[knowledge base]

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Every group is isomorphic to a group of permutations

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Abstract

CAYLEY'S THEOREM, the SYMMETRIC GROUP THEOREM, and their underlying definitions are presented in this white paper (knowledge base = <http://omkb.org>).

keywords: Cayley's theorem, symmetric group, isomorphism, abstract algebra, knowledge base

The most updated version of this white paper is available at

<https://osf.io/63pmy/download>

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Cayley's Theorem

1.

$$(G, \star) \cong (P, \circ_\alpha)$$

$(G, \star) :=$ group

$(P, \circ_\alpha) :=$ permutation group

$\cong$ isomorphism

$\star, \circ_\alpha :=$ binary operations

$\alpha : P \rightarrow P$ (permutation $:=$ bijective function)

$\circ_\alpha :=$ composition of permutations

[1, 2]

Symmetric Group on A

2.

$$(S(A), \circ)$$

$S(A) = \{f \mid f : A \cong A\} :=$ set of permutations of A

$A :=$ nonempty set

$f : A \cong A$ (bijective function)

$\circ :=$ operation of composition

[1, 2]

Prerequisites

3. Ordered pair

$$(a, b) = \{\{a\}, \{a, b\}\}$$

a := first coordinate

b := second coordinate

[1, 3]

4. Cartesian product

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

A, B := sets

$A \times B$:= Cartesian product

(a, b) := ordered pair

[3]

5. Binary operation

$$\star : S \times S \rightarrow S$$

S := set

$S \times S$:= Cartesian product

[1]

6. Group

$$(G, \star)$$

(a) Associativity: $\forall x, y, z \in G, (x \star y) \star z = x \star (y \star z)$

(b) Identity: $\exists e \in G : \forall x \in G, e \star x = x \star e = x$

(c) Inverse: $\forall x \in G \exists y \in G : x \star y = y \star x = e$

$G := \text{set}$

$\star := \text{binary operation}$

[1]

7. **Relation** from A to B

$$R \subseteq A \times B$$

$R, A, B := \text{sets}$

$x \in A, y \in B, \quad xRy \equiv (x, y) \in R$

[3]

8. **Function** from A to B

$$f : A \rightarrow B$$

$$\forall a \in A \quad \exists! b \in B \quad ((a, b) \in f)$$

$f, A, B := \text{sets}$

$(a, b) := \text{ordered pair}$

[3]

9. **One-to-one function (injection)** $f : A \rightarrow B$

$$\neg \exists a_1 \in A \quad \exists a_2 \in A \quad (f(a_1) = f(a_2) \wedge a_1 \neq a_2)$$

[3]

10. **Onto function (surjection)** $f : A \rightarrow B$

$$\forall b \in B \quad \exists a \in A \quad (f(a) = b)$$

[3]

11. **Bijjective function** := one-to-one + onto

[3]

12. **Permutation of A** := bijection from A to itself

[1, 2]

13. **Homomorphism**

f^h

(a) $f^h : G \rightarrow H$

$\forall x, y \in G : f^h(x * y) = f^h(x) \circ f^h(y)$

f^h := function

G, H := sets

$*, \circ$:= binary operations

$(G, *), (H, \circ)$:= groups

[1, 4, 5]

14. **Isomorphism** := bijective homomorphism

[1, 4, 5]

Open Invitation

*Review, add content, and **co-author** this white *white paper* [6, 7].*

*Join the **Open Mathematics Collaboration**.*

Send your contribution to `mplobo@uft.edu.br`.

Open Science

The **latex file** for this *white paper* together with other *supplementary files* are available in [8].

Ethical conduct of research

This original work was pre-registered under the OSF Preprints [9], please cite it accordingly [10]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

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<https://cos.io>

+ **Open Science Framework**

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