Title:

The robust beauty of the majority rule by a small number of selected members

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Abstract

The principle of majority rule is a major decision-making strategy widely used in human and animal society. While majority rule yields highly accurate performance compared to individuals in the same situation, its performance is strongly undermined by a small correlation among individual judgments. We focus on the alternative, selected majority rule, which is the majority rule by a small number of committees selected from the large group. The present study investigated relative advantages of a smaller group of selected majority rule with computer simulation, systematically controlling 4 parameters—group size, distribution of individual performance (mean and variance), and correlation among decisions. The result showed that compared to simple majority rule and the single expert, the selected majority rule robustly yields higher performance against changes in these parameter values. Further, the absolute number of committee members contributes to robustness against the influence of group size. This robustness is undermined if the number of committee members increases proportionally to the number of total members, even though the committee members are a minority. Thus, an absolute number of group size is more critical than the proportion of selected members. This study indicates that the rationale for the majority decision of smaller committees is often fixed against the total number of members in the organization in terms of decision accuracy.

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Introduction

In both human and animal societies, many important decisions are made collectively. While we could potentially list numerous logically possible rules to aggregate individual opinions and decisions into a single group decision, majority rule, a group decision rule to choose the option supported by majority members, is often conventionally and ubiquitously adopted in human society as well as in the animal kingdom. Examples range from everyday casual small group decisions such as where to go out for dinner, democratic parliamentary votes for amending constitutions, nest-site choice in eusocial insects to synchronizing direction and timing of animal group movement (Conradt & List, 2009; Novak & Elster, 2014). The use of majority rule even appears in young children, and many researchers believe it is beneficial to use majority rule for controversial issues (Mann et al. 1984; Moessinger, 1981).

Though majority rule could be implemented by all available group members, it is often also the case that judicial or political decisions are made by small numbers of representatives or experts (Conradt & List, 2009; Galesic, Barkoczi & Katsikopoulos, 2016). For example, the US Supreme Court consists of nine representative members. The numbers of a monetary policy committee members of central banks (also called the bank's board or council) varies according as per the country, but most committee sizes are relatively small, ranging from 3 (Switzerland) to 9 (UK) members (Fracasso et al. 2003). Similarly, majority decision making by selected experienced individuals appeared in animals as well; for examples only adult individuals are involved in collective decision making in red deer and yellow baboons (Conradt & Roper, 2003). In terms of decision accuracy, group decision by a small number of experts may yield higher performances than whole groups, including members with lower performance. However, finding experts is not always easy, and this search process itself arises as a theoretical problem to be concerned (Goldman, 2001). Finding the best members is difficult in a realistic environment, especially when there is weak association between past performance and actual abilities (Denrell & Liu, 2012). Considering errors mistakenly nominating members with lower ability as experts, majority rule with a selected small number of individuals may yield lower performance compared to simple majority rule.

The present study investigated the performance of selected majority rule against simple majority rule with all available group members. At first, we review classic studies that evaluated performance of the simple majority rule and introduce studies that argued that selected majority rarely outperforms the simple majority rule. However, it is generally accepted that wisdom of crowds attained by majority rule is significantly deteriorated by correlations among individual decisions, and this study highlights that the conclusion of inferiority of the selected members will be moderated by this factor. We conducted computer simulations in which three population-level statistical factors were systematically varied—means and variances of individual performance and correlation among individual decisions.

Wisdom of crowds and error canceling effect of majority rule. The positive evaluation of majority rule based on its performance has been discussed in the context of 'wisdom of crowds' (Landemore, 2013; Surowiecki, 2004). 'Wisdom of crowds' is a term coined to indicate a situation wherein each member's decision-making performance is relatively humble, yet group performance aggregating individual opinions yield higher performance. This phenomenon has long been discussed in humanities-related fields like classical literature and social sciences (Landemore, 2012; Ober, 2013), and the first empirical demonstration was conducted by Galton (1907a, 1907b). In Galton's demonstration, each participant independently infers the true value of an ox's weight, and later

aggregates the individual judgments statistically. Galton found that median (Galton, 1907b) and mean (Galton, 1907a) of the whole group was closer to the true value and more accurate than any individual experts' estimations. This result is later replicated by psychologists and biologists in other domains, such as inference of room temperature or the number of marbles in jars (for review see Krause et al. 2010; Lorge et al. 1958).

