The spatial structure debate in spatial interaction modeling: 50 years on

Taylor M. Oshan¹

Abstract

Spatial interaction and spatial structure are foundational geographical abstractions, though there is often variation in how they are conceptualized and deployed in quantitative models. In particular, the last five decades have produced an exceptional diversity regarding the role of spatial structure within spatial interaction models. This is explored by outlining the initiation and development of the notion of spatial structure within spatial interaction modeling and critically reviewing four methodological approaches that emerged from ongoing debate about the topic. The outcome is a comprehensive coverage of the past and a sketch of one potential path forward for advancing this longstanding inquiry.

Keywords

spatial analysis, spatial interaction, spatial structure, distance-decay, misspecification

Introduction

Spatial interaction (SI) typically refers to the aggregate flows of people, information, or goods across space as they move between a set of locations (Batty 2008; O’Kelly 2015; Wang 2017; Fotheringham 2017). As such, quantitative models of SI provide a mechanism to understand and predict components of SI systems and are typically constructed upon the hypothesis that flow volumes are a function of the potential at origins, the attractiveness of destinations, and the cost of overcoming the separation between origins and destinations. The simplicity and generalizability of this framework has motivated the use of SI models in diverse settings, and consequently, such tools have consistently remained of interest to geographers and allied disciplines (Ullman 1980; Haynes and Fotheringham 1984; Fotheringham and O’Kelly 1989; Sen and Smith 1995; Roy 2004; Batty 2013; Patuelli and Arbia 2016). Another important geographical concept is that of spatial structure, which characterizes the organization, distribution, or relative arrangement of entities embedded within a spatial system (Haggett 1965; Cliff et al. 1975a; Bennett and Haining 1985; Lloyd 2014). Although spatial structure and SI are foundational abstractions, the manner in which

they are theoretically conceived and practically deployed in quantitative models is highly varied. In particular, the assertion is put forth and examined here that there is exceptional diversity regarding the role of spatial structure in SI modeling. This is achieved by outlining the initiation and development of the notion of spatial structure within SI modeling and characterizing a portion of the current landscape on the topic that pertains to cross-sectional analyses.

The investigation of spatial structure in SI models appears to have been launched by Curry’s *A spatial analysis of gravity flows* (1972). In response, an active debate ensued over the following decade about the role of the spatial structure of locations in SI systems and models. However, the inquiry eventually fractured and the SI modeling framework evolved into several different methodological directions, each with unique analytical traditions and underlying

¹Center for Geospatial Information Science

Corresponding author:

Taylor M. Oshan, University of Maryland Center for Geospatial Information Science, Department of Geographical Sciences, College Park, Maryland, 20740, USA.

Email: toshan@umd.edu

assumptions. At the same time, advances in the ability to collect, store, and disseminate data has produced many novel flow datasets, often with increasingly finer spatial and temporal resolutions (Arribas-Bel 2014; Lovelace et al. 2015; Manley and Dennett 2019). Though these new data provide unparalleled opportunities to increase our understanding of SI processes, the realization of these opportunities is at least partially governed by the methodological framework chosen to perform an analysis. More specifically, the stance adopted here is that how spatial structure is conceptualized and operationalized in SI models has implications for how we build knowledge about SI phenomena.

To develop this view, traditional SI models are first introduced that do not explicitly consider spatial structure in their formulation. Then, a summary of the main concepts and ideas are provided by tracing the investigation of spatial structure in SI modeling in approximate chronological order. Concurrently, several proposed approaches for incorporating spatial structure in SI models are critically reviewed, their contribution to the SI knowledge-building-process are assessed, and their limitations are highlighted. Finally, additional themes and comparisons of approaches are discussed and suggestions are made for moving the SI research agenda forward before ending with some concluding remarks.

Gravity-type spatial interaction models

Early SI models were motivated through analogies to the physical law of gravity where the number of flows between two locations is given by the product of the populations of the origin and destination divided by the distance between them (Carrothers 1956; Hua and Porell 1979; Farmer and Oshan 2017). The formulation was subsequently generalized to,

$$T_{ij} = k \frac{V_i^\mu W_j^\alpha}{d_{ij}^\beta} \quad (1)$$

where T represents an $n \times m$ matrix of flows between n origins (subscripted by i) to m destinations (subscripted by j), V and W are $n \times p$ and $m \times p$ vectors of p origin and destination attributes, respectively, d is an $n \times m$ matrix of the costs to overcome the physical separation between i and j (usually distance or time), k is a scaling factor that ensures the model reproduces

the total number of flows present in the data, and μ , α , and β each represent a vector of exponential parameters. When data for T , V , W , and d are available, it is possible to calibrate or estimate the parameters, which summarize the effect that each model component contributes towards explaining the system of known flows (T). Various flow forecasts are also possible by tweaking different model components or parameters (Fotheringham and O'Kelly 1989).

Derived through entropy maximization, Wilson's 'family' of gravity-type SI models provided further generalization and formalization (Wilson 1967, 1971). Depending upon the information incorporated about the total inflows and outflows (i.e., constraints), the following members of the family are,

Unconstrained

$$T_{ij} = k V_i^\mu W_j^\alpha f(d_{ij}) \quad (2)$$

Production-constrained

$$T_{ij} = A_i O_i W_j^\alpha f(d_{ij}) \quad (3a)$$

$$A_i = \left(\sum_j W_j^\alpha f(d_{ij}) \right)^{-1} \quad (3b)$$

Attraction-constrained

$$T_{ij} = B_j V_i^\mu D_j f(d_{ij}) \quad (4a)$$

$$B_j = \left(\sum_i V_i^\mu f(d_{ij}) \right)^{-1} \quad (4b)$$

Doubly-constrained

$$T_{ij} = A_i B_j O_i D_j f(d_{ij}) \quad (5a)$$

$$A_i = \left(\sum_j B_j D_j f(d_{ij}) \right)^{-1} \quad (5b)$$

$$B_j = \left(\sum_i A_i O_i f(d_{ij}) \right)^{-1} \quad (5c)$$

where O_i and D_j are the total number of flows emanating from an origin or terminating at a destination, A_i and B_j are balancing factors that replace k and ensure that O_i and D_j are preserved in addition to the total system flows,

and d_{ij} takes on a functional form, referred to as the distance-decay function. The distance-decay function is most commonly either a power function or an exponential power function, and β is expected to take on a negative value that indicates the decaying nature on the effects of increasing physical separation on the propensity for flows to occur. The exponential distance-decay function is the natural result of the maximum-entropy derivation while the power distance-decay function is justified when the analyst believes there is a logarithmic evaluation of transport costs and is also the specification that arises from the original physical analogy. Furthermore, these two forms have also gained acceptance through empirical consensus where the exponential function is often more appropriate for shorter interactions such as intra-urban trips and the power function is more appropriate for longer distance trips such as migrations flows (Fotheringham and O'Kelly 1989).

