Some Results for Area on $\mathbb{R}^2$ by Triangle Partition

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Abstract. In 2012, Affaf was proved that the area on $\mathbb{R}^2$ can be calculated by Riemann Integral via triangle partition. He get new formula for calculate the area but he not identification when this condition happened. Next, in 2015, Juni calimed that convex function satisfy the desired condition and generalize for fontinuous function. This paper will give some of the results obtained by triangle partition.

I. INTRODUCTION

Area on $\mathbb{R}^2$, under function f and above x -axis (vice versa), from $x = a$ up to $x = b$ given by

$$\int_a^b f(x)dx$$

The formula was given by partition area to be rectangles and bring it to riemann sum. The tehniqe is very familiar. In his paper, Affaf [1] also discuss the topic but through different path, that is by using triangle partition. Other than that, in order to determine area on $\mathbb{R}^2$, his tehniqe running the partitions on curve (called primary curve) not on x -axis as previous tehniqe. Ones again which distinguish Affaf's thenique and the earlier tehniqe, that is determine one point on line or curve (called primary point) which restrict the area.

Main idea of his tehniqe is by using area of triangle on $\mathbb{R}^2$ which using distance point to line. In here is known that determine area of triangle can be getted by using line equation of triangle side. It is which Affaf called as "tangent" on primary curve for determine area on $\mathbb{R}^2$.

Figure 1. Illustration of Affaf's tehniqe

II. AFFAF'S AND JUNI'S RESULT

Here will be discussed briefly how Affaf build his result. Firts, he find that for determine area of triangle if three points known will be done by one of three line equation and third point other. He state this result into belowing theorem.

Theorem 2.1. Given three point $A(x_a, y_a)$, $B(x_b, y_b)$, and $C(x_c, y_c)$, also suppose that $x_b > x_a$ If line which poin A and B have equation $y + mx + n = 0$, then area of triangle ABC is given by

$$\frac{1}{2} |y_c + mx_c + n| (x_b - x_a)$$

Next, Affaf recognize that this result can be carried to integral sum for identification area on $\mathbb{R}^2$. with change partition to be triangle But, absolute value of “side equation” and whether partition triangle can be covered all of area be problem. For solving problem of absolute value, Affaf define position of point to line as follow

Definition 2.2. Given point $A(x_a, y_a)$ and line l with equation $y + mx + n = 0$. We define:

1. $A > l \Leftrightarrow y_a + mx_a + n > 0$
2. $A = l \Leftrightarrow y_a + mx_a + n = 0$
3. $A < l \Leftrightarrow y_a + mx_a + n < 0$

From the definition, Affaf have two small result below.

Lemma 2.3. Given line g and l which parallel both. If $A(x_a, y_a) \in g$ and $A > l$, then:

1. $B > l, \forall_{B \in g}$
2. $B < g, \forall_{B \in l}$

Lemma 2.4. Given line g and l with $m_g > m_l$ and $g \cap l = \{(x_0, y_0)\}$, then:

1. $A > l, \forall_{A \in g}$ and $x_a > x_0$
2. $A < l, \forall_{A \in g}$ and $x_a < x_0$
3. $B < g, \forall_{B \in l}$ and $x_b > x_0$
4. $B > g, \forall_{B \in l}$ and $x_b < x_0$

With the definition, Affaf build some result which used for prove his final result, that is the following theorem, Theorem 2.3 as below.

Theorem 2.5. Given continuous differentiable function f on $[a, b]$ and line g where $f = g$ in $[a, b]$ happen only for $x = a$ and $x = b$ Then, formed function h , that is

$$h(x) = y_a - f(x) + xf'(x) - x_a f'(x)$$

for some $(x_a, y_a) \in g$. We have

1. If $h(x) \geq 0$ for $x \in [a, b]$, then area which restricted by g and f given by L , that is

$$\frac{1}{2} \int_a^b h(x) dx$$

2. If $h(x) \leq 0$ for $x \in [a, b]$, then area which restricted by g and f given by L , that is

$$-\frac{1}{2}\int_a^b h(x)dx$$

In final result, Affaf not identify for primary function f like what such that the condition will be happened, that is $h(x) \geq 0$ in $[a, b]$ or $h(x) \leq 0$ in $[a, b]$. In 2015, Juni [2] claimed primary function f that satisfy the condition if f is convex. She get the following result.

Theorem 2.6. Given continuous differentiable function f on $[a, b]$ and line g where $f = g$ in $[a, b]$ happen only for $x = a$ and $x = b$. Then, formed function h , that is

$$h(x) = y_a - f(x) + xf'(x) - x_a f'(x)$$

for some $(x_a, y_a) \in g$. We have

1. If f is convex above for $x \in [a, b]$, then area which restricted by g and f given by L , that is

$$\frac{1}{2}\int_a^b h(x)dx$$

2. If f is convex below for $x \in [a, b]$, then area which restricted by g and f given by L , that is

$$-\frac{1}{2}\int_a^b h(x)dx$$

From Juni's result, we extended this condition for any continuous function by split the function into convex function.. We state the result in following theorem.

Theorem 2.7. Given continuous function f on $[a, b]$. Suppose points $a = x_0 < x_1 < x_2 < \dots < x_n = b$ on x -axis such that f is convex on $[x_{i-1}, x_i]$ for every $i = 1, 2, 3, \dots, n$. Then area that restricted by f and x -axis from $x = a$ up to $x = b$ given by L , that is

$$\sum_{i=1}^n \left(L_i - \frac{1}{2} \int_{x_{i-1}}^{x_i} h(x)dx \right)$$

Where L_i is i^{th} area of trapezoid.