This wisdom of crowd effect is interpreted as a purely statistical phenomenon (Hogarth, 1978; Lorge et al., 1958; Rauhut & Lorenz, 2011; Stroop, 1932). Referring to the central limited theorem, Hogarth (1978) statistically explained that group judgment (i.e., averaging of individual judgments) is expected to be more accurate than a typical member of the group. Let us illustrate Hogarth's (1978) explanation using a scenario analogous to Galton's experiment (Galton, 1907a, 1907b). Suppose a continuous variable which has true value, such as the weight of an ox, is estimated as t . Let an individual i 's estimates of the ox's weight be x_i , which consists of true value and random error, ϵ_i , normally distributed around a mean of 0 and statistically independent among individuals ($x_i = t + \epsilon_i$). As the error ϵ_i is independent and identically distributed, according to the central limit theorem, group judgment $\bar{x}_i$ (mean of individual judgment x_i) is more accurate than a typical individual judgment when group members are large enough (Hogarth, 1978). This phenomenon can be understood as cancelation of an error term; by law of large numbers, as group size (N) increases, mean of an error term among N -many members ($\frac{1}{N} \sum_i^n \epsilon_i$) approaches the expected value of an error term, which is 0 in this case (Hogarth, 1978).

Discussed above is a version of the wisdom of crowd effect, which is applied only when a target of estimation is a continuous value (e.g. the weight of an ox), but a similar wisdom of crowd effect also arises when the target is discrete, such as aggregating answers of yes/no binary questions using majority rule. Condorcet first argued wisdom of crowds formed by majority rule as a mathematical theorem known as Condorcet Jury Theorem (CJT) (Condorcet, 1785, 1994). CJT is based on two premises: (1) there are two options, correct and incorrect, and each individual in a group finds a correct option with a probability p , which is higher than 0.5; (2) judgments of individuals are statistically independent. As the number of group members, N , increases, probability of a group to find the correct answer, P_G , approaches 1. This wisdom of crowd effect in majority rule can also be understood as the error cancelation as with averaging individual estimations of continuous values. Assume that the probability an individual can find a correct option is $p = 1 - \epsilon$, where $0 < \epsilon < 0.5$. It has been discussed that the wisdom of the crowds of the majority rule can be explained as an effect of cancelling out error ϵ (Hastie & Kameda, 2005; Moore & Shannon, 1956; Sorkin et al. 1998, 2001).

Majority rule compared to the selected experts

One constraint of CJT is in the assumption of homogeneity in individual performance so that accuracy of detecting the correct option, p , is identical across group members. By definition, the performance of the majority's decision robustly outperforms any single individual group member under the assumption (e.g., Surowiecki (2004). However, once heterogeneity in individual accuracy is assumed, majority rule does not necessarily outperform the best member in the group, while it can beat typical group members (Conradt & List, 2009; Grofman, Owen & Feld, 1983; Luan et al. 2012). Conradt & List (2009) raised an example when the best member of the group outperforms the performance of the majority rule. First, assume a group of five members whose accuracy, p , is all 0.75. In this case, group performance based on majority rule, P_G , is equal to 0.90 as is calculated in the following equation:

$$\sum_{i=3}^5 \binom{5}{i} 0.75^i \cdot 0.25^{5-i} \approx 0.90.$$

Next, assume that accuracy of four out of the five groups members is 0.60 and one member is 0.75. In this case, the performance of majority rule is approximately 0.73, which is lower than the performance of the best members ($p = 0.75$):

$$0.75 \cdot \sum_{i=2}^4 \binom{4}{i} 0.60^i \cdot 0.40^{4-i} + 0.25 \cdot \sum_{i=3}^4 \binom{4}{i} 0.60^i \cdot 0.40^{4-i} \approx 0.73 .$$

As illustrated in the equation, when the performance of individuals is not homogenous, the most capable member may outperform the performance of the majority rule.