The unconstrained model (or total-trip constrained model) given in (2), is equivalent to equation 1 and does not conserve the total inflows or outflows. The production-constrained and attraction-constrained models are given in (3a) and (4a). They conserve either the number of total outflows or inflows at each location and are therefore useful for building models that allocate individuals either from a set of origins or to a set of destinations. Finally, the doubly-constrained model is given in (5a), which conserves both the inflows and the outflows at each location during model calibration. These models may also be derived using the economic behavioral framework of utility theory (McFadden 1974; McFadden 1977) and have been linked to discrete choice models (Anas 1983; Fotheringham 1986). An exposition of the family of SI models, their applications, and various extensions is provided by Wilson (2010a).

Though non-linear optimization was initially the preferred calibration technique for the family of models, they are frequently linearized and included within an ordinary least squares regression specification by taking the logarithm of both sides of a given model. For the unconstrained model*, this yields the so-called log-linear or log-normal gravity model,

$$\ln T_{ij} = k + \mu \ln V_i + \alpha \ln W_j - \beta \ln d_{ij} + \epsilon \quad (6)$$

where β is still expected to take on a negative value and ϵ is a random normal error term. A Poisson log-linear regression calibration framework is also popular because it more realistically models discrete entities (i.e., flows of people), avoids the issue of zero flows that cannot be logged, and is not subject to transformation bias for the dependent variable (Flowerdew and Aitkin 1982; Flowerdew and Lovett 1988). Using binary indicator variables for each origin and/or destination within Poisson regression yields the constrained variants of the family of SI models, producing results equivalent to those obtained using non-linear optimization (Tiefelsdorf and Boots 1995).

A popular extension to the family of models involves calibrating separate models for subsets of flows from each origin to all destinations, which is also known as an origin-specific local model. A set of parameter estimates for each model term is then obtained for each origin and is mappable in order to explore potential spatial variation (Oshan 2016). The importance of origin-specific SI models in the discovery and understanding of spatial structure effects is noted below.

The spatial structure debate

Throughout the 1970's and 1980's there was much debate about the role of the structure of a spatial system on the results from SI models. The intense interest in the matter can be best captured in a series of publications, associated comments and replies, and subsequent reviews (Curry 1972; Johnston 1973; Cliff et al. 1974; Curry et al. 1975; Cliff et al. 1975b; Johnston 1975; Cliff et al. 1976; Griffith 1976; Sheppard et al. 1976; Sayer 1977; Fotheringham and Webber 1980; Griffith and Jones 1980, Fotheringham 1981, Sheppard 1984). A primary concern surfaced that somehow an unidentified aspect of the spatial system was influencing distance-decay parameter estimates and producing *unintuitive* patterns in the local distance-decay parameter estimates from origin-specific SI models (i.e., relatively weak

*The unconstrained model is used as an example throughout this manuscript because it provides the most convenient linkage between the various extensions presented in subsequent sections. Similarly, this manuscript focuses on regression-based estimation techniques because they are often at the heart of the debate and they provide a consistent framework to discuss all of the models described here.

negative or positive distance-decay). Since these unintuitive patterns manifested in a spatial manner, the unidentified effect became known as the spatial structure effect[†]. Therefore, as evidence towards a spatial structure effect accrued, a consensus formed that whatever was causing the unlikely patterns in the estimated distance-decay parameters could be skewing their behavioral interpretation. Over the next few decades, several theories pertaining to the spatial structure effect were proposed, but the SI corpus remains far from a consensus in terms of what causes spatial structure effects and how to best account for them.

Though the debate initially included a variety of culprits[‡] that could be contaminating distance-decay parameters, two primary theories emerged. The first was that of Curry, Griffith, and Sheppard (CGS), who argued that it was spatial autocorrelation amongst the locational attributes that causes the spatial structure effect (Curry 1972; Curry et al. 1975; Sheppard et al. 1976). However, both Cliff et al. (1974; 1975b; 1976), and Fotheringham (1981) discuss how the concept of spatial autocorrelation amongst locational attributes is an intermediary for the collinearity between explanatory variables and is an unnecessary concept in this context because it does not provide any explanation for the unintuitive patterns in the distance-decay parameter estimates. Second, Johnston (1973, 1976) put forth the hypothesis that the spatial pattern observed in local distance-decay parameters estimates was caused by the variation in the distributions of distances between each origin and the set of destinations, which would later be described as the *conditional distance distribution* (Tiefelsdorf 2003). Johnston tested the conditional distance distribution hypothesis using a controlled simulation, but the experiment was not entirely convincing because the data were generated assuming that all of the flows between each origin and destination are constant, implying no relationship between the number of flows and distance (Sheppard 1979; Fotheringham 1981). Therefore, neither the theory of CGS nor Johnston sufficiently exposed the spatial structure effect or how to address it.

A second wave of the debate witnessed several additional approaches to further explore the spatial structure effect. Griffith and Jones (1980)

doubled-down on the idea of spatial autocorrelation in locational attributes by exploring the correlations between different components of doubly-constrained SI models across census tracts for several individual cities. The main goal was to answer the question, ‘is the rate of distance decay in SI models independent of the spatial structure associated with the corresponding origins and destinations?’ Through a series of loosely connected experiments it was surmised that an ‘indirect relationship exists between distance-decay and the geometric pattern, with this relationship being reasonably sensitive to changes in the geometry of destinations’. And even though little-to-no spatial autocorrelation was found in the locational attributes for each cities’ system of locations, it was concluded that ‘there exists a fundamental geometric dimension relating to the geographic distribution of workers/jobs’, without explicitly defining the nature of bias in the distance-decay parameter estimates. It also appears that Griffith and Jones identified but were unable to untangle three distinct issues: (i) spatial autocorrelation of the locational attributes; (ii) spatial autocorrelation amongst the migration flows themselves; and (iii) the uncertainty that arises from modeling individual trips as aggregate flows between areal units.

In contrast to Griffith and Jones (1980), the approach of Fotheringham and Webber (1980) proposed to account for spatial structure by explicitly modeling the interdependence between SI and locational attributes. For example, in a study of migration between urban centers, to accurately obtain parameters for the effect of urban centers’ size on SI, there also needs to be a sufficient model for growth of urban centers as well. Since growth is theorized to depend upon SI, these two things should be modeled with a simultaneous equation system (SES) that is calibrated using two-stage least squares. In general, parameter estimates will be biased and

[†]Before consensus formed around the terminology of spatial structure, many terms were used to describe the unidentified effect, including map pattern, locational structure, interdependence, and spatial autocorrelation. Furthermore, these terms would sometimes be used to refer to empirical data and other times the resulting model parameter estimates, which may have caused confusion and intensified the debate.

[‡]Early accounts of the unidentified spatial structure effect, such as (Curry 1972), included a plethora of potential factors, such as the shape of the study area, aggregation and representation bias, spatial autocorrelation in the locational attributes, model estimation techniques, and interdependence between SI and locational distributions, though no specific links were made.

inconsistent if any of the independent variables are a function of the dependent variable, which is a type of endogeneity misspecification. Though [Fotheringham and Webber \(1980\)](#) derived the exact nature of this misspecification, it was only for the unconstrained SI model and contained the restrictive assumptions that the system is highly sensitive to the proposed feedback mechanism, the interdependence is instantaneous and the system is in some steady-state equilibrium.

