Zero Curve Construction Model

- Zero curve is essential for valuing financial products. Zero curves are derived or bootstrapped from observed market instruments that represent the most liquid and dominant interest rate products for certain time horizons.
- Normally the curve consists of an input set of deposit rates, futures contracts and swap. The effective date of the first futures is located either on a control point or between two control points on the portion of the swap curve obtained from the deposit rates.
- For any two consecutive futures, the maturity of the earlier futures contract is the effective date of the subsequent futures contract.
- The last Libor reset date of the first swap is located either on a control point or between two control points on the portion of the swap curve obtained with the deposit rates and futures contracts.
- For any two swaps, the Libor reset and payment dates of the swap with shorter maturity are also the Libor reset and payment dates of the swap with longer maturity.
- All swaps payments are made in arrears, that is, at the end point of the corresponding Libor rate accrual period.
- We denote by B(t0, T) the price of the zero coupon bond, with unit face value, at the valuation date2 t0 and maturity T. We denote by d(T) the discount factor from the maturity T back to t0. We note that d(T) = B(t0, T). The discount factor d(T) and the simple interest spot rate s(T) are related by the following formula

$$d(T) = \frac{1}{1 + s(T) \times (T - t_0)}.$$

- The short end of the swap curve, under three months, is derived from deposit rates. Let R(T) denote the deposit rate at date T. The discount factor d(T) at the valuation date is then calculated as

$$d(T) = \frac{1}{1 + R(T) \times (T - t_0)}.$$

- The control points obtained from the deposit rates are the maturities of the deposits. Market quotes for deposits with maturities of one day, and one, two and three months are usually available.
- The middle range of the swap curve, up to two years, is derived from an interest rate futures strip. A futures strip is simply a sequence of futures contracts with successive expiration dates. For interest rate futures (e.g., Eurodollar futures), the prices are quoted according to the following convention:

$$\text{Futures price} = 100 - \text{Futures rate}(\%) \times 100.$$

- The futures rate, f, is related to the forward rate, F, by a futures-forward convexity adjustment3, C, as in the formula, F = f −C. The maturities of the futures contracts constitute control points of the bootstrapped curve, following the control points obtained from deposit rates.
- The long end of the swap curve is derived directly from observable swaps. These are plain vanilla interest rate swaps with fixed rates exchanged for floating interest rates. The swap rate is the fixed rate that makes the value of the swap equal to zero. More specifically, consider a forward starting swap with the following reset and payment date sequence
- The forward swap rate, rs, can be shown to be

$$r_s = \frac{B(t_0, T_0) - B(t_0, T_n)}{\sum_{i=1}^{n} B(t_0, T_i) \times \Delta T_i} = \frac{d(T_0) - d(T_n)}{\sum_{i=1}^{n} d(T_i) \times \Delta T_i},$$

- The present value of an interest rate swap can expressed as
  - From the fixed rate payer perspective, $PV = PV_{float} - PV_{fixed}$
  - From the fixed rate receiver perspective, $PV = PV_{fixed} - PV_{float}$

Reference:

https://finpricing.com/lib/EqSpread.html