac{-1}{\chi-1}} \right] \end{aligned}$$

and this can even be further simplified if we can observe n_1 and n_0 , because we can use the individual's solution for $n_s = \left(\frac{\alpha \hat{u}_2}{\theta_s} \right)^{\frac{1}{\chi-1}}$, and therefore we have:

$$\frac{\partial \hat{V}}{\partial S} = (1 - \alpha)\hat{u}_2(n_1 - n_0).$$

Therefore, our welfare derivative now becomes:

$$\frac{d\hat{W}}{db} = \frac{dW}{db} - \frac{\hat{u}_2}{v_c^0(c_v^0, l_0)}(1 - \alpha)(n_0 - n_1)\frac{dS}{db}. \quad (9)$$

We already know that $n_0 - n_1 = 0.339$, and that $\frac{dS}{db} = \frac{S}{b} \varepsilon_{Sb}$, so the main challenges are calculating $\frac{\hat{u}_2}{v_c^0(c_v^0, l_0)}$ (the social utility generated by a child in dollar terms), and choosing a value of α .

To estimate $\frac{\hat{u}_2}{v_c^0(c_v^0, l_0)}$, we simulate the calibrated model from (Lawson 2017) for the childrens' generation, using the parameters from that paper and assuming that wages (and the debt limit A) continue to grow at $g = 0.04$ per period for the child. We also increase the mean of η_i to keep the percentage of children who go to university at $\hat{S} = 0.388$, and calculate the expected lifetime utility of the child, which is then discounted back to the start of the model to give us a total of $u_2 = 11.9821$ in utils. The critical level $\bar{u}_2$ is somewhat arbitrary, but to be very conservative, we assume that it is the level of utility obtained by an uneducated child who works full-time at the 2007 federal minimum of \$5.85. Such a worker obtains consumption of 11.7 with $l_{0i} = 1$, which generates a total discounted present value utility of $\bar{u}_2 = 3.4298$.

Finally, at the baseline values in (Lawson 2017), the marginal utility from consumption for an uneducated worker is $v_c^0(c_v^0, l_0) = 0.045$, which means that $\frac{\hat{u}_2}{v_c^0(c_v^0, l_0)} = 190.1049$. Our conservative estimate, therefore, is that the discounted present social value of the utility of a child is about \$190 thousand dollars per year over 4 years, or about \$760 thousand dollars in total. Therefore, the term we subtract from the welfare derivative $\frac{dW}{db}$ is $64.45(1 - \alpha)\frac{S}{b} \varepsilon_{Sb}$, and we will present results for a value of $\alpha = 0.8$;¹⁵ the optimal policy results can be found in Table 4.

¹⁵ This is still a fairly high value for α , but for lower values the optimal grant drops to zero, which is an artificial lower bound caused by the parametric assumptions that we make, including the assumption that $S = 0$ if $b = 0$.

Table 4. Optimal Policy Results using (9)

ε_{sb}	$\frac{\partial \widehat{S}}{\partial a_1}$	
	0	0.0021
A. Estimate of $\frac{d\widehat{W}}{db}$ at $b = 2$		
0.1	-0.0709	-0.0242
0.2	-0.0853	-0.0633
B. Optimal Student Grants		
0.1	\$536	\$1169
0.2	\$822	\$1021
C. Welfare Gains from Moving to Optimum		
0.1	\$285 (9.2%)	\$46 (1.5%)
0.2	\$248 (8.0%)	\$145 (4.7%)

The results are dramatically different from those in the previous subsection on environmental externalities: the baseline welfare derivatives turn negative, the optimal grants decrease by about \$5300 to \$7300 per year, and the total welfare gains are now small because the optimal grants are not too far from $b = 2$. This is despite the fact that we made several conservative assumptions in the calibration, including the assumption that the critical level of utility is that of a full-time minimum wage worker – which implies that anyone worse off would have, from the perspective of the planner, a life that is not worth living – and our assumption that parents only underestimate their child's contribution to global welfare by 20%. To make any of these assumptions less conservative, we would need to use a calibration-and-simulation approach, to model S more flexibly. In principle, however, we could generate almost any magnitude of negative b – that is, of education taxes – with empirically plausible assumptions on the parameters.