After reviewing much of the existing literature on origin-specific SI models and spatial structure, [Fotheringham \(1981\)](#) brought an additional perspective to the debate by suggesting that the presence of spatial structure effects should be diagnosed by directly analyzing the unintuitive patterns in the local distance-decay parameter estimates. By doing so, it was noticed that the patterns in the parameters seem to be driven by each location's accessibility to the other locations in the system. It was then deduced that spatial autocorrelation in the locational attributes specifically served as a surrogate for multicollinearity between accessibility and distance. A discourse between [Sheppard \(1982\)](#) and [Fotheringham \(1982\)](#) refined this argument by noting that spatial autocorrelation in the residuals can indicate that there is one of several types of misspecification that can bias parameter estimates (e.g., omitted variables or incorrect function form); however, observing spatial autocorrelation in the independent variables is not sufficient to diagnose such misspecification(s). Furthermore, the concept of spatial autocorrelation was now unnecessary in this context: the spatial structure effect described by [Fotheringham, \(1981\)](#) had been identified as a misspecification arising due to the omission of a model term that captures location accessibility. It was demonstrated that the inclusion of a variable that measures each destination's accessibility to all other destinations can greatly reduce the spatial variation in local distance-decay parameter estimates and remove unintuitive patterns, and was therefore dubbed the Competing Destinations (CD) model ([Fotheringham 1983](#)).

Over the next few decades, the CD framework was expanded, reinforced, applied, and critiqued. In parallel, several additional methods to account for spatial structure were also proposed. The next section elaborates upon each methodology.

Accounting for spatial structure

In the wake of the debate, four generalizable approaches were proposed to account for the spatial structure effect: the CD model, the Box-Cox transform, spatial econometric models, and eigenvector spatial filtering. As each approach is introduced and discussed, three characteristics are highlighted to inform a contemporary research agenda. First, the misspecification a method is meant to correct in order to remove the spatial structure effect is identified. Since multiple occurrences of misspecification are possible, it is important to clarify those being addressed by each method. Second, connections are drawn from each approach to the spatial structure debate summarized hitherto and henceforth to follow the progression of theorization and conceptualization within the SI modeling paradigm. Finally, some limitations of each method for investigating SI processes are evaluated.

The Competing Destinations model

The core tenets of the CD model are that: (i) the relative configurations of destinations with respect to each other can affect an individual's propensity to select a particular destination; and (ii) including a destination accessibility term in SI models can capture this factor and mitigate the observed spatial structure effect ([Fotheringham 1983](#)). Accessibility is often defined based on a combination of physical locations and attribute values. While spatial systems typically do have attribute spatial variation that may contribute to the spatial structure effect, it is ultimately driven by relative spatial organization and how individuals perceive the overall system. For example, in the event that attributes are uniform or random across observations, the spatial structure effect would be due solely to the configuration of locations in space.

The behavioral foundation of the CD model is that spatial decisions, such as location choice, often arise from a hierarchical two-stage or multi-stage decision-making process as a spatial information processing strategy when there are many choices to evaluate ([Hirtle and Jonides 1985](#); [McNamara 1986](#); [Walker et al. 1990](#); [Fotheringham and Curtis 1992](#); [Curtis and Fotheringham 1995](#); [Fotheringham and Curtis 1999](#)). Individuals first select one or more clusters

of locations and then subsequently choose an individual destination from within these clusters. The effect is that as the accessibility of a particular destination, j , to all other potential destinations increases, j will experience an increase or decrease in the number of flows than it would otherwise expect if it existed independently (i.e., not a member of any clusters) (Fotheringham and O'Kelly 1989).

Destination accessibility can be defined as A_{ij} in equation 8 and added to the SI models introduced here, where A_{ij} is thought of as the likelihood that other destinations are also considered along with destination j . For an unconstrained log-linear SI model, for example, this results in:

$$\ln T_{ij} = k + \mu \ln V_i + \alpha \ln W_j - \beta \ln d_{ij} + \delta \ln A_{ij} + \epsilon \quad (7)$$

$$A_{ij} = \sum_{\substack{k=1 \\ (k \neq i, k \neq j)}}^n W_k d_{jk}^\sigma \quad (8)$$

where δ is the parameter corresponding to destination accessibility, A_{ij} , which is the sum of the attractiveness, W , at each alternative destination k weighted by its distance to each alternative destination d_{jk} , and σ is a parameter that controls the scale over which destinations compete with each other, which in practice is often set to -1 or calibrated iteratively with the parameters in equation 7. Adopting the destination accessibility term, A_{ij} , is also advantageous to avoid the undesirable independence from irrelevant alternatives[§] (IIA) property that exists in many spatial choice modeling frameworks (Fotheringham 1986). Importantly, the set of locations defining the competing destinations is likely unique for different contexts, need not be the entire set of locations, can include locations that are not included in the original SI dataset, or could rely on bespoke criteria (Thill 1992; Thill and Horowitz 1997; Pellegrini et al. 1997).

A failure to account for destination accessibility when spatial structure effects are present generally biases origin-specific distance-decay parameter estimates upwards for accessible origins and downward for inaccessible origins. The strength of this bias is shown by Fotheringham (1984) to depend on the strength of two relationships: that between the volume of flows and destination

accessibility and that between destination accessibility and distance from the origin to each destination. It is this bias that causes the unintuitive spatial patterns observed in origin-specific distance-decay parameter estimates, such as positive estimates for the most accessible origins (Fotheringham 1981). Furthermore, the destination accessibility bias can be categorized in terms of its relationship to the volume of flows where competition effects are associated with a negative relationship (i.e., negative δ) and agglomeration effects are associated with a positive relationship (i.e., positive δ). The type of effect that arises is typically dependent upon the type of SI process being modeled, though competition is more often observed in empirical settings. The proliferation of the competition effect may also be driven by the behavioral tendency of individuals to underestimate the overall size of large objects, such as the attractiveness of clusters (Stevens 1975; Fotheringham and Curtis 1999).

The determinants of potential distance-decay parameter bias due to destination accessibility were further outlined such that bias may arise due to a combination of

1. a relationship between A_{ij} and T_{ij} ;
2. a direct relationship between A_{ij} and d_{ij} ;
3. an indirect relationship between A_{ij} and d_{ij} due to a relationship between A_{ij} and W_j and a relationship between d_{ij} and W_j .

Bias source (3) can occur independently of bias source (2), since it can occur even when there is no relationship between A_{ij} and d_{ij} . In addition, if bias sources (2) and (3) do not occur, but source (1) does occur, then distance-decay parameter estimates will not be biased; however, A_{ij} is still a relevant explanatory variable that can increase the accuracy of the model. These sources of bias can be similarly defined for the parameter estimates on other variables, though it is likely that the bias will be stronger for distance-decay whenever there is a stronger relationship between A_{ij} and d_{ij} than between A_{ij} and locational attributes (Fotheringham 1984).

