A Unified Theory of Psychophysical Laws in Hearing

Fan-Gang Zeng

Center for Hearing Research, Departments of Anatomy and Neurobiology, Biomedical
Engineering, Cognitive Sciences and Otolaryngology – Head and Neck Surgery
University of California Irvine
fzeng@uci.edu

Abstract

Psychophysical laws quantitatively relate perceptual magnitude to stimulus intensity. While most people have accepted Stevens's power function as the psychophysical law, few believe in Fechner's original idea using just-noticeable-differences (jnd) as a constant perceptual unit to deduce psychophysical laws. Here I present a unified theory in hearing, starting with a general form of Zwislocki's loudness function (1965) to derive a general form of Brentano's law. I will arrive at a general form of the loudness-jnd relationship that unifies previous loudness-jnd theories. Specifically, the "slope", "proportional-jnd", and "equal-loudness, equal-jnd" theories, are three additive terms in the new unified theory. I will also show that the unified theory is consistent with empirical data in both acoustic and electric hearing. Without any free parameters, the unified theory uses loudness balance functions to successfully predict the jnd function in a wide range of hearing situations. The situations include loudness recruitment and its jnd functions in sensorineural hearing loss and simultaneous masking, loudness enhancement and the midlevel hump in forward and backward masking, abnormal loudness and jnd functions in cochlear implant subjects. Predictions of these loudness-jnd functions were thought to be questionable at best in simultaneous masking or not possible at all in forward masking. The unified theory and its successful applications suggest that although the specific form of Fechner's law needs to be revised, his original idea is valid in the wide range of hearing situations discussed here.

Introduction

Psychophysical laws attempt to relate the amplitude of a physical stimulus to its perceived magnitude, such as loudness as a function of sound intensity or brightness as a function of light intensity. The classic approach to uncovering psychophysical laws was advanced by Fechner in the mid 18th century (Fechner, 1966). Fechner assumed that the just-noticeable-difference (jnd), expressed as Weber's fraction ($\Delta I/I$), produced an equal increment in loudness sensation (ΔL). Integrating this equation, namely $\Delta L = \Delta I/I$, he produced the famous Fechner's law: loudness is a logarithmic function of intensity ($L = \log I$).

Not only was Fechner's log law replaced Stevens's power law (Stevens, 1961), his general approach was also questioned due to failure to integrate the jnd functions of two different sounds to predict their respective loudness functions (Miller, 1947; Newman, 1933). Thus, it was not too surprising that the Fechnerian approach in relating the stimulus jnd to the subjective magnitude was abandoned by some researchers. What was surprising is the grounds on which the Fechnerian approach was abandoned. For example, Stevens (1961) argued that the direct magnitude estimation technique obsolesced intensity discrimination as a measure of the stimulus-sensation relationship. He viewed the discrimination measure as "an engineer talking... the scatter of some dial settings." In a completely opposing view, Viemeister and Bacon (1988) stated that loudness estimate data were a measure with "probably strong involvement of non-sensory factors, (and) we did not attempt to relate these data to those for intensity discrimination."

There have been other researchers who continued to advance the Fechnerian approach in searching for a unified theory relating the intensity discrimination to loudness function. Fechner's original assumption was sometimes referred to as the "slope" theory, because it predicted that the steeper the loudness function, the smaller the jnd or Weber's fraction for a constant increment in loudness. This simple slope prediction turned out to be not true at least in cases of loudness recruitment, where cochlear hearing loss or partial masking elevated the hearing threshold but produced abnormally steep loudness growth so that normal loudness was perceived at high sound levels (Fowler,

1936). To account for the failure of Fechner's slope theory, several researchers proposed a "proportional-jnd" theory, in which the jnd size needed to be normalized by the total jnd number within a stimulus's dynamic range (Lim, Rabinowitz, Braida, & Durlach, 1977; Riesz, 1933; Teghtsoonian, 1971). On the other hand, the "equal-loudness, equal-jnd" theory argued that the jnd had no relation to the slope of the loudness function, but rather was determined by the total loudness (J. J. Zwillocki & Jordan, 1986). Despite significant effort in testing these loudness-jnd relationships, no consensus has been reached yet (Allen & Neely, 1997; R. Hellman, Scharf, Teghtsoonian, & Teghtsoonian, 1987; Houtsma, Durlach, & Braida, 1980; Johnson, Turner, Zwillocki, & Margolis, 1993; Rankovic, Viemeister, Fantini, Cheesman, & Uchiyama, 1988; Schlauch, Harvey, & Lanthier, 1995; Schlauch & Wier, 1987; Stillman, Zwillocki, Zhang, & Cefaratti, 1993).

