Dynamical Outcome Modeling of Fragmented Intensive Longitudinal Data

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Abstract

The advent of new sensing technologies has enabled the collection of high frequency individual data. Not only can this allow for the identification of personalized behavior models, it also opens up the possibility of developing just-in-time interventions that leverage the information collected to determine when and which micro intervention should be provided. However, there are significant challenges in the analysis of this type of data. First, due to the high rate of data collection, one can no longer assume that the stimulus (independent excitation or micro intervention) only has an instantaneous effect on the outcome, one has to also allow for delayed effects. Moreover, one is also frequently faced with fragmented data; i.e., poor placement of sensors, non-wear of the data collecting device and/or external disturbances can lead to intervals of time where the data collected is not reliable; i.e., missing or corrupted. To deal with these challenges, we leverage concepts from the areas of dynamical systems and signal processing to develop tools that i) can identify models that take into account the delayed stimuli effects and ii) are able to handle fragmented data. In this paper, we provide both the mathematical foundation of the tools proposed and the description of a package that implements them. We also discuss ways to interpret the results obtained.

Keywords: intensive longitudinal data, dynamical models, sparsity

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Introduction

The development of new technology such as cameras, cell phones, activity trackers and health monitors, has enabled the collection of large amounts of intensive longitudinal personal data. This opens the exciting opportunity of developing targeted adaptive micro treatments that use these data to determine which is the best treatment option and when it should be delivered. This approach to behavioral treatments can be considered to be related to the concept of *precision medicine* where treatment is tailored to the biological characteristics of the individual (e.g.; see Denny and Collins, 2021) and, more generally, to the concept of *precision health* where both the status of the individual and the surrounding environment are used to decide which (if any) intervention should be provided (Hekler et al., 2020).

This revolution in technology means that high frequency data can now be passively collected on many aspects of the status of an individual. One can collect data on health behaviors such as physical activity (Conroy et al., 2019; Hojjatinia et al., 2020), eating (Thomaz et al., 2015), alcohol consumption and smoking (Kumar et al., 2013, Kumar et al., 2012, Fairbairn and Kang, 2019; McGarraugh, 2009), on biological quantities such as heart rate, ECG, blood oxygen (Trister et al., 2016), respiration rate, and glucose, on mood and stress (Canzian and Musolesi, 2015; Saeb et al., 2015), and on location, social environment and social contacts (Choudhury, 2014; Choudhury et al., 2008; H. Lu et al., 2012; Wyatt et al., 2007). Hence it is possible to collect valuable real time information that can be used for tailoring treatments that can be instantly delivered using devices such as cell phones.

However, intensive longitudinal data, like the ones mentioned above, are very different from the data collected in “traditional studies” and cannot, in our view, be properly analyzed using the tools that are currently available to behavioral scientists. First, data on an individual can now be collected every day, every hour or even every

minute. Hence, the traditional assumption that the stimulus at time t only has a possible effect on the outcome at time t does not hold. In other words, a framework is needed that allows for modeling “memory;” i.e., a framework that can model the fact that a stimulus can have a delayed effect where it may not only have an immediate influence on the outcome, it can also affect future outcomes. Moreover, such a framework should allow for undetermined and/or variable lags. For example, a stress event not only has an immediate effect on the probability of smoking but it can also have a lingering effect that increases the probability of smoking for some time interval following the stress event (Hojjatnia, Daly, et al., 2021). One should note that at this point, there has been some attempts to extend existing approaches in order to deal with these delayed effects. An example of these is the use of lags in multilevel models (Gistelinck et al., 2020; H. Goldstein et al., 1994; Hamaker and Grasman, 2015). However, these approaches only consider models with a “rigid structure” that do not allow for complex stimulus responses. Moreover, we found that there is limited interpretation of the dynamic characteristics of the models obtained.

Second, intensive longitudinal data is often fragmented; i.e., it is common to have intervals with no measurements or unreliable ones. For example, in the study (Kumar et al., 2015), events such as physical activity or incorrect placement of sensors confound estimates of stress resulting in fragmented information across the day on its probability estimates. Moreover, in studies using wearable devices, data fragmentation can occur due to non-wear (Cho et al., 2021).

To address the two challenges mentioned above, in this paper, we propose to use concepts from the general area of dynamical systems to develop models that take “memory” and fragmentation into account. Dynamical systems are a powerful tool for modeling interaction between longitudinal variables and have been used in areas as diverse as aerospace (Stevens et al., 2015), agriculture (Cox, 1997) and economics (Weber, 2011). Dynamical systems has also been used as a modeling tool in the area of psychology to model development and social interactions (Gottman et al., 2005; Molenaar et al., 2014).

To identify these dynamical models, we use concepts from control systems engineering and sparse signal reconstruction to develop tools for parsimonious modeling of interactions between intensive longitudinal data streams. We focus on binary outcomes but the approach proposed can be extended to other types of outcomes such as multi-level or continuously valued data.