It becomes clear that the CD model is addressing a very particular misspecification that arises

[§]The IIA property implies that if choice a is a substitute for choice b and choice b is a substitute for choice c , then choices a and c are also substitutes for each other. However, this may be very unlikely if choice a and choice c are very far apart.

organically due to collinearity amongst variables that would ordinarily be included in most, if not all, SI models. Therefore, it is attempting to correct for the omission of a correlated spatially patterned variable. Baxter (1983; 1985) provides a more general framework for analyzing omitted variable bias in SI models, though it is less amenable to interpretation. For example, it does not provide strong empirical evidence for any particular form, and it is not clear how each form would arise, as has been done for the CD framework. As a result the CD model has been applied in many domains including urban modeling (Fotheringham 1985; Fotheringham and Knudsen 1986), the study of telecommunications flows (Guldmann 1999), access to health facilities (de Mello-Sampayo 2016), crime location analysis (Bernasco 2010), determinants of trade (de Mello-Sampayo 2017a,b), and recreation and tourism (Matthews et al. 2018), though it has enjoyed particular popularity within migration modeling (Ishikawa 1987; Ishikawa 1990; Fik et al. 1992; Pellegrini and Fotheringham 1999; Fotheringham et al. 2000; Pellegrini and Fotheringham 2002; Yano et al. 2003; Fotheringham et al. 2004; Kalogirou 2015), commuting-to-work research (Thorsen and Gitlesen 1998; Gitlesen and Thorsen 2000; Gitlesen et al. 2010), and retail analysis (Fotheringham and Knudsen 1986; Guy 1987; Fotheringham 1988; Pellegrini et al. 1997; Birkin et al. 2010). The underlying theory of the CD model has also been explored via simulation studies (Lo 1991a; Fotheringham et al. 2001; Ubøe et al. 2008) and it has been connected with other key concepts, such as central place theory (Fik and Mulligan 1990) and trip chaining (Bernardin et al. 2009).

A criticism of the CD model's explicit focus on spatial configuration is that it overlooks other important factors. For example, Gordon (1985) suggests that further work investigating the spatial patterns of distance-decay parameters should focus on functional and economic differences between locations rather than solely physical accessibility. Lo (1991a; 1991b; 1992) takes up this call and argues that spatial structure should be renamed to *destination interdependence*, where this interdependence is composed of physical aspects and economic aspects. The physical aspect, which Lo calls *locational substitutability*,

is synonymous with the effects associated with the CD model, whereas *economic substitutability* is offered as an example of the economic aspects that might still be misspecified in CD models. Economic substitutability refers to consumer preferences toward destination activities and services. If the degree that destinations provide activities that are substitutable, or conversely, that are complimentary, varies and this is not accounted for, then SI models may produce biased parameter estimates. Similarly, Pooler (1998) proposes an alternative competition effect, termed *spatial influence* which also theorizes a hierarchical decision-making process, but where macro-level groups of destinations are based primarily on attributes (i.e., aspatial) rather than their locations in space. However, in the exposition of both the theories of economic substitutability and spatial influence, no generalizable framework or solution is proposed to account for the omitted variables that are causing model misspecification. One reason may be that such misspecifications are idiosyncratic, whereas potential exists for a (locational) spatial structure effect in any SI scenario. This also suggests that some criticisms of the CD model may be overstated, such as Hu and Pooler (2002) who assert that any spatial variation within local distance-decay parameter estimates means that the model is misspecified. In contrast, spatial variation could be due to variations in how distance is perceived or due to aggregation error, both of which can occur even when the model is correctly specified. Furthermore, extensions have been proposed to account for more complex location choice relationships within the CD model framework, such as disaggregated SI retail models that include store brand and household type (Newing et al. 2015).

The Box-Cox transform

It would be approximately thirty years before Johnston's argument about functional forms in SI models was picked up by again by Tiefelsdorf (2003), who illustrates how a degree of spatial variation in local distance-decay parameter estimates can arise systematically due to conditional distance distributions *when the functional form of distance is misspecified*. Recall that power and exponential functional forms of distance-decay are

the most often employed. Johnston's original argument was that spatial variation in distance-decay parameter estimates can arise if the underlying data-generating process follows the intervening opportunities model but a gravity-type SI model is calibrated on the data. This is essentially an extreme case of functional form misspecification, which Tiefelsdorf (2003) generalizes upon to show that any inconsistencies between the true data-generating functional form on distance and that which is specified in the model can result in distance-decay parameter estimates with spatial variation. Subsequently, it is recommended that the correct functional form of the distance term can be obtained by using the Box-Cox transformation, which encompasses a spectrum of functions depending on a tuning parameter that is selected by maximizing the model likelihood ratio. Tiefelsdorf (2003) claims that this allows the spatial structure effect that is caused strictly by distance to be accounted for, which cannot otherwise be accounted for by the CD model. However, it is not clear that functional form misspecification causes *unintuitive* interpretations of local distance-decay.

Several additional factors generate uncertainty about the effect of conditional distance distributions and the use of the Box-Cox transformation approach. First, although the distance term receives the most attention, any of the terms within a SI model could be subject to functional form misspecification. This suggests more work is needed to understand how the Box-Cox transformation approach performs if other terms are subject to functional form misspecification or distance is correlated with other attributes. Second, there are not many empirical examples of the Box-Cox transformation approach[¶], making it difficult to gauge the prevalence of conditional distance distribution misspecification and assess the magnitude of the associated spatial variation compared to other sources of misspecification. Researchers have often opted instead to choose theoretically motivated functional forms, other selection techniques, to shift focus to travel time or monetary costs, or to include technical, cultural, or political separations that may co-influence flows along with distance (Fischer et al. 2006; Chun 2008; Vries et al. 2009; Ortúzar S et al. 2011; Martínez and Viegas 2013; Thomas and Tutert 2013; Mukherji and Silberman 2019; Sohn et al.

2019; Buczkowska et al. 2019). Despite these ambiguities, Tiefelsdorf (2003) is commonly cited to caution against spatial dependence in SI models and to motivate spatial econometric or Eigenvector spatial filtering techniques (LeSage and Pace 2008; Fischer and Griffith 2008; Griffith and Fischer 2013; Fichet de Clairfontaine et al. 2015; Margaretic et al. 2017).

Spatial econometric models

Traditional spatial regression models incorporate spatial dependence (i.e., spatial structure) in (i) the dependent variable, (ii) the error term, (iii) the independent variables, or (iv) some combination of (i-iii) when modeling relationships across two-dimensional geographic units (Halleck Vega and Elhorst 2015). Spatial dependence is implied by the presence of spatial autocorrelation, which may violate common independence assumptions at the core of many modeling techniques. Curry (1972) first theorized the role of spatial autocorrelation in SI models, though it was amongst two-dimensional location attributes and not the flows themselves. Subsequently, Griffith and Jones (1980) posited that spatial interaction itself is spatially autocorrelated due to the fact that the spatial interaction system was spatially structured. Almost three decades later, the complexity of representing dependence between spatial interaction flows compared to two-dimensional spatial units was recognized and approached (Fischer and Griffith 2008).