Here I present a unified theory, starting with a general form of Zwillocki's loudness function (1965) to derive a general form of Brentano's law, and I will arrive at a general form of the loudness-jnd relationship that unifies previous loudness-jnd theories. Specifically, I find that the previous "slope", "proportional-jnd", and "equal-loudness, equal-jnd" theories, are three additive terms in the new unified theory. I also show that the new theory is capable of predicting loudness and jnd data a wide range of hearing situations, including sensorineural hearing loss, simultaneous masking, forward masking, and electric hearing.

A. Derivation of A Unified Theory

1. Derivation of a general form of Brentano's or Ekman's law

I start with the general form of a loudness function proposed by Zwillocki (1965), which can predict the loudness of tones in quiet, in simultaneous masking, and in sensorineural hearing loss.

$$L = k[(I + cI_0)^\theta - (cI_0)^\theta] \quad (1)$$

where I_0 is the threshold, c represents some internal noise level, and k is a constant. For convenience, I define loudness at threshold as: $L_o = k(cI_0)^\theta$.

Differentiating equation (1) to obtain:

$$\frac{\Delta L}{\Delta I} = \theta k(I + cI_0)^{\theta-1} = \theta k \frac{(I + cI_0)^\theta}{I + cI_0} \quad (2)$$

Adding and subtracting the same component in the above equation, I obtain:

$$\frac{\Delta L}{\Delta I} = \theta k \frac{(I + cI_0)^\theta - (cI_0)^\theta + (cI_0)^\theta}{I + cI_0} = \theta \frac{L + L_0}{I + cI_0} \quad (3)$$

Rewrite the above equation, I obtain the general form of Brentano's law or

Ekman's law, namely, $\frac{\Delta L}{L} = \frac{\Delta I}{I}$, (see Stevens (1961) for discussion of these laws):

$$\frac{\Delta L}{L + L_0} = \theta \frac{\Delta I}{I + cI_0} \quad (4)$$

Equation (4) means that Weber's law applies to both the stimulus domain and the sensation domain, except that one also needs to factor in the threshold term.

I then assume that Weber's fraction in the sensation domain is a "constant", which is inversely proportional to the number of jnds (N) within the stimulus dynamic range:

$$\frac{\Delta L}{L + L_0} = \frac{1}{N} \quad (5)$$

In other words, a small change in loudness is proportional to the overall loudness and inversely proportional to the number of jnds:

$$\Delta L = \frac{1}{N}(L + L_0) \quad (6)$$

Equation (6) turns out to be important in relating the jnd function to the loudness function, and will be shown to provide a theoretical basis for the "proportional-jnd" and the "equal-loudness, equal-jnd" theories.

2. Prediction of the jnd function from the loudness balance function

Suppose that the loudness function for a tone in quiet is: $L=f(I)$, and that the loudness balance function between the tone in quiet and the tone in masking has been obtained: $I=g(I_m)$. By definition, at $I=g(I_m)$, loudness is balanced so that the loudness function can be derived for tone in masking:

$$L_m = L = f(I) = f[g(I_m)] \quad (7)$$

Differentiating the above equation to obtain:

$$\frac{\Delta L_m}{\Delta I_m} = f'(I)g'(I_m) = \frac{\Delta L}{\Delta I} g'(I_m) \quad (8)$$

Rewrite the above equation:

$$\Delta I_m = \Delta I \frac{1}{g'(I_m)} \frac{\Delta L_m}{\Delta L} \quad (9)$$

Replace ΔL with equation (6) to obtain:

$$\Delta I_m = \Delta I \frac{1}{g'(I_m)} \frac{N}{N_m} \frac{L_m + L_{mo}}{L + L_o} \quad (10)$$

To predict the jnd in the form of Weber's fraction at the same intensity (I), one can divide the above equation by (I):

$$\frac{\Delta I_m}{I} = \frac{\Delta I}{I} \frac{1}{g'(I_m)} \frac{N}{N_m} \frac{L_m + L_{mo}}{L + L_o} \quad (11)$$

Taking a logarithmic transformation, one can calculate the jnd in terms of Weber's fraction in dB (WFdB):

$$WF_m dB = WF dB - 10 \log g'(I_m) + 10 \log \frac{N}{N_m} + 10 \log \frac{L_m + L_{mo}}{L + L_o} \quad (12)$$

I consider the above equation as a unified theory of psychophysical laws in hearing because the last three terms in the equation contain the three previous theories that attempted to relate the jnd function to the loudness function. The $10 \log g'(I_m)$ term represents Fechner's original "slope" theory; the $10 \log (N/N_m)$ term represents Riesz's "proportional-jnd" theory; and the final term represents Zwislocki's "equal-loudness, equal-jnd" theory.

