Problems on Robin Inequality and Riemann Hypothesis

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Abstract

In this piece of work we explore the sum of divisors functions as coefficient in a series of real (complex) numbers. We provide a brief conclusion and evoke the connection to Robin inequality which is equivalent to Riemann Hypothesis.

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Robin Inequality (See [1] and [2]) *Riemann Hypothesis is equivalent to*

$$\sigma(n) < ne^{\gamma} \ln \ln n, \quad n > 5040, \quad \gamma = 0.57721...$$

$\sigma(n)$: *The sum of divisors function*

The one can readily deduce the following equation

$$\sum_{n=1}^{\infty} \frac{z^n}{(1-z^n)^2} = \sum_{n=1}^{\infty} \sigma(n)z^n \quad (1)$$

Where z real (complex) number, The LHS converges everywhere except at $z = 1$

Problem I:

Write

$$\sum_{n=1}^{\infty} \frac{z^n}{(1-z^n)^2} = \sum_{n=1}^{\infty} f(n)z^n$$

Find ϵ_n such that

$$|\sigma(n) - f(n)| < \epsilon_n, \quad n = 5041, 5042, \dots$$

Then prove that:

$$\sigma(n) < f(n) + \epsilon_n < e^\gamma n \ln \ln n, \quad n = 5041, 5042, \dots$$

Or

$$\sigma(n) < f(n) + \epsilon < e^\gamma n \ln \ln n, \quad \epsilon = \min\{\epsilon_n, \quad n = 5041, 5042, \dots\}$$

□

Problem II:

From (1) and for a fixed $N > 5041$

$$\sigma(N) = \sum_{n=1}^{\infty} \frac{z^{n-N}}{(1-z^n)^2} - \sum_{\substack{n=1 \\ n \neq N}}^{\infty} \sigma(n)z^{n-N}$$

Show that:

$$\sum_{n=1}^{\infty} \frac{z^{n-N}}{(1-z^n)^2} - \sum_{\substack{n=1 \\ n \neq N}}^{\infty} \sigma(n)z^{n-N} < e^\gamma N \ln \ln N$$

□

References

- [1] Salah, Jamal Y. "On Riemann hypothesis and Robin's inequality." *International Journal of Scientific and Innovative Mathematical Research (IJSIMR)* 3.4 (2015): 9-14.
- [2] Young-Ju Choie, Nicolas Lichiardopol, Pieter Moree and Patrick Solé, On Robin's criterion for Riemann Hypothesis, *Journal de Théor. Nombres. Bordeaux* 19 (2007), no 2, 357 - 372.