The two spatial regression specifications that are most frequently used and studied in the spatial econometric literature are the spatial autoregressive model (SAR) or spatial lag model and the spatial error model (SE) (Halleck Vega and Elhorst 2015), which satisfy (i) and (ii) above, respectively. Several early attempts were made to extend the SE model to spatial interaction models (Brandsma and Ketellapper 1979; Bolduc et al. 1989, 1992, 1995); however, these extensions seem to have suffered from issues of interpretability, estimation, or both. Other attempts at modeling spatial structure effects in spatial interaction models using a SE

[¶]Two examples are (Willigers et al. 2007) and (Martínez and Viegas 2013), though the focus is on accuracy rather than misspecification.

framework^{||} have produced more encouraging results (Porojan 2001; Lee and Pace 2005; Fischer and Griffith 2008), though a standard specification has not arisen nor has a consistent spatial effect been identified. This may be due to idiosyncrasies of the processes being modeled or because in the SE framework, spatial dependence is treated as a nuisance and is not of substantive interest. Nevertheless, without a specific underlying theoretical misspecification, it is difficult to assess the quality of any particular method or to build intuition about the role of spatial structure.

Recent work has also proposed an extension to the SAR model to accommodate flow data (LeSage and Pace 2008; LeSage and Fischer 2014; LeSage and Thomas-Agnan 2015). To include a spatially dependent process in the unconstrained spatial interaction model, LeSage and Pace (2008) suggest the following specification

$$\begin{aligned}\ln T_{ij} &= \rho_i M_{iy} + \rho_j M_{jy} + \rho_{ij} M_{ijy} + \psi X + \epsilon \\ M_i &= I_n \otimes M \\ M_j &= M \otimes I_n \\ M_{ij} &= M_i \otimes W_j = M_i \otimes M_j = M \otimes M\end{aligned}\quad (9)$$

where ψX becomes $k + \mu \ln V_i + \alpha \ln W_j - \beta \ln d_{ij}$, M_i , M_j , and M_{ij} are contiguity-based spatial weight matrices that define neighborhoods from the perspective of origins, destinations, and origin-destination pairs, respectively, ρ_i , ρ_j , and ρ_{ij} are the corresponding autoregressive parameters, I_n is an $n \times n$ identity matrix with non-zero entries on the diagonal representing n locations, and $\otimes$ denotes the Kronecker product. In creating M_i and M_j , the role of the Kronecker product is to map the spatial relations amongst n locations encoded in M to spatial relations between n^2 flows, which can then be combined via the Kronecker product to create M_{ij} . Note that here it is assumed that there is an equal number of origins and destinations and that all origins are also destinations and, therefore, non-zero entries in M_{ij} are denoted by scenarios where both M_i and M_j are non-zero in the case of binary contiguity. This specification is motivated by the theory that the movements that people decide to make are based upon their knowledge of neighboring flows in a previous time period. Assuming that the exogenous variables are

relatively stable over time and that the cross-section of flows may be taken as the steady-state equilibrium of a long-run process, then LeSage and Pace (2008) demonstrate mathematically that a SAR data-generating process may be an adequate representation of spatial interaction. It was then subsequently applied to study migration (LeSage and Pace 2008; Murayama and Nagayasu 2019; Sardadvar and Vakulenko 2020), trade and commodity flows (LeSage and Polasek 2008; Chun et al. 2012; Metulini et al. 2018b; Moura et al. 2019; Yin et al. 2020), tourism (de la Mata and Llano 2013; Marrocu and Paci 2013; Alvarez-Diaz et al. 2020), commuting, transit, and labour choices (LeSage and Fischer 2014; LeSage and Thomas-Agnan 2015; Kerkman et al. 2017), and airline passenger travel (Margaretic et al. 2017; Kim et al. 2019).

Importantly, when locations serve as both origins and destinations and are represented by the same variable (i.e., population), a change in a single location's attribute value can influence many flows in the system, which can further diffuse across the system if observations are spatially dependent. That is, it is not possible to interpret how a change in a single origin attribute (destination attribute) would affect flows originating (terminating) from that origin (destination) without also considering how that change would also effect flows that terminate at that origin (originate at that destination). Therefore, scalar summary measures that capture the multiple changes across the system, which are known as *effects estimates*, should be used rather than treating the regression parameter estimates as representing the relations between system components as is typically done (LeSage and Fischer 2014; LeSage and Thomas-Agnan 2015). The SAR SI model allows for these type of feedback effects or spillovers, and can be thought of an instance of the feedback misspecification originally proposed by Fotheringham and Webber (1980). Including the spatial lag(s) of the dependent variable is theorized to account for endogenous effects that may arise due to changes in shared resources such as transportation infrastructure that can cause reactions to defuse through an entire system (i.e., global spillovers) (LeSage

^{||}Multilevel specifications have also been suggested to incorporate latent spatially structured effects (LeSage et al. 2007; LeSage and Llano 2013; Zhang et al. 2020)

and Fischer 2014). However, this interpretation is dependent upon a set of locations serving as both origins and destinations with a common set of attributes, which might not be appropriate in some SI systems.

Since the focus of the work of LeSage and Pace (2008), LeSage and Fischer (2014), and LeSage and Thomas-Agnan (2015) is on the interpretation of spatial interaction parameter estimates, it is surprising that little is mentioned about the interpretation of the parameter estimate on the distance variable (i.e., distance-decay). LeSage and Thomas-Agnan (2015) acknowledge distance and spatial dependence may be competing to explain the same variation in the dependent variable and notice that the distance-decay parameter estimate is smaller in magnitude in a SAR spatial interaction model than in a corresponding basic spatial interaction model. Similar results can be noted in additional applications of the SAR spatial interaction model (LeSage and Pace 2008; de la Mata and Llano 2013; Kerkman et al. 2017), especially, with increasingly complex definitions of spatial structure (LeSage and Polasek 2008). LeSage and Pace (2008) note that the distance-decay for models with spatial lags (i.e., SAR model) to those without spatial lags are not comparable, since those with lags require the effects estimate. Curiously, the effects estimate for distance-decay is otherwise not typically reported or discussed in these studies. In a study of air passenger data from Margaretic et al. (2017) who use a model with lags being based on either the origin or on the destinations, a counter-claim is provided that variables characterizing an origin-destination dyad, like distance, do not require the effects estimate for interpretation. Hence, it is not immediately clear how to interpret distance-decay in these spatial econometric models of SI.