B. Validation of the Unified theory

1. Prediction of the jnd functions in simultaneous masking

Simultaneous masking not only elevates a pure tone's threshold but also affects its loudness perception, similar to loudness recruitment in sensorineural hearing loss. Both loudness balance and intensity discrimination functions have been measured in the same group of listeners for pure tones in quiet and in simultaneous noise maskers (Houtsma et al., 1980; Rankovic et al., 1988; Schlauch et al., 1995).

Figure 1 shows relative contributions of the three special terms to predictions of the jnd data in simultaneous masking. Fig. 1A shows the loudness balance functions (lines) and the derived slope values (X's represent the 15 dB SPL/Hz masking and O's represent the 40 dB SPL/Hz masking condition). Fig. 1B shows loudness growth functions derived from Zwislocki's model (Eq. (1)), with X's and O's representing the loudness ratio between the corresponding masking and quiet conditions. Figure 1C shows measured jnd functions in quiet (solid line), 15-dB masking (dashed line), and 40-dB masking (dotted line). The X's and O's represent the predicted jnd values in the above two masking conditions based on Equation (12). In addition to using the slope values in 1A and loudness ratio values in 1B, Equation (12) uses a normalization factor of 4 dB and 8 dB for the 15-dB and 40-dB masking conditions, respectively. The 4 dB and 8 dB normalization factor was equivalent to the assumption that the quiet condition had 2.5 times and 6.3 times more jnd steps than the 15-dB and 40-dB masking condition, respectively. There was no free parameter in this prediction. In terms of relative contributions to the successful prediction, the slope theory (flat symbol pattern in 1A) contributed the least, while the loudness ratio theory (the same downward pattern in 1B and 1C) and the proportional jnd theory (parallel separations in 1C) did the most.

Figure 1. Predictions in simultaneous masking, with data (lines) being from Schlauch et al. (1995).

Fig. 1A shows loudness balance functions between a tone in quiet (y axis) and a tone in noise (x axis): The solid line represents the control condition where the same tone was balanced between the two ears in quiet, the dashed line represents the balance function for a tone being masked by a 15-dB SPL/Hz broadband noise, and the dotted line represents the loudness balance function for a tone with a 40-dB SPL/Hz noise. The symbols represent slope values for the balance function. Fig. 1B shows derived loudness growth functions. The symbols represent loudness ratio values between masked tones and tones in quiet. Fig. 1C shows the measured jnd functions (lines) and predicted jnd values (symbols).

2. Prediction of the jnd function in forward masking

Loudness and its jnd functions of a stimulus can also be affected forward and backward masking. Loudness is enhanced and intensity discrimination is degraded in forward and backward masking, particularly at middle intensities (Plack & Viemeister, 1992; Zeng & Turner, 1992; Zeng, Turner, & Relkin, 1991). A previous attempt to relate the “midlevel hump” (the jnd function) to loudness enhancement was not successful (Zeng, 1994).

Figure 2 shows the loudness balance function between a 25-ms tone in quiet and in the presence of a 90-dB SPL, 100-ms forward masker (2A), the derived loudness growth function (2B), and the measured as well as predicted jnd functions in quiet and masking (2C). The slope theory (2A) predicted that forward masking would produce smaller than normal jnds for standard levels below 50 dB SPL but larger jnds for levels above 50 dB SPL. The “equal-loudness, equal-jnd” theory (2B) predicted the midlevel hump jnd function due to enhanced loudness in forward masking. A 7-dB normalization factor, or five times less jnd steps in forward masking, was used in the final successful prediction (2C) that combined all three special theories in Equation (12). It would be interesting to know if the present unified theory could predict a similar jnd function observed for clicks under notched noise conditions (Carlyon & Moore, 1984).