At this point, one should note that although the proposed approach can be applied to any longitudinal data, one of the major advantages is that it can leverage the data collected to identify *personalized models*. These computational models can then be used to simulate the response of the individual to a variety of stimuli. Moreover, by leveraging tools from control engineering, it can be used to design efficient decision rules for just-in-time adaptive interventions. This personalized model approach has been demonstrated in previous work on digital messaging intervention to promote physical activity (Hojjatinia, Hojjatinia, et al., 2021)

The objective of this paper is not only to describe tools developed by the authors for longitudinal data analysis but also to elaborate on the foundations of the proposed approach. To this end, we start by describing the conceptual underpinning of the algorithm developed. This requires the review of some basic concepts related to linear time-invariant dynamical systems. Then, a description of the Matlab package developed is provided with a detailed description on how the data should be provided and what the output is. Finally, we provide examples on how to use the package and, more importantly, how to interpret the results obtained. We should note at this point that, since the final result of this analysis is a dynamical system that relates stimulus to outcome, we have a “computational model” of interaction and we can use it to simulate responses to different stimuli.

Relation to Current State of the Art

Intensive Longitudinal Data Makes Just-in-Time Adaptive Interventions Possible

High frequency data, which is also defined by volume, velocity, and variety (Laney et al., 2001), makes it possible (a) to detect and predict the moments of vulnerability, opportunity, and receptivity and (b) to design intervention programs in order to make decisions about whether to intervene or not. It has been suggested that vulnerable states are dynamic processes that incorporate static and episodic factors; i.e., influences that can change over time (Nahum-Shani et al., 2014). Just-in-Time Adaptive interventions (JITAIs) are an emerging type of adaptive interventions capable of administering adaptive treatments in terms of type, timing, and intensity by monitoring individual's behavior changes using intensive longitudinal data including ecological momentary assessments (EMAs).

Behavioral JITAIs have three key features including providing the support in real time when the individuals are at risk of engaging in a negative health behavior or have the opportunity to engage in a positive one, adapting the timing and content of the support according to collected data like location of the user, and triggering the support by the system, not necessarily based on users' feedback (Nahum-Shani et al., 2014). The third feature is the one that distinguishes JITAIs from "just-in-time" interventions. That is, using sensors in the system to collect person-level data in addition to the EMAs provides the opportunity for adaptation and triggering the support dynamically (Spruijt-Metz and Nilsen, 2014). These three features that represent the ability of JITAIs to detect and deliver support at the right "moment" has led them to be implemented in various health related concerns such as physical activity (Hardeman et al., 2019), stress management (Sarker et al., 2016), fluid consumption (Conroy et al., 2020), weight loss (Forman et al., 2019), sun protection (Huh et al., 2021), addiction (S. P. Goldstein et al., 2017), and smoking cessation (McClernon and Choudhury, 2013) among others.

As a specific example, (Ben-Zeev et al., 2014) developed a smartphone behavioral intervention called FOCUS that can provide illness management support to individuals with schizophrenia. In this platform, four components of a JITAI that are decision points, intervention options, tailoring variables, and decision rules are as follows: (1) intervention decisions were made after each random prompt for self-report (if the individual did not respond to a prompt, it was interpreted as being not receptive and no decision was made); (2) intervention options consisted of recommendation on self-management techniques, and feedback and positive reinforcements; (3) tailoring variables were obtained via active assessments or EMAs by sending prompts three times a day to self-report; and (4) each decision point corresponds to a decision rule and the value of the tailoring variables determined the intervention options.

Although JITAIs use longitudinal data, decision rules for an intervention are mostly tailored on less frequent data from, for example, self reports. Driven by infrequent sampling in most JITAIs, there are assumptions about the change between assessments or the latency of predictions. Furthermore, JITAIs require large amounts of data, usually aggregated from all the participants to be able to develop a decision rule. Moreover, to the best of our knowledge, they do not take into account the "memory effect" of the tailoring variables. In this study, we address these limitations by identifying effective decision rules that are person-specific algorithms developed using high frequency sensor data. Applying these methods, "memory effect" is considered to provide decision rules considering not only the present moment of the tailoring variables but also their past moments that might lead to an event. As an example, if we want to investigate the effect of urge on eat, our aim is to identify person-specific dynamic models of eating responses to the urge. In this context, the tailoring variable is urge collected by high frequency sensors and decision points are the moments that urge level leads to eating (considering the past and present moments of urge).

In short, despite advances in designing just-in-time adaptive interventions, these techniques are still in their early stages (Nahum-Shani et al., 2016); some problems to be

addressed consist of: (1) What personal and/or contextual conditions represent risk? (2) How fast do these conditions change? (3) What intervention options can address these conditions? In other words, what are the tailoring variables and decision rules for selecting intervention options to modify proximal outcomes at a given moment?

Established Modeling Strategies to Inform Decision Rules for a JITAI

One method used in literature to analyze the time-varying relations between longitudinal variables is multilevel modeling of time-varying relations (Kim et al., 2018). Multilevel modeling (MLM) is an analytical method for analyzing intensive longitudinal data with time-varying and time-invariant covariates and an extension of traditional regression models that can handle within- and between-individual effects in the same model (Grimm et al., 2016; Schwartz and Stone, 1998; Snijders and Bosker, 2011). MLM addresses the limitation of traditional analysis techniques like analysis of covariance that aggregate data over time (e.g. repeated measures analysis of variance) and explores the effect of time-varying covariates as within- and between-person effects (Schwartz and Stone, 1998). Computation of the within-person effect can be done by creating a series of deviation scores from a personal average on the covariate. Therefore, the average amount of changes in an outcome associated with momentary deviation from a personal mean is represented by the effect of a time-varying covariate. While multilevel modeling provides a basis and is extendable to analyze time-varying effects (see Kim et al., 2018 for time-varying association between physical activity and affect), it is limited to simple change patterns that can be modeled with a relatively few parameters; e.g., linear or quadratic trends, or it becomes complicated in terms of implementation and interpretation.