A general critique of spatial econometric specifications, such as the SAR model, is that they have identification issues, rely upon strong assumptions, and are prone to overfitting (McMillen 2003; Pinkse and Slade 2010; Partridge et al. 2012; Gibbons and Overman 2012; Corrado and Fingleton 2012; McMillen 2012; Halleck Vega and Elhorst 2015). As a result, several additional perspectives have been put forth to complement the use of these models, such as the use

of natural experiments (Gibbons and Overman 2012), incorporating stronger underlying theory for a model (Corrado and Fingleton 2012), semi-parametric and nonparametric smoothing methods (McMillen 2012), and the use of simpler specifications when there is no strong theoretical basis for more complex ones (Gibbons and Overman 2012; Halleck Vega and Elhorst 2015).

Eigenvector spatial filtering

Eigenvector spatial filtering (ESF) is a technique that accounts for spatial autocorrelation based on the interpretation that the eigenvectors of a projected contiguity-based** connectivity matrix are the set of possible orthogonal and uncorrelated map patterns (Griffith 1996; Griffith 2011) given a particular definition of connectivity. Further, the first Eigenvector, E_1 , is the set of real numbers that produces the map pattern with the largest achievable Moran's I correlation coefficient (MC), the second Eigenvector, E_2 , is the set of real numbers that produces the map pattern with the largest achievable MC while remaining uncorrelated with E_1 , and continues on such that E_n , achieves the largest negative MC and is uncorrelated with the preceding $(n - 1)$ Eigenvectors. The projected connectivity matrix, C , that is most frequently†† decomposed into such Eigenvectors is defined as

$$(I - 11'/n)M_n(I - 11'/n) \quad (10)$$

where I is an $n \times n$ identity matrix, 1 is an $n \times 1$ vector of 1's, $'$ denotes the matrix transpose operation and M_n is the binary connectivity matrix for n mutually exclusive and exhaustive spatial units (i.e., locations) that partition the study space. By selecting a subset of the eigenvectors derived from C and creating a linear combination, it is possible to produce a synthetic variable that serves as a proxy for missing spatially autocorrelated exogenous variables (Griffith 2004), which can be

**Distance-based spatial weights have been used within the ESF technique, though it requires a distance cut-off which denotes the point at which all further relations become zero entries of the spatial weight. Furthermore, distance-based examples are based on research in the field of ecology and have not been employed in spatial interaction models (Borcard and Legendre 2002; Legendre et al. 2002; Borcard et al. 2004; Dray et al. 2006; Griffith and Peres-Neto 2006; Blanchet et al. 2008).

††While equation (10) is the most commonly found projection, others have been defined that ensure symmetric spatial relationships (Chun 2008). In addition, M may be standardized using different coding schemes (Boots 1999; Chun 2008).

included in a linear regression as follows (Chun and Griffith 2011):

$$Y = X\beta + E\gamma + \epsilon \quad (11)$$

where Y is a dependent variable representing areal units, X is a set of explanatory variables, E is a set of selected eigenvectors, β and γ are coefficient vectors, and ϵ is a vector of normally distributed random errors. This specification has been shown to produce results where the error term does not violate common assumptions of independence (Griffith 2000, 2002, 2004).

After the ESF framework was established, it was subsequently extended from spatial data aggregated to n areal units to SI flow data that occur between n^2 pairs of origins and destinations such that $X\beta$ becomes $\mu \ln V_i + \alpha \ln W_j - \beta \ln d_{ij}$ for various types of spatial processes, such as migration (Chun 2008; Chun and Griffith 2011; Liu and Shen 2017; Gu et al. 2019), commuting (Griffith 2007, 2009a,b, 2011; Griffith and Chun 2015), patent citations and research collaboration (Fischer and Griffith 2008; Scherngell and Lata 2012; Griffith and Fischer 2013; Griffith et al. 2016), and trade (Patuelli et al. 2015; Metulini et al. 2018a). While this cluster of work signifies a standard protocol for applying and interpreting the ESF SI methodology, a closer look at the literature indicates some ambiguities and inconsistencies with the technique in the SI context.

First, the primary motivation for using an ESF SI framework is often unclear and is stated as either that flows are *a priori* dependent upon each other or that the ESF's can serve as a proxy for spatially patterned omitted variables or both. Much of the research using an ESF in a SI model tends to cite Curry (1972) or Griffith and Jones (1980), which have already been discussed as convoluting several modeling violations and spatial analysis concepts. Second, Black's (1992) notion of network autocorrelation is often cited as inspiration to capture flow autocorrelation in ESF SI models but spatial contiguity is then used to define dependence. Thus, the use of Black's network autocorrelation term in the ESF SI context is a misnomer because Black actually defines proximity in terms of network connectivity and not in terms of spatial proximity. Further, network autocorrelation was examined by Black to define additional substantive

geographical variables such as regional indicators or accessibility terms, which were demonstrated to successfully reduce the measured network autocorrelation to insignificant levels. Thus, the underlying concepts and motivations of using an ESF in SI models are not always clear.

The ESF framework is also dependent upon a number of specification decisions that vary across existing research within SI modeling. For example, there is variation in how spatial relationships are defined (i.e., C and M matrices) and made operational (i.e., selecting a subset of eigenvectors E), the type of SI model the ESF is applied to (i.e., equations 2-5a), and the underlying probability model. Each of these issues is an important part of the modeling framework that may impact the model results. Table 1 captures the diversity of the ESF SI methodology specifications and their associated results, which presents the details of 22 SI models (with and without an ESF) that were extracted from 13 research articles produced during the first decade after the technique was introduced. Any entries of *not reported* indicate that the necessary information could not be found in a particular publication while bolded entries indicate that a particular detail of the ESF model was unclear and could not be concluded with certainty.

Since table 1 is organized roughly in chronological order, it is possible to detect some temporal patterns. Initially the ESF methodology was applied to unconstrained SI models, though eventually the focus shifted primarily to doubly-constrained models. An associated trend is that originally there was a separate ESF for origin variables and destination variables; however, this was eventually abandoned in favor of a single ESF that is specified using a combination (i.e., sum or product) of origin proximity relationships and destination proximity relationships. This seems to mark a shift in the primary motivation for using an ESF from correcting for spatial autocorrelation in the explanatory variables to spatial autocorrelation in flows themselves, though this distinction is not typically made. Further examining Table 1 reveals that outside of these patterns, a standard protocol is not apparent.