If the performance of all group members is identical, by the definition of CJT, the majority rule with all group members always outperforms selected majority rule. If this assumption does not hold, the performance of selected majority may outperform the simple majority rule. Goodin & Spiekermann (2012) investigated the relationship between the simple majority and the selected majority rules based on models developed by Grofman and his colleagues (Grofman (1978), Grofman et al. (1983): Grofman Dummkopf-Witkopf Theorem). Grofman (1978) considered trade-offs between group sizes and individual performance and showed that $N+y$ many group members with individual performance $p-x$, perform equivalent to N many group members with individual performance p when the following condition is satisfied:

$$y = N \left[\frac{0.25x(2p-1-x)}{p(1-p)(p-x-0.5)^2} \right].$$

Consider that a group of five members whose performance is 0.75 makes a decision with the majority rule. The above condition indicates that in order for another group of individuals whose performance is 0.6 to achieve equivalent performance, group size needs to be 40. Based on this formula developed by Grofman (1978), Goodin & Spiekermann (2012) discussed the disadvantage of the majority rule based on a small number of selected members. As an illustration, they considered the group of 1,001 members whose performance is 0.55. In order for 11 representatives to perform equivalent to the larger group, it needs to find members whose performance is 0.80. Goodin & Spiekerman (2012) argued that finding 11 members whose performance is 0.80 is relatively hard compared to finding 1,001 members whose performance is 0.55. Goodin & Spiekerman (2012) further argued that if the group size increases from 1,001 to 100 million, the performance of representatives needs to be almost 1.

Majority rule in an inheritably interdependent world

As discussed in the previous section, the classic argument about superiority of majority rule over selected majority is supported by the CJT, which assumes homogeneity in individual performance. Once this assumption is eliminated, there is possibility that selected majority with superior group members outperforms simple majority. Thus, the analyses by Goodin & Spiekermann (2012) could not be applied to heterogeneous population. Further, the model of Goodin & Spiekermann (2012) was also based on another assumption, that individual decisions made by group members are all statistically independent. The importance of the independence of judgment has been discussed especially in the modern context of the wisdom of crowds (Dietrich & List, 2004; Ladha, 1992; Lorenz et al. 2011). For instance, CJT predicts that group performance always increases with adding new members to a group, but this addition of members leads to a different conclusion if the assumption of statistical independence is removed. Owen (1984) revised CJT and introduced statistical dependence among individual decisions by assuming that each individual follows the opinion leader of the group in probability α whose performance is also p . As stated, the probability of finding the correct option is identical across group members as assumed in the original CJT. With this new assumption introduced by Owen (1984), correlation among individual judgments emerges (α^2 in terms of the Pearson correlation coefficient) due to social influence.

[Figure 1 to be placed here]

Figure 1 shows how the performance of majority rule changes as a function of group size and degree of correlations. It is shown that even a small degree of correlations among individual judgments (i.e., $\alpha = 0.3$ and Pearson's $r = \alpha^2 = 0.09$) deteriorates the performance of majority rule and accuracy of group decisions decrease as group size increases. It is established that statistical non-independence hampers wisdom of the crowd (Ladha, 1992; Lorenz et al., 2011; Owen, 1984), and this has been one of the main topics in a modern research on the wisdom of crowds.

While statistical dependence of judgments is often assumed to be derived from social influence among group members (Lorenz et al., 2011; Owen, 1984), statistical dependence can emerge even when individuals makes decisions without any social interaction with other members (Dietrich & List, 2004; Ladha, 1992). A typical case of such situations is where common or correlated cues (information) are used for making individual decisions (e.g. predicting tomorrow's weather based on today's weather or glancing at the night sky: Dietrich & List, 2004; Gigone & Hastie, 1997; Ladha, 1992). Figure 2 illustrates such cue-based decision-making situations.