In general, although multilevel modeling is useful for studying developmental processes, they are not ideal in modeling longitudinal data concerning daily affective, symptom measurements, or affective ratings in an observational study (Hamaker and Grasman, 2015). For such data, there is an interest in the dynamics of an stationary

process, i.e., changes in the process over time but not as a function of time. Multilevel autoregressive model can be used to handle this kind of data. Using this approach, the individual- and population-level autoregressive coefficients provide an overall description of the dynamic process for an individual and the average dynamic pattern, respectively (Liu, 2017). Even though multilevel autoregressive model considers the effect of previous data points on the later data points to model the process, the first- and second-order models (i.e. considering only one or two lags) have usually been used in the studies (Gistelinck et al., 2020; H. Goldstein et al., 1994). Additionally, when using this technique, the order of the model has to be predetermined. The dynamical system modeling approach applied in this study is capable of identifying how far to look back in time automatically. Moreover, multilevel modeling cannot leverage the interpretation tools that are provided via dynamical system modeling.

Time-varying effect model (TVEM) can address the limitation of MLM model on the simplicity of change patterns, since the shape of modeled relation does not require an a priori constraint and it only assumes a smooth pattern of change (Lanza and Linden-Carmichael, 2021; Shiyko et al., 2012). TVEMs are statistical approaches and extensions of linear regression models with the main difference being the model coefficients. That is, TVEMs estimate regression coefficients (i.e., intercepts and slopes) as flexible, nonparametric functions of time (Li et al., 2017). Success of TVEM in handling the challenges of EMA data and determining the time-varying effects has led it to be used in variety of behavioral and psychological science researches to explore the relation between smoking, drinking, marijuana, and physical activity, and various covariates such as depression, stress, age affect, and behavioral cognitions over time (Flannery et al., 2018; Koslovsky et al., 2020; Lanza and Vasilenko, 2015; Lanza et al., 2014; Maher and Dunton, 2020; Maher et al., 2016; Merrill et al., 2017; Shiyko et al., 2012; Snippe et al., 2017).

Although TVEM predicts the regression coefficients and allows the coefficients to be nonlinear functions of time, it has four limitations: (1) TVEM might not be able to

capture the unknown, latent variables that are also related to the outcome (Dziak et al., 2015). In other words, as TVEM is extended from a generalized linear model, it cannot determine more complex underlying within-time associations (Lanza et al., 2016) and is limited to contemporaneous relations in a single model. Similar to MLM, one can run multiple models to examine lagged associations, but that is not part of the core model. Tools applied from control systems engineering can fill this gap by incorporating the delayed effect of tailoring variables in the model. (2) TVEM requires a meaningful zero point. This works fine in intervention-based approaches or when age or some developmental transition provide the zero point, but it is more challenging with purely observational work outside of an intervention context. (3) TVEM does not include random effects and constrains all people to the same functional form. Dynamical models used in this paper are not constrained by the equivalence across individuals. (4) The impact of missing data is unexplored using this technique (Shiyko et al., 2012). The problem of missing data is handled effectively in the present study.

MixTVEM is a finite mixture version of TVEM that allows the population to have heterogeneous pattern of change (Dziak et al., 2015). Still, this model does not take into account the "memory effect". Another extended version of TVEM is differential time-varying effect model (DTVEM) that is capable of identifying time lags and estimating these lags within a state-space framework (Jacobson et al., 2019). To investigate, identify, and fit group-based vector autoregressive models of unknown lag structures, DTVEM combines some of smoothing and estimation tools for fitting generalized additive mixed models and state-space estimation routines. Although these capabilities make DTVEM more practical in comparison to TVEM, it still has two limitations. First, it requires the lag level to be predefined. Dynamical system modeling approach proposed in this study identifies the length of lagged effect automatically through solving the optimization problem. Second, similar to TVEM, DTVEM does not consider the person-specific differences in exploring time-varying associations, which is addressed in the current study.

To explore the dynamic characteristics of intensive longitudinal datasets that are in the form of time series, one approach is to analyze them in the context of stochastic differential equation models. These models are capable of extracting the key dynamics of data set as well as providing a platform to relate them to different covariates (S. M. Chow et al., 2016; Z. H. Lu et al., 2015). Dynamic factor analysis models are another type of models that can describe the structure and time-lagged relationships of unobserved psychological concepts (S.-M. Chow et al., 2004). The limitation of standard dynamic factor analysis models is that they can only be used for stationary time series. In other words, all modeling parameters have to be constant for the entire study period. However, nonstationary data are prevalent in practical studies (Houtveen and Molenaar, 2001). Additionally, these models require large amount of data that might not always be provided. Similarly, techniques like reinforcement learning methods require large datasets. This problem and their incapability in handling noisy data make these algorithms inappropriate for this kind of analysis (Liao et al., 2020).