As before, we can conclude that population externalities have a large impact on optimal tuition subsidy policy in this case: if we are concerned about the population ethics consequences of imperfect altruism, but we lack efficient direct policy tools to affect fertility, any economic argument for increased subsidies to college tuition vanishes if such subsidies would lead to reductions in fertility.

4. A General Toolbox for Welfare Analysis

The analysis of this paper has focussed so far on the specific setting of college tuition subsidies, because this is a policy that can have straightforward and intuitive effects on fertility decisions. However, a variety of other social and economic policies could also have important impacts on fertility: obviously there are direct fertility incentives from “baby bonus” policies such as that studied by (Milligan 2005), but also maternal/parental leave and benefit programs (see (Hyatt and Milne 1991), (Phipps 2000), and (Cannonier 2014) for analyses of their effects on fertility), and there are a few studies of the impact of programs such as social assistance on fertility (Robins and Fronstin 1996). On a related note, (Currie and Schwandt 2014) find that higher unemployment reduces conceptions, which suggests that a policy like unemployment insurance that affects the duration and severity of experiences of unemployment could also have an impact on fertility.

Of course, if fertility is chosen in a socially-optimal way, then none of these potential interactions between public policy and fertility have first-order welfare implications. However, if environmental or population ethics externalities exist, then any number of social policies that impact fertility could be affected by population externalities.

As a result, we will now demonstrate how the analysis from the previous section of the paper can be generalized to a broader range of policies. To do so, let us consider a version of the general model presented in section 3 of (Chetty 2009). We will focus on a static representative agent model, but the analysis easily generalizes to heterogeneous agents and multi-period problems, as mentioned by (Chetty 2009). To make the intuition of the basic model as easy to understand as possible, we begin with a version without any population externalities; in other words, there are no future generations to consider (or at least their utility is unaffected by choices made by the current generation).

We assume that the representative agent makes a vector of J choices denoted by $x = \{x_1, \dots, x_J\}$, receiving utility $U(x)$. The vector x can include choices such as the binary decision to attend university, but it also includes consumption and labour supply in different states, such as graduate, non-graduate, and student, along with potentially a wide variety of other choice variables.

The government, meanwhile, operates a variety of programs and taxes that act upon various choices made by the agent, but as in (Chetty 2009) we assume that the government wants to evaluate one particular program, which we assume is a monetary incentive acting upon choice 1. The government places this monetary incentive p (which could be a subsidy to education, or any other one-parameter policy program) on x_1 , and finances it with a tax t on x_J , where a government budget

constraint (which may include other exogenous government spending as in the education model above) defines the budget-balancing values of t given p and the agent's choices.

The agent also faces a set of $M < J$ constraints on their choices, given by $\{G_1(x, p, t) = 0, \dots, G_M(x, p, t) = 0\}$, and so the representative agent solves:

$$\max_x U(x) \text{ s. t. } G_1(x, p, t) = 0, \dots, G_M(x, p, t) = 0. \quad (10)$$

Therefore, the equation for social welfare can be written as:

$$W(p, t) = \max_x \left\{ U(x) + \sum_{m=1}^M \lambda_m G_m(x, p, t) \right\}. \quad (11)$$

When we take the derivative of W with respect to p , we can use the envelope condition resulting from individual maximization to conclude that $\frac{\partial W}{\partial x} = 0$, and therefore the welfare derivative is:

$$\frac{dW}{dp} = \sum_{m=1}^M \lambda_m \left[\frac{\partial G_m}{\partial p} + \frac{\partial G_m}{\partial t} \frac{dt}{dp} \right] \quad (12)$$

where $\frac{dt}{dp}$ comes from the total derivative of the government budget constraint. We then use the individual maximization results:

$$U_j(x) = - \sum_{m=1}^M \lambda_m \frac{\partial G_m}{\partial x_j} \quad \forall j$$

where $U_j(x)$ is the derivative of U with respect to x_j . Next, we add an assumption that (Chetty 2009) calls Assumption 1, which is that p affects the constraints in the same way as the good (x_1) on which it is placed, and the tax affects the constraints in the same way as the good (x_j) on which it is levied. This implies that there exist functions $k_p(x, p, t)$ and $k_t(x, p, t)$ such that:

$$\begin{aligned} \frac{\partial G_m}{\partial p} &= -k_p(x, p, t) \frac{\partial G_m}{\partial x_1} \quad \forall m \\ \frac{\partial G_m}{\partial t} &= k_t(x, p, t) \frac{\partial G_m}{\partial x_j} \quad \forall m. \end{aligned}$$