[Figure 2 to be placed here]

When the assumption of statistical independence among individual decisions is removed, the argument by Goodin & Spiekermann (2012) may not be supported since adding new members decreases the performance of majority rule, and the performance of few selected experts may yield higher performance. In fact, several theoretical studies that revised CJT reported that a smaller group can outperform a larger group even in the situation where members have the same competence (Galesic et al. 2016; Kao & Couzin, 2014; Owen, 1984).

Wisdom of crowds of selected experts

Above discussions proceeded by relaxing assumptions in the CJT, homogeneity in individual performance and statistical independence among individual decisions, and sketched a possibility that selected majority can outperform simple majority. In the present study, we conducted a series of computer simulations to thoroughly investigate conditions under which small numbers of experts can outperform the simple majority rule. Our study is distinct from previous studies in explicitly incorporating the processes of identifying experts from group members into a model.

Prior to the current study, Mannes et al. (2014) compared performances of a small sub group of experts with a whole group in a context of averaging judgments of continuous values. In their theoretical study, they controlled parameters related to individual accuracy and correlation among individual judgments and showed that small group of experts outperforms robustly under various combinations of parameter values (Mannes et al. 2014; they further took into account the problem of identifying a small number of selected best members). Though our study is close to research settings investigated by these studies, there are considerable theoretical gaps between averaging and majority rule as Mannes, Soll, & Larrick (2014) remarked that majority voting is an interesting topic to explore in their conclusion. In our study, we used computer simulations for incorporating the process of inferencing experts into a model without adding a new parameter of finding experts, as Mannes et al. (2014) implemented in their theoretical (see the section of *Selecting M-many Experts* for details). The rationale behind this decision is in that

the accuracy of finding experts (capable members) is not independent of other variables. For instance, finding capable members should be difficult when the variance in individual accuracy is small. In our computer simulations, we did not implement the probability (or ease) of finding capable members as an exogenous variable. Instead, we assumed that capable members are selected based on past performance, which will be later explained in detail.

Previous studies that extended the CJT demonstrated that, besides group size, three statistical features influence performance of the majority rule: mean (Miller, 1996) and variance (Grofman et al. 1983; Kanazawa, 1998) of the distribution of individual accuracy in finding a correct answer, and correlation (Boland, et al. 1989; Galesic et al. 2016; Owen, 1984) among resulting individual decisions. We also controlled these parameters together systematically through agent-based computer simulations. In the beginning of the next section, we will summarize how each of the three key parameters (mean and variance of individual performance and correlation among members' decisions) influence performance of majority rule, best member rule, and processes of selecting experts from the group. It will also be explained that selected majority rule of experts contains both features of majority rule and the best member rule.

Illustrating the Roles of Key Parameters in Computer Simulation

Consider a normal distribution of P_i , a probability of an individual i to find a correct option out of the two options. We illustrate how mean and variance of distribution and correlation among resulting individual decisions influence performance of majority and best member rules.

Mean. Let p be the mean of the distribution. The mean has significant impact on the majority rule compared to the best member rule. According to CJT which assumes $p = p_i$ for all individuals i , in order for the majority rule to outperform an individual performance, individual competence p_i needs to be higher than 0.5. Further, consider a group of five individuals ($N = 5$) whose competence is $p_i = 0.55$ or 0.75. When the group competence is $p_i = 0.55$, the performance of majority rule is $p_i = 0.59$, which is 0.04 point higher than an individual. In contrast, when group competence is $p_i = 0.75$, competence of majority is $P_G = 0.90$, which is 0.15 point higher than an individual. As demonstrated by this calculation, means of group competence influence how groups can improve their performance compared to typical individuals by using the majority rule. Thus, higher mean is advantageous for majority rule.

