



[original idea]

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Complex complex numbers

Open Mathematics Collaboration^{*†}

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Abstract

We apply in the complex numbers the same line of thought that led to the very creation of the complex themselves. In addition, we consider multiple imaginary numbers and generalize both ideas altogether.

keywords: number theory, complex numbers, gamma numbers, multiple imaginary numbers

The most updated version of this paper is available at

<https://osf.io/485kj/download>

Notation

1. indexes = $i, j, k, \ell \in \mathbb{N} = \{1, 2, 3, \dots\}$

Real

1. $\nexists x \in \mathbb{R} : x^2 = -1$

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Complex

2. $z = a + bi; \quad a, b \in \mathbb{R}; \quad z, i \in \mathbb{C}; \quad i^2 = -1$

3. $\exists z \in \mathbb{C} : z^2 = -1$

4. $\bar{z} = a - bi$

5. $\nexists z \in \mathbb{C} : z\bar{z} = -1$

Open questions

6. $\exists \xi \in \mathbb{CC} : \xi\bar{\xi} = -1?$

7. $\mathbb{CC} = ?$

8. $\mathbb{R} \cap \mathbb{CC} = ?$

9. $\mathbb{C} \cap \mathbb{CC} = ?$

10. $\gamma = ?$

11. $\gamma_1\gamma_2 = ?$

12. $\gamma_1 + \gamma_2 = ?$

13. $\bar{\gamma} = ?$

14. $\gamma\bar{\gamma} = ?$

15. $\xi = \text{ring?}$

16. $\xi = \text{commutativity?}$

17. $\xi = \text{associativity?}$

18. How is \mathbb{CC} *inserted* into the **fundamental theorem of algebra**?

Beyond complex?

- 19. Let $\xi = a + bi + c\gamma \in \mathbb{CC}$.
- 20. $a, b, c \in \mathbb{R}$
- 21. $i \in \mathbb{C}$
- 22. $\gamma \in \mathbb{CC}$ such that $\xi\bar{\xi} = -1$.
- 23. $\bar{\xi} = a - bi + c\bar{\gamma}$
- 24. $\xi\bar{\xi} = a^2 + b^2 + c(a + bi)\bar{\gamma} + c(a - bi)\gamma + c^2\gamma\bar{\gamma}$
- 25. Let: $d = a^2 + b^2$, $z_1 = c(a + bi)$.
- 26. $\xi\bar{\xi} = d + z_1\bar{\gamma} + \bar{z}_1\gamma + c^2\gamma\bar{\gamma}$
- 27. Suppose $\xi\bar{\xi} = -1$.
- 28. $z_1\bar{\gamma} + \bar{z}_1\gamma + c^2\gamma\bar{\gamma} = -1 - d$
- 29. Dividing both sides by c^2 ,

$$z\bar{\gamma} + \bar{z}\gamma + \gamma\bar{\gamma} = -\frac{(1+d)}{c^2},$$

where $z = z_1/c^2$.

- 30. Let $e = -\frac{(1+d)}{c^2}$.
 - 31. Since $c^2, d > 0$, then
- $$z\bar{\gamma} + \bar{z}\gamma + \gamma\bar{\gamma} = e < 0.$$

Special case

- 32. $1 + d = c^2$
- 33. $c^2 = a^2 + b^2 + 1$

34. $e = -1$

35. $\gamma\bar{\gamma} = -1$

Brainstorming

36. $(\xi_i = z_i + c_i\gamma) \underline{\vee} (\xi_i = z_i + c_i\gamma_i)?$

37. $(c_i \in \mathbb{R}) \vee (c_i \in \mathbb{C}) \vee (c_i \in \mathbb{CC})?$

38. $(\gamma\bar{\gamma} = -1) \vee (\gamma\bar{\gamma} < 0)?$

39. From (36) and (37), $\xi_i = z_i + z'_i\gamma_j?$

Multiple imaginary numbers

40. This section presents an alternative idea for extending the complex numbers, not necessarily involving ξ .

41. Let $z = a + b\iota_i$; $a, b \in \mathbb{R}$; $z, \iota_i \in \mathbb{C}$; $\forall i, j : \iota_i\iota_j < 0$.

42. $\iota_1 = i$, so that $\iota_1^2 = i^2 = -1$.

Complex complex numbers with multiple imaginary numbers

43. $\xi_i = z_i + z'_i\gamma_j$

44. $z_i = a_i + b_i\iota_k$

45. $\iota_k\iota_\ell < 0$

46. $a, b \in \mathbb{R}$

47. $z_i, \iota_k \in \mathbb{C}$

48. $\xi_i, \gamma_j \in \mathbb{CC}$

Open Invitation

Review, add content, and **co-author** this paper [1, 2].

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Open Science

The **latex file** for this paper together with other *supplementary files* are available [3].

Ethical conduct of research

This original work was pre-registered under the OSF Preprints [4], please cite it accordingly [5]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

References

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