Using these results and assumptions, we can write the welfare derivative as follows:

$$\begin{aligned}
 \frac{dW}{dp} &= \sum_{m=1}^M \lambda_m \left[-k_p(x, p, t) \frac{\partial G_m}{\partial x_1} + k_t(x, p, t) \frac{\partial G_m}{\partial x_j} \frac{dt}{dp} \right] \\
 &= -k_p(x, p, t) \sum_{m=1}^M \lambda_m \frac{\partial G_m}{\partial x_1} + k_t(x, p, t) \frac{dt}{dp} \sum_{m=1}^M \lambda_m \frac{\partial G_m}{\partial x_j} \\
 &= k_p U_1(x) - k_t \frac{dt}{dp} U_j(x).
 \end{aligned}$$

To be able to implement this welfare derivative empirically, we need to be able to evaluate each of these terms. The model itself generally provides functional forms for k_p , k_t , and the derivative of the government budget constraint $\frac{dt}{dp}$, whereas the marginal utility terms require a further step which varies depending on the model: they need to be written in terms of empirically observable quantities, using direct estimates of marginal utility or using the structure of the model to make a substitution. For example, in the education model from section 2 of this paper, $\frac{d\tau}{db}$ takes the place of $\frac{dt}{dp}$, $u'(c_u)$ and v_c^* are the heterogeneous-agent versions of $U_1(x)$ and $U_j(x)$, and $k_p = S$ and $k_t = \bar{Y}$.

We now switch our focus to population externalities; in particular, we will suppose that one of the x_j choices is fertility, and to avoid confusion we will call the fertility decision x_f . As we have stated throughout the paper, if x_f is chosen optimally by the agent, then the analysis above is completely unchanged. However, if the agent does not properly optimize x_f due to an externality, the analysis above will be incomplete, because we have failed to account for the possibility that changing p will affect the agent's choice of x_f .

To keep the analysis simple, let us assume that there is an external effect (which could be positive or negative) of x_f that is not accounted for by the individual; that is, the individual continues to solve the individual optimization problem (10), but the welfare function takes the following form:

$$\widehat{W}(p, t) = \max_x \left\{ U(x) + \sum_{m=1}^M \lambda_m G_m(x, p, t) \right\} + e(x_f)$$

where $e(x_f)$ is the externality caused by fertility. It is important to note that, to keep the analysis simple, we assume additive separability, and we assume that this exter-

nality is only a function of x_f , with no interactions with the other x_j . That is, we assume that the externality is a function of the number of children born, but not of any characteristics of those children that might be a function of other choices represented by x_j . The functional form of $e(x_f)$ could depend on exogenous values of utility per child, or even on a critical value of utility in a setting of critical-level utilitarianism, but $e(x_f)$ cannot depend on an endogenous measure of utility per child. If we wanted to permit interactions between x_f and other x_j , we would need to consider the impact of p on those other choices as well, and we will return to a brief analysis of how this would affect our results later in this section.

The welfare derivative now takes the following form:

$$\frac{d\hat{W}}{dp} = \sum_{m=1}^M \lambda_m \left[\frac{\partial G_m}{\partial p} + \frac{\partial G_m}{\partial t} \frac{dt}{dp} \right] + e'(x_f) \frac{dx_f}{dp}$$

so we simply need to add the extra term representing the effect of p on the externality $e(x_f)$. All of the subsequent steps above for $\frac{dW}{dp}$ are unchanged, as the individual maximization equations and the consequences of the assumption from (Chetty 2009) are unaffected by the introduction of a population externality. Therefore, the final welfare derivative is:

$$\frac{dW}{dp} = k_p U_1(x) - k_t \frac{dt}{dp} U_j(x) + e'(x_f) \frac{dx_f}{dp}$$

We find, therefore, that it is relatively simple to incorporate population externalities into a sufficient-statistic welfare analysis of a social program: we simply need to add an extra term that captures the effect of the program on fertility multiplied by the marginal effect of such a fertility impact on social welfare.