Variance. A degree of dispersion of individual competence in a group can be described by variance (or standard deviation). A variance of competence influences both, the performance of majority rule, and the best member rule (Owen et al. 1989). For majority rule, an extended model of CJT that relaxed homogeneous assumption that $p = p_i$ for all i demonstrates that performance of majority increases as variance increases, when the mean competence $\bar{p}$ is fixed (Kanazawa, 1998). Consider two groups of three individuals whose competence varies among three individuals while mean competence is identical: a group with smaller variance ($p_i = 0.69, 0.70, 0.71$) and a group with larger variance ($p_i = 0.51, 0.70, 0.89$) with the same mean competence ($\bar{p} = 0.70$). A performance of majority in the group with larger variance is $p_i = 0.80$, which is higher than performance in a group with smaller variance, $p_i = 0.77$. Therefore, majority rule performs better with larger variance. Performance of the best member rule, however, tends to be more powerfully influenced by variance. As demonstrated in the example, the performance of the best member of a group with smaller variance is 0.71, only increasing 0.01 from the mean competence ($\bar{p} = 0.70$). In contrast, in the group with larger variance, performance is 0.89, increasing by 0.18 compared to the group

of smaller variances. Thus, higher variance is advantageous for both, majority rule and best member rule, yet influence is stronger for best member rule.

Correlation. As discussed, correlation among individual decisions decreases performance of the majority rule, but for the best member rule, correlation has no influence as long as the true best member is always chosen. However, as our computer simulations will demonstrate, this assumption may not hold true when the members must identify the best member from a past performance.

As explained, mean, variance and correlation have different influences on performance of both, majority rule and best member rule. The next section evinces that the selected majority rule contains both features of the best member and majority rules, and influences of three parameters on the relative advantage of the selected majority rule over the simple majority rule is not straightforward. Next, we will investigate parameter space where selected majority rule outperforms other decision rules.

Computer Simulations

Basic Settings

For a simulation, we assume that a group consisting of N -many individuals make binary choices for 100 trials, where one option is correct and the other is incorrect. At the beginning of each round, all group members make choices individually. Then, a group makes a decision by aggregating individual choices following a certain group decision rule. Therefore, the performance of a group decision solely depends on decisions of individuals and aggregating rules for group decision making. The performance of individuals is given by a probability distribution. The mean competence of an individual i (p_i) is given by a value randomly sampled from normal distribution $N(\mu, \sigma^2)$ and does not change across the trials. Values outside the range between 0.40 and 0.99 were truncated and resampled. μ and σ^2 were controlled as simulation parameters.

First, we created 1,000 groups of N -many members. Each member's competence, p_i is sampled from the probability distribution. Based on individual competence p_i , a decision matrix is created for each group. A decision matrix consists of N rows of group members and 100 rows of trials, where the value of each cell X_{it} signifies decision outcome of member i in trial t . The value of a cell is either correct (1) or incorrect (0). The correlation of each group is controlled by randomly replacing the value of X_{iy} and X_{iz} in the decision matrix, where y and z signify the trials randomly selected. Repeating this procedure enabled the production of populations of a wide range of correlation while competence of each individual p_i is kept identical (See supplementary material for an example of this procedure). We repeated this procedure until average correlations among individual decisions reached a certain level. Correlation was calculated by Yule's coefficient Q . The population is clustered into 10 groups based on the value of Yule's Q (i.e. 0.0 to 0.1, 0.1 to 0.2, 0.3, to 0.4... and 0.9 to 1.0).

Group decision making. Performance of each decision rule was calculated from a decision matrix as illustrated above. In this simulation, we calculated performance of 3 decision rules: simple majority rule, selected majority rule, and best member rule. In the decision process of simple majority rule, all N -many group members participated in voting. In the case of selected majority rule, N -many members were selected as a committee (see below for details) and only committee members participated in voting. In both the simple majority and the selected majority rules, a group can make a decision correctly if more than half the participants (N for simply majority, M for selected

majority) choose a correct option. For the best member rule, a group followed the choice of a single selected member (see below for the details). We also recorded the performance of the randomly chosen member as the baseline.