Finally, we would mention a recent approach that aims at modeling intervention effects by leveraging “intensive” randomization of trial participants. This approach is referred to as micro-randomized trials (MRTs) (Boruvka et al., 2018; Klasnja et al., 2015; Nahum-Shani et al., 2018; Smith et al., 2017). In this type of trials individuals are randomized hundreds or thousands of times over the course of the study. The goal of these trials is to optimize mobile health interventions by modeling the effect of intervention components and assessing whether the intervention component effects vary with time or the individuals current context. Through MRTs one can gather data to build optimized JITAIs. Although this approach is efficient at obtaining models of context dependent intervention effects, it requires a large amount of data and, in our opinion, is not suitable for modeling individual behaviors.

In summary, although different models have been applied by researchers to study the behavior and substance consumption patterns of the individuals or groups of users due

to different stimuli over time, they all suffer from some limitations. In the approach proposed in this paper, we address the limitations mentioned above by taking into account the following criteria: First, the algorithms provided can leverage intensive high frequency longitudinal data gathered from sensors and activity monitors, providing the opportunity to investigate a range of moments immediately leading up to and following an event. Previous studies using intensive longitudinal data mostly relied on self-reports that might be too infrequent to capture and/or predict the moments of vulnerability, opportunity, and receptivity. Since mHealth tools and activity monitors provided the means to collect high frequency data, i.e. as frequent as every minute, existing analytic tools are not well-suited. Second, this study is based on the dynamic behavior changes of individuals. More precisely, we seek to identify the person-specific models describing the relation between different variables over time. Additionally, we consider the "memory effect" in the system modeling. In other words, dynamical modeling is done according to the concept that the outcome at a specific moment is not only related to the stimulus at the present moment, but it also is connected to the past moments of both the stimulus and the outcome. Additionally, the factor of how far to look back in time is not an a priori constraint. This parameter is identified by the algorithm and may be different for each individual. Third, to the best of our knowledge, no study has practiced a systematic way of handling the missing data in the datasets. Existing literature only relies on planned missing data designs by planning for reducing the number of data in time series as missingness by blocking irregularly spaced data (Silvia et al., 2014). This study provides a tool to handle the fragmented data; i.e., intervals of proper measurements accompanied by intervals of missing data.

Dynamical Modeling of Binary Outcomes

In this section, we provide a precise description of the assumptions on the data to be used and of the mathematical concepts central to the proposed modeling approach. The reader solely interested in application of our dynamical modeling algorithm can skip much

of the discussion of the mathematical underpinnings on the proposed algorithm.

Stimuli and Outcomes

For simplicity, in this paper it is assumed that our main objective is to model the relation between a longitudinal stimulus/cause/micro-intervention and a longitudinal outcome. No constraints are imposed on the possible values of the longitudinal stimulus. However, we assume that the outcome is binary; i.e., it takes value one if the event of interest occurred and zero otherwise.

As an illustrative example, throughout this paper we consider a hypothetical study on the effect of urge on eating. This running example assumes that we can measure/estimate urge to eat and we consider this as our stimulus longitudinal variable. The outcome considered is a binary longitudinal variable that, for each time instant, is one if eating occurs and zero otherwise.

Assumptions on Data

As mentioned in the Introduction, the objective of this paper is to provide tools for estimating the dynamic relation between a longitudinal stimulus and a longitudinal binary outcome. We assume that measurements are made at intervals of length T minutes. Hence, we represent the data as

$$\text{stimulus}(kT) \quad \text{and} \quad \text{outcome}(kT) \tag{1}$$

where k is an integer and kT represents the time at which the measurement was made.

We do not enforce continuous measurement of stimulus and outcomes. Data available is only assumed to be composed of what we refer to as “chunks.” More precisely, it is divided into sets where we have continuous measurements of the stimulus. Hence, for chunk i starting at time k_iT and of size n_i we assume that the values of

$$\text{stimulus}(k_iT), \text{stimulus}((k_i + 1)T), \dots, \text{stimulus}((k_i + n_i - 1)T) \tag{2}$$

are provided. It should be emphasized that we do not assume that, in these chunks, outcome is continuously measured; i.e., we can have a significant number of gaps in its knowledge. To provide consistent coding in outcome representation, at a given time kT , we allow three possible values for the outcome data

$$\text{outcome}(kT) = \begin{cases} 1 & \text{if outcome is True at time } kT \\ 0 & \text{if outcome is False at time } kT \\ -1 & \text{if outcome is Unknown at time } kT \end{cases} \quad \text{for all } k = k_i, k_i+1, \dots, k_i+n_i-1 \quad (3)$$

Back to our running urge and eating example. If we have the following data

$$\text{outcome}(T) = 1, \quad \text{outcome}(2T) = -1, \quad \text{outcome}(3T) = 0 \quad (4)$$

it means that eating occurred at time T , we do not know if eating occurred or not at time $2T$, and eating did not occur at time $3T$.

Remark 1 *The starting point of a chunk is not necessarily a point where a stimulus started. It just means that data is available on the stimulus continuously on a given interval, even if this information is that no stimulus has occurred.*

An example of such “data chunks” is provided in Figure 1. In this figure we have three chunks with urge available for every sample time in them (assumed sampling time interval $T = 1$ min). As discussed before, we do not assume that the outcome, in this case eating event, is known for all time instants, hence, we see a code of -1 when no information on eating is available. Note that no data is considered outside the chunks and inside each chunk, the length of the stimulus and outcome time series is the same and the stimulus and outcome variables are temporally aligned.