The effect of the program on fertility is given by $\frac{dx_f}{dp}$, which is a quantity that needs to be evaluated empirically; in our analysis, we use a value of $-0.339 \frac{s}{b} \varepsilon_{sb}$, which is equal to -0.0132 at baseline if $\varepsilon_{sb} = 0.2$. In other words, in our case, each \$1000 of tuition subsidy lowers fertility by about 0.013 children per person.

$e'(x_f)$, or the marginal effect of a change in fertility on social welfare, is much harder to evaluate convincingly, as we have documented in this paper: there are plausible reasons for it to be large and negative (environmental externalities), and plausible reasons for it to be large and positive (population ethics considerations). In our analysis, it takes a value of $-25.13 U_1(x)$ in the environmental externality

case and $38.02U_1(x)$ in the population ethics case.¹⁶ Therefore, for a researcher wanting to evaluate the impact of population externalities on an optimal social policy in a second-best world in which it is hard to directly target fertility, our recommendation is to incorporate a term corresponding to $e'(x_f) \frac{dx_f}{dp}$ into their welfare derivative, using an estimated value of $\frac{dx_f}{dp}$ from the relevant literature, and a range of plausible values for $e'(x_f)$ – perhaps corresponding to our range of values of approximately $\{-25, 40\} \times U_1(x)$.

If, as discussed above, the externality was also a function of other choices x_j , we would have to add additional terms to the welfare derivative. For example, if the externality was $e(x_f, x_c)$ where x_c represents inputs into child quality, then instead of simply adding $e'(x_f) \frac{dx_f}{dp}$ to the equation for $\frac{dW}{dp}$, we would have to add $\frac{\partial e(x_f, x_c)}{\partial x_f} \frac{dx_f}{dp} + \frac{\partial e(x_f, x_c)}{\partial x_c} \frac{dx_c}{dp}$. In the case of environmental externalities, this could occur if, for example, education led to an increase in the emissions of the parent or the child; in the case of imperfect parental altruism, education might generate a positive externality through x_c if education raises inputs into child quality. In each case, the conclusions of our welfare analysis would become more ambiguous, but we implicitly assume that $\frac{\partial e(x_f, x_c)}{\partial x_c} \frac{dx_c}{dp}$ is small relative to $\frac{\partial e(x_f, x_c)}{\partial x_f} \frac{dx_f}{dp}$ throughout, to focus on the fertility margin.

5. Conclusion

Our goal in this paper has been to demonstrate that public economists can incorporate population externalities in their analyses of public policy, and that the impact of such externalities can be highly significant. If fertility – and thus population – is not chosen optimally, and if policy measures to directly target fertility are costly and inefficient, then optimal policies in other areas could be strongly affected. We provide a demonstration for the policy of tuition subsidies for college: if such subsidies increase enrollment and graduation, and if that increase in education has a negative causal effect on fertility, then there could be an important population externality channel in our evaluation of the optimal subsidy.

We show that the optimal tuition subsidy is very sensitive to the population externalities we consider: optimal tuition subsidies should be much larger if we want to use education policy to help combat climate change, and optimal tuition subsidies should be much smaller if we want to use education policy to correct

¹⁶ Our final welfare derivative in sections 2 and 3 was normalized into dollars rather than utils, and so we effectively measure $e'(x_f)$ as a constant multiplied by the $U_1(x)$ used for normalization.

imperfect parental altruism. We do not know which of these – if either – is the right answer, but we have shown that answering that question, and evaluating the sign and magnitude of population externalities, is very important.

The final section of the paper demonstrates how to generalize our approach to a broader set of policy questions: to any sufficient-statistic analysis of optimal policy, a new term can be incorporated into the welfare derivative that can be expressed as the product of the effect of the policy on fertility and the effect of that fertility change on welfare.

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