Selecting M -many Experts (committee size). Best member rule and selected majority rule requires the process of selecting experts. Here, we considered the process that individuals selected experts for those who yielded higher performance in past decisions. In order to implement this procedure, we separated 100 trials into learning sets (50 trials) and training sets (50 trials). In the best member rule, a group selected the highest performer in the learning set as the expert or best member, and then followed decisions of the selected member. In the selected majority rule, a group selected M -many higher performers in the learning set. Note that the best member in our study is not the member who has the highest p_i value in a group. Accordingly, this process does not require any external variables (such as prestige) and matched psychological and anthropological behavior observed in the experiment or field setting, known as ‘copy the most successive’ or ‘pay-off bias’ (Cheverud & Cavalli-Sforza, 1986; Laland, 2004; Mesoudi, 2008). Given that M -many members are selected based on performance in the training set, our definition of best member allows fair comparison at least in our context. Note also that selected experts (based on past records of decisions) have been used in the study comparing decisions of group average and best member (Mannes, Soll & Larrick, 2014). The whole process was replicated 100 times, and average value of each strategy was calculated.

Results

We present the result of simulations in two steps. First, we have an initial expectation about how the three key parameters (mean, variance, and correlation) are likely to influence majority rule and the best member (see the last section of Introduction). For the first section, we focus on how selected majority rule, the rule with features of both majority rule and best member, were influenced by three key parameters. Thus, group size was fixed in relatively small numbers ($N = 11$) to focus solely on the effects of mean, variance, and correlation. In the following sections, we present results of larger group sizes to confirm if the result in the first section can also be applied to larger groups. In increasing group sizes, we explored the following two versions. In one version, we fixed committee sizes and increased the group size. In the other version, we increased committee size proportionally to the group size. Both versions were considered in order to explore if exact number or proportion is effective in yielding higher performances for selected majority rule.

In order to illustrate the relative performance of the selected majority over best member and majority rules, we picked up three environments (i.e., parameter combinations of mean and variance) in a group of $N = 11$ while the number of selected experts (committee size) was either $M = 3$ or 7. The three environments were as follows. (A): Advantageous environment to the majority rule, which has higher mean ($\mu = 0.85$) and lower variance ($\sigma = 0.05$). As discussed in the introduction, higher mean increased performance of majority rule, and lower variance sharply diminished performance of best member rule. (B): Advantageous environment to the best member rule, which has lower mean ($\mu = 0.55$) and higher variance ($\sigma = 0.20$). This environment is opposite to the environment (A). Finally, we illustrated (C): Natural Environment, which has low mean ($\mu = 0.55$) and low variance ($\sigma = 0.05$). This environment is not advantageous for both rules, and its relative advantage depends on correlation.

[Figure 3 to be placed here]

In scenario (A) of the advantageous environment for majority rule ($\mu = 0.85$, $\sigma = 0.05$; Fig. 2A), majority rule has

relative advantages compared to best members who were outperformed in 6 points (from $Q = 0.0$ to 0.6). The performance of majority rule decreased as correlation decreased. When the value of Q is larger than $0.7\sim 0.8$, the best member rule outperformed the majority rule. The performance of the selected majority similarly decreased as the correlation increased. However, the selected majority rules were more robust compared to majority rule and yield higher performance when the correlation is high. The selected majority rule with $M = 3$ experts suffers less from the correlation compared to when $M = 5$. In general, unlike with majority rule, selected majority tends to exhibit less extreme performance decline due to the correlation, and is more likely to outperform the best member. In scenario (B) which is advantageous for the best member rule ($\mu = 0.55$, $\sigma = 0.20$; Fig. 3B), the simple majority rule could not outperform any rules other than the random member. Although both selected majority rules with $M = 3$ and 7 could not outperform the best member rule when correlation exists, the smaller the number of selected members, the more it approaches the performance of the best member rule. Thus, selected majority is more robust in this setting. Finally, in the scenario of (C) ambiguous environment ($\mu = 0.55$, $\sigma = 0.05$; Fig. 3C), majority rule outperformed the best member rule when the correlation is low. As correlation increases, performance of majority rule decreases while best member rule increases. However, in these situations, both the selected majority rules with $M = 3$ or 7 are less influenced by the correlation and could maintain relatively higher performances; although the selected majority cannot defeat the best member rule and majority rule when correlation is either in very low or high environments. In summation, the selected majority rule, which has both features of the best member rule and the simple majority rule, exhibited robustness against changes in three parameter values.

