HARDNESS OF MINE SWEEPER PROBLEM

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1. DEFINITION AND CLAIM

Definition 1. MINE SWEEPER Problem:

Given an undirected graph where some vertices have a natural number associated with them, decide whether there is a way to mark some of the non-numbered vertices such that the number in each numbered vertex is equal to the amount of its marked neighbours.

We reduce DOMINATING SET problem to ours. DOMINATING SET is shown to be hard by [1].

Claim 2. *We have that* DOMINATING SET $\leq_p$ MINE SWEEPER

2. REDUCING DOMINATING SET TO MINE SWEEPER

In this section, we prove the claim 2.

Proof. Describing the construction: Given an instance (G, k) of DOMINATING SET, we construct our instance of MINE SWEEPER as follows.

- Add a new vertex s , connect s with all the vertices of $V(G)$. Label s with k .
- For each $v \in V(G)$, add a new vertex s_v . Connect s_v to all the vertices of $N(v) \cup \{v\}$ (the neighborhood of v and v itself). Connect s_v to $|N(v)| - 1$ new dummy vertices. Label s_v with $|N(v)|$.

Call the constructed graph G' . The produced MINE SWEEPER instance is G' itself.

Correctness of the construction: We claim that (G, k) is a **YES** instance of DOMINATING SET iff. G' is a **YES** instance of MINE SWEEPER.

If (G, k) is a **YES** instance of DOMINATING SET, we have a dominating set $S \subseteq V(G)$ with $|S| = k$. By simply marking all the vertices of S , we satisfy the label of s in G' . Then, for each $s_v \in V(G')$, according to the number of marked vertices (up to now) in $N(v) \cup \{v\}$, we mark some of the dummy vertices connected with s_v . Note that, since S is dominating, $N(v) \cup \{v\} \cap S \neq \emptyset$, so at most $|N(v)| - 1$ dummy vertices needs to be marked to satisfy the label of s_v , so we always have enough number of dummy vertices connected to s_v .

Conversely, if G' is a **YES** instance of MINE SWEEPER, we denote by S the non-dummy vertices (which also means that these are in $V(G)$) in the solution to this instance. We now show that S is a dominating set of G and $|S| = k$, which concludes our proof. Indeed, to satisfy the label of s , we have that $|S| = k$. For each vertex s_v , at most $|N(v)| - 1$ dummy vertices connected to it are marked. So in

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order to satisfy its label, namely $|N(v)|$, either v or some vertex in $N(v)$ is marked. So, in particular, if v is not marked, i.e. $v \notin S$, there must be some marked vertex in $N(v)$, i.e. $N(v) \cap S \neq \emptyset$. This suffices to prove that S is a dominating set of G . $\square$

3. CONCLUSION

Garey and Johnson [1] shape their theory based on previous primal works of Cook, Levin and Karp. Johnson [2] moved on with the guide to this theory. As long as we study a mathematical conjecture, we should encourage ourselves to have enough labour hours on popular mathematics books like these. Then, reading some articles on theory of computing like [3] is a good practice. Only after that, could we think of the ultimate final for all mathematical sciences.

REFERENCES

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