In the software package provided with this paper, we include a few data sets illustrating the type of data considered in this paper. The structure of these data is described in Section DOME Matlab Package. One should note at this point, that chunks of

very small size do not provide much information in terms of modeling. Hence, although these could be provided to the algorithm, they would only increase computational time without improving results. Although it is hard to precisely define a priori what a small size chunk is, a good rule of thumb is to not use chunks whose size is smaller than the expected complexity (as defined in the next section) of the model of behavior.

In the next section, we discuss the mathematical model used to represent the dynamical relation between stimulus and outcome. The readers not interested in the mathematical details of the proposed approach can “jump” to the section describing the code.

Dynamical Model

As mentioned in the Introduction, the objective of the analysis tools provided in this paper is to go beyond the “usual approach” to longitudinal data analysis where it is assumed that the stimulus only has an immediate effect on the outcome. This “immediacy assumption” might hold if the sampling time T is very large. However, in the case of intensive longitudinal data, this assumption usually does not hold; i.e., the value of the outcome observed at present is not only influenced by the present value of the stimulus but also by past values of it. For example, an urge event might not only result in an immediate increase on the probability of eating but might also increase the chance of eating in the near future.

To model this “memory effect” in the interaction of longitudinal variables, we propose to use the concept of dynamical systems. More precisely, to be able to handle binary outcomes, we propose to use an interconnection of a linear time invariant dynamical system (which has memory in it) in series with a non-linearity that provides the estimated value of the outcome. The proposed modeling technique is depicted in Figure 2. In this model, when at a specific time the output of the linear system is at or above one, then the outcome at that time is one and, hence, the event of interest is deemed to have happened.

If the output of the linear system is below one, then the outcome is zero and, according to the model, the event does not occur.

Traditionally, linear dynamical systems are modeled using difference equations. However, if one uses this type of models, the resulting problem is computationally complex, especially if one has missing data. Instead, in this paper, we use an impulse response representation that leads to more efficient computational algorithms. To be able to explain such an approach, let us first dwell on some results based on linear systems.

On Linear Systems

Before we provide a complete description of the proposed approach, let us first concentrate on the linear system whose input is the stimulus and the output is the intermediate variable $y(kT)$. The reader is referred to (Lathi, 2009) for a more in depth discussion.

From the definition of a linear system, the intermediate outcome $y(kT)$ can be decomposed as

$$y(kT) = y_{\text{int}}(kT) + y_{\text{stim}}(kT) \quad (5)$$

where $y_{\text{int}}(kT)$ is the intrinsic response of the linear system and $y_{\text{stim}}(kT)$ is the stimulus response. In control engineering jargon, $y_{\text{int}}(kT)$ is the zero-input response and $y_{\text{stim}}(kT)$ is the zero-state response. So let's look at what each of these terms represents.

The intrinsic response $y_{\text{int}}(kT)$ is the part of the response that is connected to the memory of past events, before the stimulus was applied. It is the outcome that one would expect to observe if no stimulus is applied. The stimulus response $y_{\text{stim}}(kT)$ is what the outcome is if there are no “significant past events”. In other words, it assumes that the system was in its “resting state” and describes what is the corresponding response to the stimulus.

But how can we determine these? It turns out that these two parts of the response have a very specific form. Let $p_1, p_2, \dots, p_{n_p}$ be the so-called poles of the system (again see

Lathi, 2009 for a precise description of linear system's concepts). These are complex numbers and different poles represent different types of responses that the linear system can exhibit. Poles close to the unit circle represent responses that “last” for a large amount of time. Poles close to zero represent very fast responses that “die down” very quickly. Also, if a pole contains an imaginary part different from zero and/or if its real part is negative, then there are responses that oscillate.

Given the poles of a linear systems, its intrinsic response has the form

$$y_{\text{int}}(kT) = c_{\text{int},1} p_1^k + c_{\text{int},2} p_2^k + \cdots + c_{\text{int},n_p} p_{n_p}^k \quad (6)$$

where the constants $c_{\text{int},1}, c_{\text{int},2}, \dots, c_{\text{int},n_p}$ depend of the state of the person before the stimulus occurred. The form of the stimulus response is slightly more complex. Let $c_{\text{stim},1}, c_{\text{stim},2}, \dots, c_{\text{stim},n_p}$ be constants associated with the way the person responds to stimuli. We define the so-called *impulse response* as

$$h(k) = c_{\text{stim},1} p_1^k + c_{\text{stim},2} p_2^k + \cdots + c_{\text{stim},n_p} p_{n_p}^k. \quad (7)$$

Assume that the stimulus was applied at time zero¹. The results below can be applied to any starting time. Then, the stimulus response is

$$y_{\text{stim}}(kT) = h(0) \text{ stimulus}(kT) + h(1) \text{ stimulus}((k-1)T) + h(2) \text{ stimulus}((k-2)T) + \cdots + h(k) \text{ stimulus}(0). \quad (8)$$

This operation is called convolution and is commonly represented as

$$y_{\text{stim}}(kT) = (h * \text{stimulus})(kT). \quad (9)$$

As one can see, memory of the system appears in two ways: i) in the term y_{int} where the information about what happened before the stimulus was applied is stored and ii) in the fact that the stimulus response y_{stim} depends not only on the present value of the

¹ This is a consequence of the assumption on time-invariance.

stimulus but also on its past values. For example, if we consider time $t = 3T$, then

$$y_{\text{stim}}(3T) = h(0) \text{ stimulus}(3T) + h(1) \text{ stimulus}(2T) + h(2) \text{ stimulus}(T) + h(3) \text{ stimulus}(0) \quad (10)$$

and, hence, the stimulus response at $t = 3T$ does not depend only on stimulus at $t = 3T$ but also on all its past values at times $2T$, T and 0 .