We then explored the effect of group size under a variety of combinations of parameters. First, we considered the effect of the group size, by fixing the number of a committee ($M = 3$ or 7) and increased group size ($N = 41$ or 101). Figure 4 shows the result of the simulation. The performance of majority rule was undermined more severely by correlation as group size increased. However, the performance of selected majority rule suffered less from increases in correlation. For example, when group size is 101 , the smaller selected majority rule ($M = 3$) outperforms the best member more on each point of correlation, in all the 16 combinations of mean and variance ($\sigma = 0.05, 0.10, 0.15$ or $0.20 \times \mu = 0.55, 0.65, 0.75$ or 0.85). Thus, especially when the group size is large, a small group of experts is more robust against influences of correlation and outperforms the best member rule for a wide range of parameters of mean and variance.

[Figure 3 to be placed here]

As seen in Figure 4, we increased group size (N), while group number was fixed (M). In this situation, selected majority rule with a small number of committee members ($M = 3$ or 7) outperforms both majority rule and best member rule for a wide range of parameters. However, we have not yet explored cases where committee size (M) also increases as group sizes increases. Thus, we also increased committee size (M) in proportion to group size (N), and observed if selected majority rule can also outperform the other two rules in these cases. We fixed the proportion of committee size to group size (M/N), as $3/11$ (for a smaller committee), and $7/11$ (for a larger committee), then we increased the group size to 33 (smaller committee size: $M = 9$; larger committee size $M = 27$) and also to 99 (smaller committee size: $M = 27$; larger committee size $M = 63$). Figure 5 presents results of the simulation. Unlike the previous simulation in which the committee size was fixed with small members (Figure 4), the result indicated that when the committee size increased, performance of the selected majority rule was vulnerable to correlations like the majority rule (Figure 5). This result suggested that in order to expect higher and stable performances for the selected majority rule, the committee size should be fixed to small

numbers such as 3 or 7 regardless of whole group size ($N = 11$, or $N = 101$), and the committee size should not be expanded proportionally to the population.

[Figure 4 to be placed here]

Discussion

Present study investigated the performance of the selected majority rule by systematically controlling three key parameters: performance of individual distribution (mean, variance) and the correlation among the judgments. The result showed that the selected majority rule of small members robustly yielded high performance under a wide range of parameter space, while performance of majority rule fluctuated depending on parameter values. The majority rule and best member can outperform selected majority but only in certain limited parameter space. For example, when people are certain that competence of all members are almost similar and judgements are not strongly correlated (e.g., people tend to use different cues independently), the majority rule should be the preferable decision rule. However, in most other cases, selected majority rules can robustly outperform the majority rule. Our result is consistent to the previous study which also found that small numbers of groups robustly yielded good performance in cases where individual estimation are aggregated by averaging individual judgment (Mannes, Soll & Larrick, 2014). Mannes, Soll, & Larrick (2014) evinced an average of five experts among the group can robustly yield a wide range of environments. Thus, agreeing with the previous study on aggregating judgment with average, our result demonstrated the advantage of group performance with small numbers of experts.

Further, our result indicated that a small committee size is important for good performance of the selected majority rule, and committee size should not be enlarged proportionally to the whole population. This may be the reason why decisions made by experts are ubiquitously observed in animal and human society (Conradt & List, 2009), and also why representatives such as monetary committees of central banks are limited to small numbers across countries (Fracasso et al. 2003) even though the whole population size may vary in situations. The present result is contrary to the theoretical implication of Goodin & Spiekermann (2012) that proposed that as group size increases, majority rule with all members is more advantageous to the majority with selected committee members. Our result showed that the observation of Goodin & Spiekermann (2012) is only confirmed in an area of limited parameter regions where only three conditions are simultaneously satisfied: low mean competence, low variance, and low correlation. While Goodin & Spiekermann (2012) raised an example for 1,000 or 100 million population size as advantageous conditions for simple majority rule, our result showed that population sizes had the opposite effect for most cases: as population size increases, selected majority rule becomes more advantageous to the simple majority rule with whole members.