Figure 2. Predictions in forward masking, with data (lines) from Zeng (1994). Fig. 2A shows loudness balance functions between a tone in quiet (y axis) and a tone in forward masking (x axis): The solid line represents the control condition where the same tone was balanced between the two ears in quiet, while the dashed line represents the balance function for a tone in forward masking. The * symbols represent slope values for the balance function. Fig. 2B shows derived loudness growth functions. The symbols represent loudness ratio values between the masked tone and the tone in quiet. Fig. 2C shows the measured jnd functions (lines) and predicted jnd values (symbols).

3. Predictions of the jnd functions in electric hearing

In electric hearing where hair cells are missing and the auditory nerve fibers are directly stimulated by electric currents, loudness generally has a narrow dynamic range of 10-20 dB (Zeng & Galvin, 1999). Zeng and Shannon (1994) found that, in cochlear implant users, loudness grows as a traditional power function of electric current for stimulus frequencies lower than 300 Hz, but as an exponential function for stimulus frequencies higher than 300 Hz. These two different loudness growth functions would produce a logarithmic loudness balance function between low- and high-frequency electric stimuli. Figure 3A shows, indeed, such a logarithmic balance function (solid lines) between a 100-Hz stimulus (sinusoid or pulse amplitude on y-axis) and a 1000-Hz sinusoid (x-axis).

$$E_{1000\text{Hz}} = \theta \log E_{100\text{Hz}} \quad (13)$$

where θ is the slope of the logarithmic loudness balance function. Differentiating the above question to derive the following JND function between the high- and low-frequency electric stimuli:

$$\Delta E_{1000\text{Hz}} = \theta \frac{\Delta E_{100\text{Hz}}}{E_{100\text{Hz}}} \quad (14)$$

Zeng and Shannon (1999) measured jnds of these stimuli in the same implant subjects (symbols in Fig. 3B) and found that not only did this jnd function hold but more importantly the jnd function was nearly constant (the solid line in Fig. 3B). Given the same power loudness growth function for the 100-Hz electric stimuli, it is not surprising that their Weber's fraction was also constant. But why was the absolute difference ($\Delta E_{1000\text{Hz}}$) constant for the 1000-Hz stimulus? Zeng and Shannon (1999) showed that this constant absolute difference was a result of the exponential loudness growth function.

$$L_{1000\text{Hz}} = \exp(E_{1000\text{Hz}}) \quad (15)$$

Differentiating the above equation to obtain:

$$\frac{\Delta L_{1000\text{Hz}}}{\Delta E_{1000\text{Hz}}} = \exp(E_{1000\text{Hz}}) = L_{1000\text{Hz}} \quad (16)$$

Rewriting the above equation to obtain:

$$\frac{\Delta L_{1000\text{Hz}}}{L_{1000\text{Hz}}} = \Delta E_{1000\text{Hz}} \quad (17)$$

Equation (17) means that Brentano's ratio is also constant in electric stimulation. The only difference between Equation (17) and Equation (4) is that (17) does not contain a threshold term, probably due to a lack of spontaneous neural activity in the deafened ear (Kiang & Moxon, 1972).

Figure 3. Loudness balance (A) and JND functions (B) in cochlear implant users. A. Loudness balance functions were obtained between 100-Hz sine or 100-Hz pulse and 1000-Hz sine electric stimuli, adapted from Fig. 2D and 2E in Zeng and Shannon (1994). Symbols represent individual data and the solid line represents a logarithmic balance function. The dashed line represents a linear balance function, which clearly was not the true. B. JND data (symbols) and predicted functions (lines) using the same stimuli from the same subjects in A, adapted from Fig. 4 in Zeng and Shannon (1999).

C. Discussion

None of the individual components in the present unified theory are new. Previously studies have proposed these individual theories and actually evaluated them separately (e.g., Allen & Neely, 1997; W. S. Hellman & Hellman, 1990; Schlauch, 1994; Schlauch et al., 1995; J. J. Zwislowski & Jordan, 1986). The present study is novel in two respects. First, the present study integrates the previously disconnected individual components through a unified theoretical framework, namely, the general form of Brentano's law in Equation (4). Second, the present study offers a new formula, namely, Equation (12), which specifically combines these individual terms to successfully predict the loudness and jnd relationships in simultaneous and forward masking, as well as in cochlear implant users. The present unified theory and its successful applications suggest that although Fechner's law needs to be replaced by the general form of Brentano's law, Fechner's original idea using jnds to derive psychophysical laws is valid at least in the wide range of hearing situations examined here.

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