As mentioned above, the poles $p_1, p_2, \dots, p_{n_p}$ are complex numbers that are related to the dynamical behavior of the system. The more poles are used, the more complex the system's behavior is. Hence, in the formulation used in this paper, we always aim at using as few poles as possible so that we get the simplest explanation of the data and do not over-fit it. A final point about poles: we will assume that all the poles have magnitude less than or equal to 1. This ensures that the system is stable and does not “diverge” if long chunks of data are provided.

Dynamical Outcome ModEling (DOME)

Recall that the data is provided in chunks of continuous stimulus measurements. To be able to precisely describe the computational problem to be solved, we use the superscript i to denote the i -th chunk and its size by n_i . We assume that the total number of chunks is N_{chunks} . As mentioned before, and for simplicity of exposition, we assume that each chunk starts at time 0. The starting time does not change the outcome of the approach presented.² Hence, the data available for chunk i is denoted as

$$\text{stimulus}^i(0), \text{stimulus}^i(T), \dots, \text{stimulus}^i((n_i - 1)T) \quad (11a)$$

$$\text{outcome}^i(0), \text{outcome}^i(T), \dots, \text{outcome}^i((n_i - 1)T) \quad (11b)$$

Note that, since it is not known what happened in the times between chunks of data, we have a different intrinsic response for each chunk. This issue is magnified as the interval between measurements increases (common in EMAs which do not record continuously)

² This is a consequence of the time-invariance property of the modeling used (Lathi, 2009)

since the individual will be subjected to a large number of unmeasurable perturbations in between chunks. However, since we want to estimate a common model for all the data, the poles should be the same for all chunks and the impulse response h should also be the same. Moreover, since we want the least complex explanation of the data, we want to use the least possible number of poles. This leads to the following problem to be solved

$$\min_{p, c_{\text{int},p}^i, c_{\text{stim},p}} \text{ number of poles } p \text{ used} \quad (12a)$$

$$\text{subject to } h(k) = \sum_p c_{\text{stim},p} p^k \quad (12b)$$

$$y_{\text{stim}}^i(kT) = (h * \text{stimulus}^i)(kT), i = 1, 2, \dots, N_{\text{chunks}} \quad (12c)$$

$$y_{\text{int}}^i(kT) = \sum_p c_{\text{int},p}^i p^k, i = 1, 2, \dots, N_{\text{chunks}} \quad (12d)$$

$$y_{\text{int}}^i(kT) + y_{\text{stim}}^i(kT) > 1 \text{ if } \text{outcome}^i(kT) = 1 \quad (12e)$$

$$y_{\text{int}}^i(kT) + y_{\text{stim}}^i(kT) < 1 \text{ if } \text{outcome}^i(kT) = 0 \quad (12f)$$

$$c_{\text{stim},p} = c_{\text{stim},p}^*, \quad c_{\text{int},p}^i = \left(c_{\text{int},p}^i \right)^* \quad (12g)$$

In the last equation above, z^* denotes the complex conjugate of complex number z .

Lets carefully look at each of the terms of the problem above. First, note that the quantities that we are looking for are i) the poles to be used, ii) the corresponding coefficients for the impulse response $h(k)$, and iii) the corresponding coefficients of the intrinsic responses for each chunk. Now, (12a) just says that we are looking for the simplest explanation of the data provided; i.e., we want to use as few poles as possible.

In (12b), the impulse response of the linear system is defined. Note that this is the same for all chunks since the linear system used is the same for all of them. This leads to equation (12c), where the stimulus response for each chunk is built. Note that they all use the same $h(k)$ but compute the stimulus response according to the stimulus data provided for each chunk.

Equation (12d) provides the form of the intrinsic response for each chunk. Note that all of them use the same poles p but have different coefficients since the memory of past

events is different for each one of the chunks.

Equations (12e) and (12f) make sure that the output of the nonlinearity matches the observed outcome data; i.e., it is made sure that for each time instant the output of the linear system is above one if the outcome is one and that is below one if the outcome is zero. Finally, equation (12g) is a technical condition that ensures that the linear system identified has real output.

Although the problem above is the one that we would ideally like to solve, it is not computationally feasible because i) there are an infinite number of poles inside the unit circle and ii) minimizing the number of poles leads to a problem whose computational complexity cannot be handled by most computers, since this minimization problem is combinatorial and its complexity increases exponentially with the number of variables.