III. EXTENDED PRIMARY FUNCTION AND SOME RESULT

Prior to final result, will be constructed some results that support the final result. Firts result we state into lemma below.

Lemma 3.1. Given lin g and point $A(x_a, y_a)$. Through A is made line l and taken $A'(x'_a, y'_a) \in g$. Then

1. If $A < g, m_l < m_g, x_a \geq x'_a$, then $A' < l$
2. If $A < g, m_l > m_g, x_a < x'_a$, then $A' < l$
3. If $A < g, m_l < m_g, x_a < x'_a$, then $A' > l$
4. If $A < g, m_l > m_g, x_a \geq x'_a$, then $A' > l$

Proof. 1. For $x_a = x'_a$, clearly that $A' < l$. Now, for $x_a > x'_a$, make line l' via A with $m_{l'} = m_g$. Then according Lemma 2.3.1 we can take $A''(x'_a, y''_a) \in l'$ such taht $A'' > g$, impact $y''_a > y'_a$. Next, take $P(x'_a, y_p) \in l$. Since

$l \cap l' = \{(x_a, y_a)\}$, $m_l < m_{l'}$, and $x_a < x_{a'}$, then according Lemma 2.4.1 we have $P > l'$, that is $y_p > y_a'' > y_a'$. Consequently $A' < l$. (For 2, 3, and 4 are analogue)

Lemma 3.2. Given line l , point $A(x_a, y_a)$, and $A'(x_a', y_a') \in l$. Through A' is made line l' , Then:

1. If $A' > l, A > l', m_{l'} < m_l, x_a < x_{a'}$ then $x_a < x_0 < x_{a'}$ where $l' \cap l = \{(x_0, y_0)\}$
2. If $A' < l, A < l', m_{l'} > m_l, x_a < x_{a'}$ then $x_a < x_0 < x_{a'}$ where $l' \cap l = \{(x_0, y_0)\}$

Proof. 1. Suppose $l \cap l' = \{(x_0, y_0)\}$. Since $m_l > m_{l'}$ and $A > l'$, then according to Lemma 2.4.4 we have $x_a < x_0$. In the other side, Since $m_l > m_{l'}$ and $A' > l$, then according to Lemma 2.4.1 we have $x_0 < x_{a'}$. So, we can conclude $x_a < x_0 < x_{a'}$. For 2 analogue.

Lemma 3.3. Given line l and l' with $A'(x_a', y_a') \in l$, $A(x_a, y_a) \in l'$, $x_a < x_{a'}$, and $l' \cap l = \{(x_0, y_0)\}$, then:

1. For all point $P(x_p, y_p)$ with $x_a < x_p < x_{a'}$ which hold $P > l$ and $P > l'$, if through P is made line p with $m_{l'} < m_p < m_l$, then $(x_0, y_0) < p$
2. For all point $P(x_p, y_p)$ with $x_a < x_p < x_{a'}$ which hold $P < l$ and $P < l'$, if through P is made line p with $m_{l'} < m_p < m_l$, then $(x_0, y_0) > p$

Proof. 1. For $x_p \leq x_0$, consider position of P to 1. Since $m_p < m_l$, then according to Lemma 3.1.1 clearly that $(x_0, y_0) < p$. Next, for $x_p > x_0$, consider position of P to l' . Since $m_{l'} < m_p$, then according to Lemma 3.1.2 we have $(x_0, y_0) < p$. So we are done. For 2 analogue.

Before we state final result of this paper, for efisiensi we write $l_{f(a)}$ as tangent of function f at point $(a, f(a))$. Now, we are ready to state the final result from this paper. The final result state in Theorem 3.4 below.

Theorem 3.4. Given continuous function f on $[a, b]$. Then:

1. If f is convex above, then area which restricted by f , $l_{f(a)}$, and $l_{f(b)}$ given by L , that is

$$-\frac{1}{2} \int_a^b h(x) dx$$

2. If f is convex below, then area which restricted by f , $l_{f(a)}$, and $l_{f(b)}$ given by L , that is

$$\frac{1}{2} \int_a^b h(x) dx$$

Where $h(x) = f(x) - y_s + x_s f'(x) - x f'(x)$ and $\{(x_s, y_s)\} \in l_{f(a)} \cap l_{f(b)}$

Proof. 1. Consider that $h(x) = f(x) - y_s + x_s f'(x) - x f'(x)$ are $l_{f(x_i)}$ for all $x_i \in [a, b]$. Since f is convex above, then $m_{l_{f(a)}} < m_{l_{f(x_i)}} < m_{l_{f(b)}}$ for all $x_i \in [a, b]$. So according to Lemma 4.2.1 we have $a < x_s < b$. Furthermore, according to Lemma 4.3.1 hold $x(x_s, y_s) < l_{f(x_i)}$ for all $x_i \in [a, b]$. In another word, we know that $f(x) - y_s + x_s f'(x) - x f'(x) < 0$ for all $x_i \in [a, b]$. So, according to Theorem 2.5.2 we have that area which restricted by f , tangent on $(a, f(a))$, and $l_{f(b)}$ given by L , that is

$$-\frac{1}{2} \int_a^b h(x) dx$$

For 2, analogue. So, we have done to proof this result.

CONCLUTION

From the final result we can conclude the area which restricted by three function, that is a function convex and two of tangents into interval of his convexing can be expressed by one interval without split it.

ACKNOWLEDGMENTS

For Further study can be researched this situation to volume on $\mathbb{R}^3$. Therefore, be expected a new formula for determine volume on $\mathbb{R}^3$. In this case, need to reviewed how the triangle partition should be designed.

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