Table 1. Characteristics of Eigenvector spatial filtering methodologies applied to spatial interaction data

Source	Process	Scale (<i>n</i>)	spatial interaction model	Probability model	<i>C</i>	<i>M</i>	<i>E</i>	Pre-filter	Criterion	β effect	Significant	SE
Griffith (2007)	commuting	reported-3 (439)	UNC	Poisson	standard	contiguity	separate	> 0.25 MC	minimize RSS or MC	larger	not reported	not reported
Fischer and Griffith (2008)	patent citations	NUTS-2 (112)	UNC	log-normal	standard	contiguity	separate	> 0.25 MC	maximize likelihood	larger	no	larger
Fischer and Griffith (2008)	patent citations	NUTS-2 (112)	UNC	Poisson	standard	contiguity	separate	> 0.25 MC	maximize likelihood	larger	yes	larger
Chun (2008)	migration	states (49)	UNC	Poisson	symmetry-corrected	S-coded contiguity	product	not reported	minimize T statistic	smaller	no	smaller
Chun (2008)	migration	states (49)	PC	Poisson	symmetry-corrected	S-coded contiguity	product	not reported	minimize T statistic	smaller	no	smaller
Griffith (2009a)	commuting	NUTS-3 (439)	DC	Poisson	standard	contiguity	product	> 0.5 MC	smallest p-value (< 0.1)	smaller	yes	smaller
Griffith (2009b)	commuting	counties (254)	DC	Poisson	standard	contiguity	product	> 0.5 MC	not reported	smaller	yes	not reported
Chun and Griffith (2011)	migration	states (49)	UNC	log-normal	standard	contiguity	sum	> 0.25 MC	minimize AICc	larger	yes	larger
Chun and Griffith (2011)	migration	states (49)	UNC	Poisson	standard	contiguity	sum	> 0.25 MC	minimize Quasi-AICc	larger	yes	typically larger
Griffith (2011)	commuting	counties (73)	DC	Poisson	standard	contiguity	product	> 0.5 MC	smallest p-value	smaller	yes	larger
Scherngell and Lata (2012)	collaborations	NUTS-2 (255)	UNC	neg. binomial	standard	contiguity	separate	> 0.25 MC	not reported	smaller	yes	larger
Griffith and Fischer (2013)	patent citations	NUTS-2 (257)	DC	Poisson	standard	contiguity	product	> 0.5 MC	minimize AIC	larger	not reported	not reported
Patuelli et al. (2015)	trade	counties (64)	UNC	neg. binomial	symmetry-corrected	contiguity	separate	> 0.25 MC	statistical sig.	larger	not reported	not reported
Griffith and Chun (2015)	commuting	counties (11)	DC	Poisson	standard	not reported	product	not reported	not reported	larger	not reported	not reported
Griffith and Chun (2015)	commuting	tracts (38)	DC	Poisson	standard	not reported	product	not reported	not reported	almost identical	not reported	not reported
Griffith and Chun (2015)	commuting	counties (73)	DC	Poisson	standard	not reported	product	not reported	not reported	smaller	not reported	not reported
Griffith and Chun (2015)	commuting	NUTS-1 (17)	DC	Poisson	standard	not reported	product	not reported	not reported	smaller	not reported	not reported
Griffith and Chun (2015)	commuting	NUTS-2 (40)	DC	Poisson	standard	not reported	product	not reported	not reported	smaller	not reported	not reported
Griffith and Chun (2015)	commuting	NUTS-3 (439)	DC	Poisson	standard	not reported	product	not reported	not reported	smaller	not reported	not reported
Griffith et al. (2016)	patent citations	NUTS-2 (257)	DC	Poisson	symmetry-corrected	8 NN	sum	> +/ - 0.25 MC	smallest p-value (< 0.1)	larger	yes	smaller
Margaretic et al. (2017)	air travel	cities (279)	UNC	log-normal	symmetry-corrected	C-coded 4/5 NN	separate	> 0.25 MC	minimize AIC	larger	yes	smaller
Margaretic et al. (2017)	air travel	cities (279)	UNC	Poisson	symmetry-corrected	C-coded 4/5 NN	separate	> 0.25 MC	minimize AIC	larger	yes	smaller

- UNC = unconstrained; PC = production-constrained; DC = doubly-constrained
- Standard means the projection matrix given in equation 10 rather than one that forces symmetry
- The weight matrix *M* is assumed binary unless another coding scheme is noted

Perhaps the most varied aspect of the ESF-SI framework is the selection criterion employed to select a specific subset of eigenvectors. Forward or backward stepwise selection is employed in the SI literature, and the selection criterion may involve directly optimizing the model fit, indirectly optimizing a model fit statistic, minimizing spatial autocorrelation, or finding all eigenvectors that are collectively statistically significant (Table 1). In addition, the collection of all eigenvectors is typically pre-filtered^{‡‡} so that only those with higher levels of positive spatial autocorrelation can be selected. In one recent case though, both negatively and positively spatially autocorrelated eigenvectors are included (Griffith et al. 2016). Hence, it is unclear which eigenvectors should be *a priori* filtered from the selection process and how sensitive the model results are to various filtering schemes, which is important because Chun et al. (2016) show that too many or too few eigenvectors can cause over- or under-correction by the ESF approach.

ESF SI model results are frequently deemed more intuitive than their non-ESF counterparts, though some confusion arises based on the aggregated results observed here. For instance, the effect that adding an ESF has on the distance-decay coefficient estimate seems to vary extensively (Table 1). In some cases it results in a stronger distance-decay and in other cases it results in weaker distance-decay. In addition, the standard errors for these estimates with an ESF are sometimes larger and sometimes smaller than the standard errors from a model without an ESF, though sometimes they are not reported at all. When the standard errors are reported, it is not always the case that the change in distance-decay is statistically distinguishable (at the 95% confidence interval) from the original distance-decay estimate. This suggests some variability of the impact of the ESF on the estimated distance-decay parameter. Related to this, Hodges and Reich (2010) and Paciorek (2010) demonstrate that a spatially correlated random effect, which an ESF is theorized to approximate (Fischer and Griffith 2008), can compete with the substantive spatial effects (i.e., main effects) in a regression model and potentially result in biased estimates. An alternative interpretation posits that ESF components may suggest that a SI system is

more vulnerable or resilient to disruptions than anticipated by a traditional gravity-type model (Griffith and Chun 2015). More work is therefore necessary to explore whether adding an ESF model makes SI model interpretation more intuitive or if it potentially obfuscates distance-decay effects.

Further themes & a future research scheme

Several themes either persist throughout or begin to emerge from the foregoing sections. Most prominently, that distance-decay is a quintessential concept within quantitative human geography and much energy has been put forth to understand the intricacies associated with it. However, more recent methods in SI modeling shift the focus away from distance-decay in favor of alternative but related spatial concepts, such as accessibility, spatial dependence, network dependence, and associated notions of autocorrelation. It is easy to conflate these ideas under the banner of spatial structure, but it has been shown that the frameworks proposed to make these concepts operational are not interchangeable. Rather, further juxtaposition of the frameworks helps clarify their differences.