To circumvent these problems, we instead implement an approximation of the problem above. We start by gridding the unit circle and work only with n_p poles. Moreover, we relax the objective function using a well known approach from the literature on sparsity (Shah et al., 2012). More precisely, given unit circle grid $p_1, p_2, \dots, p_{n_p}$, we solve the following optimization problem, which we refer to as Dynamical Outcome ModEling (DOME)

$$\min_{c_{\text{int},p_j}^i, c_{\text{stim},p_j}} \sum_{j=1}^{n_p} c_{p_j} \quad (13a)$$

$$\text{subject to } h(k) = \sum_{j=1}^{n_p} c_{\text{stim},p_j} \alpha_{p_j} p_j^k \quad (13b)$$

$$y_{\text{stim}}^i(kT) = (h * \text{stimulus}^i)(kT), i = 1, 2, \dots, N_{\text{chunks}} \quad (13c)$$

$$y_{\text{int}}^i(kT) = \sum_{j=1}^{n_p} \alpha_{p_j} c_{\text{int},p_j}^i p_j^k, i = 1, 2, \dots, N_{\text{chunks}} \quad (13d)$$

$$y_{\text{int}}^i(kT) + y_{\text{stim}}^i(kT) \geq 1 + \epsilon \text{ if } \text{outcome}^i(kT) = 1 \quad (13e)$$

$$y_{\text{int}}^i(kT) + y_{\text{stim}}^i(kT) \leq 1 - \epsilon \text{ if } \text{outcome}^i(kT) = 0 \quad (13f)$$

$$c_{\text{stim},p_j} = c_{\text{stim},p_j}^*, \quad c_{\text{int},p}^i = \left(c_{\text{int},p}^i \right)^* \quad (13g)$$

$$|c_{\text{stim},p_j}| \leq c_{p_j}, \quad |c_{\text{int},p_j}^i| \leq c_{p_j} \quad (13h)$$

Careful choice of the scaling factors α_p which are constants and the use of equations (13a) and (13h) provide a computationally efficient way to “encourage” the use of the least complex explanation of the data provided. More precisely, we use

$$\alpha_p = \frac{1 - |p|^2}{1 - |p|^{2\bar{n}+2}} \quad (14)$$

where $\bar{n}$ is the average size of the data chunks; see (Yilmaz et al., 2018) for details on this choice of scaling factors. Moreover, $\epsilon > 0$ used in equations (13e) and (13f) is a small positive constant that allows for the use of non-strict inequalities which are more amenable for most optimization software packages.

Remark 2 *The introduction of the scaling factors α_p is connected to a long standing problem in the area of signal processing: when do solutions problems of the form (13a)-(13h) provide good approximations of the solutions of problems of the form (12a)-(12g)? In general there is no one answer to this problem. The scaling factors that are used in our approach have shown to have good numerical performance and, in the examples that we have run, do result in low complexity explanations of the data provided.*

DOME Matlab Package

The DOME modeling tool was implemented in Matlab (MATLAB, 2018) and can be downloaded from Github <https://github.com/Lagoa-s-lab/DOME.git>. It requires the Matlab Control System toolbox (available for downloading on the Mathworks website) and the CVX toolbox (Grant and Boyd, 2014, Grant and Boyd, 2008). It has a single command whose syntax is

```
>> Lin_sys = dome(chunk,Ts,r,prec,epsilon);
```

Lets look at the structure of each of the inputs

- **chunk** is a structured array whose size is the number of chunks and where each of the entries has two elements in its structure

`chunk(i).stim`

`chunk(i).out`

These are two arrays of the same size with the `chunk(i).stim` containing the values of the stimulus for the i-th chunk and `chunk(i).out` containing the corresponding outcomes.

Recall that each entry of `chunk(i).out` takes the value 1 if the outcome is true, 0 if it is not true and -1 if outcome is not available.

- **Ts** is the sampling period of the data, which is equal for all chunks.
- **r** is the degree of discretization of the unit circle (see discussion at the end of last section). The number of poles used is approximately proportional to r^2 .
- **prec** is a bound used to decide when a quantity is assumed to be zero. In other words, if $|c| \leq \text{prec}$ then we assume that $c = 0$.
- **epsilon** is the small constant ϵ used in the formulation at the end of last section.

We have just one output, **Lin_sys** which is the identified linear system in sys format.

An Example

We now provide an example of how to use the code. For simplicity, we can consider the stimulus as urge to eat and the outcome as eating, but this can be extended to any other variables too. Lets load the first data set provided in the package

```
>> load data_set_1;
```

This will load two variables into the workspace: **chunk** and **Ts**. As discussed before, the first one contains the longitudinal data collected that is stimulus and outcome data, i.e. urge and eating that are already divided into chunks. The second one is the sampling period used when collecting these data. This data set contains 100 samples of both the stimulus and the outcome and it is divided into three chunks of different lengths. Now execute the command that performs the estimation of the model

```
>> Lin_sys = dome(chunk,Ts,10,10^-2,10^-5)
```

The result obtained is the variable

```
>> Lin_sys
```

```
Lin_sys =
```

```
-0.009888 z^3 + 2.069 z^2 + 0.453 z - 0.09616
```

```
-----
```

```
z^3 + 0.3684 z^2 - 0.06925 z - 0.02551
```

```
Sample time: 1 seconds
```

```
Discrete-time transfer function.
```

which is a transfer function representation of the linear system identified by the algorithm.

In this case we identified a system with 3 poles or a system of order 3. In other words, the

algorithm has identified a very simple description of dynamic relation between the stimulus and the outcome i.e. urge and eating. The poles can be determined by using the command `zpkdata`

```
>> [z,p,k]=zpkdata(Lin_sys);
```

```
>> p{1}
```

```
ans =
```

```
0.2632
```

```
-0.3684
```

```
-0.2632
```

Interpreting the Results

The package DOME provides the linear part of the model that relates stimulus to outcome. In this section, we provide some ways of analyzing the results obtained. Since we are studying the dynamic response to stimuli, there are many characteristics that we can look at. For example, what is the delay in responding to stimuli, how strong is the response, how long does it last, etc... In this section we exemplify how can one find out how to precisely quantify some of these concepts.

