BV

[original insight]
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A pedagogical approach to convert left into right-hand limit

Open Mathematics Collaboration*†

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Abstract

The title of this microarticle is self explanatory. One example is included.

keywords: left-hand limit, right-hand limit, calculus

Introduction

1. In some cases, converting one left-hand into two right-hand limits turns out to be handy.

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The transformation

2. Consider the function $f : I \to \mathbb{R}$, where I is an open interval, and let $a \in I$.

3. Suppose that f is continuous, except possibly in a.

4. Thus, if the left-hand and the right-hand limits exist, we have

$$\lim_{x \to a^-} f(x) = \lim_{\epsilon \to 0^+} \left(\lim_{x \to a^+} f(x - \epsilon) \right) = \lim_{\epsilon \to 0^+} f(a - \epsilon)$$

and

$$\lim_{x \to a^+} f(x) = \lim_{\epsilon \to 0^+} \left(\lim_{x \to a^+} f(x + \epsilon) \right) = \lim_{\epsilon \to 0^+} f(a + \epsilon).$$

Example

5. Let $f(x) = \frac{|\sin x|}{\sin x}$.

6.

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{|\sin x|}{\sin x} = \lim_{\epsilon \to 0^+} \left(\lim_{x \to 0^+} \frac{|\sin (x - \epsilon)|}{\sin (x - \epsilon)} \right)$$

7.

$$\lim_{x \to 0^-} f(x) = \lim_{\epsilon \to 0^+} \frac{|\sin (-\epsilon)|}{\sin (-\epsilon)} = \lim_{\epsilon \to 0^+} \frac{|-\sin (\epsilon)|}{-\sin (\epsilon)}$$

8.

$$\lim_{x \to 0^-} f(x) = \lim_{\epsilon \to 0^+} \frac{|\sin (\epsilon)|}{-\sin (\epsilon)} = -\frac{\sin (\epsilon)}{\sin \epsilon} = -1$$

Final Remarks

9. You can try out more examples from a Calculus textbook [1].

Open Invitation

Review, add content, and **co-author** this article. *Join* the **Open Collaboration**. Send your contribution to mplobo@uft.edu.br.

Ethical conduct of research

This original work was pre-registered under the OSF Preprints [2], please cite it accordingly [3]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

References

[1] Stewart, James. *Calculus: Concepts and contexts.* Cengage Learning, 2009.

[2] COS. *Open Science Framework.* https://osf.io

[3] Lobo, Matheus P. "A Pedagogical Approach to Convert Left into Right-hand Limit." *OSF Preprints*, 28 Oct. 2019. https://doi.org/10.31219/osf.io/5vfde

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