One observation is that SI models invoked through spatial dependence and spatial autocorrelation (i.e., spatial econometrics and spatial filtering) are supported by such general mechanisms that interpretation of the results in terms of distance-decay and specific SI processes can become opaque. This may be because they were originally developed for observations across points and areal units and then subsequently adapted to flows. It may also be related to their comparatively high flexibility that may indiscriminately account for a large degree of spatial variation in data regardless of its source. A consequence is that problems of one spatial concept, that of distance-decay, become couched in a new spatial concept, that of spatial autocorrelation or spatial

^{‡‡}A LASSO routine and a random effects variant of the ESF have been proposed that suggest more parsimonious methods for selecting a subset of eigenvectors (Seya et al. 2015; Murakami and Griffith 2015), though neither of them has been applied in the spatial interaction literature and therefore do not shed light on the variations found in existing ESF spatial interaction research. Moreover, Chun et al. (2016) show that an ideal number of eigenvectors is dependent upon the amount of spatial autocorrelation in model residuals and the size of the tessellation. A method for identifying an ideal number of eigenvectors is put forth, but has not been applied in the spatial interaction literature.

dependence (Sayer 1977), rather than incrementally and iteratively developing theories and corresponding models. For example, the veracity of the spillovers assumed to exist in SAR SI processes and the diffusions quantified by the scalar summary metrics have not been investigated empirically. Another example is that the ESF method generally increases model fit, but is also prone to overfitting (Helbich and Griffith 2016; Oshan and Fotheringham 2017), and could mask the fact that other substantive covariates may be appropriate. In contrast, the CD model was conceived in the context of flows and SI decision-making and is suited to modeling more specific geographic entities and processes. Though it does not provide a panacea for incorporating spatial structure into SI models, it does encourage the analyst to critically evaluate how spatial structure is abstracted and is less prone to overfitting. This provides evidence of tradeoffs amongst interpretability, conceptual adaptability, model complexity, and the types of assumptions that are made across the spatial structure approaches and future inquiries could focus on collectively considering these issues in greater detail.

The different approaches are also comparable in how they extend or diverge from legacy SI modeling traditions. For example, the CD and ESF methods can augment any member of the family of models and are straightforward to adapt to different probability models. Though a log-normal model may be appropriate for some flows (i.e., retail expenditure), a Poisson distribution is natural for discrete entities (i.e., people), and sometimes a negative binomial distribution or zero-inflated extension is ideal (Silva and Tenreiro 2006; Burger et al. 2009; Metulini et al. 2018a). A CD or ESF variation is possible in any of these circumstances, though it should be remembered that only Poisson maximum likelihood estimates will yield results equivalent to the family of multiplicative models and ensure the constraints upheld by the balancing factors are met (Arvis and Shepherd 2013). It is also less clear how to interpret the balancing factors in constrained models in the ESF approach (Griffith and Fischer 2013) where the balancing factors and the ESF may compete to account for spatial variation, potentially altering previous interpretations as accessibilities or rents (Cesario

1977; Morphet and Shabrina 2020). In contrast, the SAR SI model is generally only available using Gaussian assumptions, which does not respect the constraints nor easily extend to other distributional assumptions.

There are also some important tendencies from the initial phases of the spatial structure debate that are less prevalent in the contemporary literature. Although local models and spatial non-stationarity were originally used to diagnose problems arising from spatial structure, there has been limited recent research using local SI models (e.g., Nakaya 2001; Nissi and Sarra 2011; Kalogirou 2015; Kordi and Fotheringham 2016; Zhang et al. 2019a,b). Specifically the SAR and ESF approaches have virtually ignored the possibility of spatial non-stationarity. Meanwhile, in the ESF literature, spatial autocorrelation is often distinguished as *local* distance effects in contrast to *global* distance effects that are captured by distances between locations (Griffith 2007, 2009a, 2011; Griffith and Paelinck 2018). An effect that occurs locally and globally implies a multiscale process that potentially varies over space (Wolf et al. 2018). However, there has yet to be a comparison or incorporation of local SI models within the context of ESFs, which could be used to shed light on the nature of scale in SI models rather than *a priori* assuming process stationarity. Initial work in this direction has begun but does not include SI models (2017; 2019).

Another trend worth reviving in light of advances in computing capabilities is the reliance on simulations for building intuition about SI models (e.g., Johnston 1973; Cliff et al. 1974; Lo 1991a; Fotheringham et al. 2001). Spatial econometric and spatial filtering approaches have been explored via simulation, but not within the context of SI models. In particular, Pace et al. (2013) demonstrate a general tension between the SAR and ESF models since the ESF filters out the spatial spillovers implied by the SAR model. An interesting extension could therefore use simulation studies to explicitly compare implementations to account for spatial structure in SI models in order to verify their underlying theory and measure their performance under different

A semi-parametric Poisson SI model incorporating a spatial autoregressive component exists, but deviates from maximum likelihood estimates (Sellner et al. 2013).

types of misspecification. This might also help clarify the nature and degree of misspecification bias for different scenarios and types of spatial systems.

An area largely untapped for advancing SI modeling is how to leverage aspects of the modern data economy. The swell of new data sources with increasing spatial and temporal resolutions provide a wealth of additional information that can supplement SI models and provide novel insights. One strategy is to refine the definition of origin and destination variables and the definition of spatial structure based on spatial and temporal context (Waddington et al. 2019). Another way forward is to explore the dynamics of human behavior by calibrating parameters for temporal subsets of data to inspect how processes change over time (Batty 2018; Oshan 2020). There is also a history of dynamic SI models that investigate how a component of the SI system responds to the evolution of other components (Harris and Wilson 1978; Fotheringham and Knudsen 1986; Nijkamp and Reggiani 1988; Wilson 2008; Birkin and Heppenstall 2011; Wilson 2010b) and these could be enhanced and validated using sensors that continuously collect mobility data to construct contextualized flows (Sila-Nowicka et al. 2016; Sila-Nowicka and Fotheringham 2019).

Ultimately, the recommendations put forth here to move the SI modeling research agenda forward are a call for more specific theories and a better understanding of the connection between theories, models, and the interpretations extracted from them. As quantitative human geography progresses, the incorporation of best practices for scientific code development, a culture of data-sharing, and the establishment of common task frameworks could also go a long way towards broadening and aligning overlapping yet sometimes diverging branches of inquiry within the SI modeling paradigm (Wolf et al. 2020).

Concluding remarks

An entry point into the vast SI corpus was provided in this review through the pursuit of a number of tasks. First, SI models and the spatial structure debate were introduced, outlining some foundational concepts, the central issues, and the intellectual lineage of the main debate arguments. Four frameworks proposed to

incorporate spatial structure in SI models were then considered. The typical motivations, theories, assumptions, applications, and interpretations of each framework were highlighted, as well as some criticisms and limitations. Connections were also made between each methodological approach and the debate at large. Finally, some synthesis was constructed around additional themes detected across the literature and some avenues for subsequent research were identified. These included more explicit evaluations of framework tradeoffs, a closer consideration of the repercussions of framework evolution on knowledge accumulation, the reincorporation of simulations, local models, and spatial non-stationarity, and the utilization of burgeoning data sources and technology. The overall outcome is a comprehensive coverage of the past and a sketch of one potential path forward for the treatment of spatial structure in SI models. This path advocates primarily for a tighter integration of the SI modeling ecosystem and a more holistic approach to the issue of spatial structure. It also calls for shifting the focus back toward modeling specific SI processes and spatial decisions. These suggestions are only some of the many possible directions to contribute towards the next 50 years of SI modeling.

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