The first step is to determine a “typical” stimulus that is expected to be observed often. In the examples provided in this paper, we selected a stress event of length 10 min, see Figure 4. This can be obtained by using the command

```
>> pulse_stim=[ones(1,10), zeros(1,40) ];
```

where the total length is the time horizon considered (in the example above 40 sample periods) so that we can see what is the response after the stimulus is applied. One should note at this point that performance can be studied for more than one stimulus. Since this

is done by simulation, one can in principle study the response to most of the stimuli that can occur.

To obtain the stimulus response for the generated stimulus, one can use the `lsim` command. More precisely, the following command can be used

```
>> ysim=lsim(Lin_sys,pulse_stim);
```

The resulting simulated stimulus response is depicted in Figure 5. Since we deem the event to happen if the output of the linear system

$$y(kT) = y_{\text{int}}(kT) + y_{\text{stim}}(kT) \quad (15)$$

is larger than 1, from a practical point of view, one can say that the stimulus response y_{stim} starts to have a significant impact when it is above 0.1. The time that it takes to reach this value is referred to as the delay in the response and denoted by t_d . This measures how long it takes for the stimulus to start having an effect. On the other hand, we have the time at which the stimulus response reaches 0.1 and stays below this value for the rest of the time. We refer to this as effective time and is denoted by t_{eff} . This is the largest time for which the stimulus has a measurable effect. Finally, one can look at the time that the stimulus has the biggest effect, in other words, when it has its peak response. We refer to this as peak time and denote it by t_{peak} . The peak value of the response is denoted by $y_{\text{stim,peak}}$. These quantities are depicted in the diagram in Figure 6.

Back to `data_set_1` and its pulse response depicted in Figure 5, one can see that the stimulus response has approximately no delay; i.e. $t_d \approx 0$. Moreover it has an effective time of approximately $t_{\text{eff}} \approx 12$ minutes. It has its peak at $t_{\text{peak}} = 1$ with a value of $y_{\text{stim,peak}} \approx 2.06$. Going back to our running example, this means that, if the individual experiences a 10 minute urge episode, there is an increasing chance of eating in the 12 minutes after the urge episode started. This chance of eating is largest at 1 minute after the urge episode starts.

One should note at this point that these are not the only characteristics that one can look at. Depending on the specific data that is being analyzed, there might be other quantities of interest.

Not all stimulus responses are as “simple” as the one just discussed. In `data_set_2` we provide a more complex data set with many chunks and many missing output measurements. Again, for testing purposes we consider a pulse of length 10 as the stimulus and the corresponding stimulus response is provided in Figure 7. In this case, we have two peaks in the pulse response, one at $t = 10$ and the other at $t = 25$. In this case, we have both an immediate and a delayed response to the stimulus. The first one starts immediately when the stimulus is applied and the second one starts around $t = 22$.

Discussion and Concluding Remarks

In this paper, we describe an approach to modeling the effect of a longitudinal stimulus on a binary longitudinal outcome that allows for delayed effects and can use fragmented data. The tools provided rely on recent developments on the area of parsimonious systems identification. We have developed an implementation of this approach and the MATLAB package developed is described and discussion of how to interpret the results obtained is also provided.

We should mention at this time that the discussed result interpretation is just one of many possibilities. The approach presented provides a computational model of the interaction between stimulus and outcome which can be used to simulate the response to many different types of stimuli. In other words, the interpretation provided here was aimed at studying the impact of a single event but the model can be used to study many other cases, such as the one where a stimulus event occurs periodically.

Also, we have not addressed directly the fact that there might be external perturbations that might influence the outcome. This can be addressed in multiple ways. If

the perturbation is known then it can be incorporated in the model as another stimulus; i.e., we can extend this framework to deal with multiple stimuli. If the perturbation is not known, then it can be modeled as noise and it can also be incorporated in the model. These do not change the basic approach described and we are planning on introducing these extensions in future versions of the DOME package.

The modeling tool provided, as any other tool, has its limitations. Given the fact that the stimulus response is modeled as a linear time-invariant system in series with a fixed non-linear term, it assumes that the effect of the stimulus on the outcome is proportional to the magnitude of the stimulus. Moreover, in its present form, it cannot address different stimuli at different time scales. Extensions of the proposed approach to address these problems are the subject of ongoing work.

In summary, we have shown that dynamical systems can be a powerful tool for modeling the interaction between longitudinal variables allowing for stimuli effect that “stretches” over time. The approach presented can leverage “real data” where one often has missing measurements and provides simple models that predict the response to stimuli. The models obtained can be used both to study how an individual responds to different stimuli but also as a first step in the process of designing just-in-time adaptive interventions. More precisely, once a stimulus response model is available, concepts from control engineering can be used to design effective robust interventions.

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Figure 1

Example of eating data with 3 chunks and $T = 1s$.

Figure 2

Dynamical system model.

Figure 3

Gridding the circle.

Figure 4

A stimulus pulse of length 10.

Figure 5

Pulse stimulus response for the system obtained from `data_set_1`.

Figure 6

Performance measures.

Figure 7

A more complex stimulus response.