In the current study, we focused on decisions where all individuals gained the same benefits by choosing correct options (such as investing in the firm to yield higher benefits, or choosing food patches with more resources). However, our study did not consider cases where individuals have conflicts of interest (Austen-Smith & Feddersen, 2009). A recent theoretical study conducted by biologists and political scientists considered the scenario where a group has a common goal with this study (e.g., arriving at the same destination), but minor conflict of interest existing among individuals (e.g., costs of routes to the destination are different across individuals: Conradt et al., 2013). In this scenario, Conradt et al. (2013) reveal that minor conflicts in individual interests increased decision

accuracy for the common goal as long as the conflict is relatively weak. It would thus be interesting to explore how conflict of interest influences decision accuracy.

Present research indicates that the selected majority rule robustly yields high performance within a wide range of parameters, and this confirmed the advantageous decisions of smaller numbers of experts (Mannes, Soll & Larrick, 2014). However, since this study is limited to cases where no conflicts of interest exist, further studies exploring this problem are needed.

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Figure 1. The performance of a group (P_G) in various group size (N) and correlation (α). The figure was created by the authors based on Owen's (1984) theoretical models (not by the approximation formula in the paper). The figure plotted only odd number of group size.

Figure 2. The schematic image of group decision making (Gigone & Hastie, 1997; reproduced by the authors). Where X signifies true state of the environment to be inferred from cues, C1, C2, C3, that are statistically correlated to the value of true state. Three individuals infer true values from values of the three cues and J1, J2, J3 signifies values estimated by each individual. G signifies the aggregated decisions of three individuals. The figure shows that even though each individual makes a decision without interacting with each other, correlations among individual decisions will emerge due to the common usage of the cues. Furthermore, even in a situation where one individual used only one cue without any overlaps of cues, correlation of judgments remains because three cues are statistically correlated with the true value X.

Figure 3. The result of computer simulations when group size is $N = 11$. (A): Advantageous environment for majority rule ($\mu = 0.85, \sigma = 0.05$). (B): Advantageous environment for the best member rule ($\mu = 0.55, \sigma = 0.20$). (C): Neutral environment ($\mu = 0.55, \sigma = 0.05$)

Figure 4. Results of fixing the number of the committee (M) and increasing whole group size (N). For the result of other committee sizes (M). See supplementary Figure S1.

Figure 5. The result where committee size (M) increased proportionally to whole group size (N). For the result of other committee sizes (M) in the larger population size ($N = 297$), see supplementary Figure S2.

Supplementary Materials of ‘The robust beauty of the majority rule by a small number of selected members’

Yo Nakawake & Masanori Takezawa

Manipulation for controlling correlation in Simulation

In our study, we controlled correlation among member judgement by randomly shuffling the decision matrix of a group. Here, we illustrated this process with an example. We will assume a group of three members whose competences (p_i) are 0.7, 0.7, 0.8 who made decisions for 10 trials (we used the small trial numbers for an illustration, although trials were fixed as 100 in our actual simulation). Based on individual competence, we made decision matrix as below (Table S1). A decision matrix consists of 3 rows of group members (Member A, B, C) and 12 trials, where a value of each cell X_{it} signifies decision outcome of member i in trial t . The value of a cell is either correct (1) or incorrect (0). First, we assume that each individual made correct decisions proportionally to his/her competence. If a member's competence is 0.6, then the expected number of correct decisions is 7.2 (0.6×12 trials). In this case we established that this individual made the correct decision for 7 trials (cutting off after the decimal point). Based on the competence of each member, we filled the decision matrix as below (Table S1), in the way that each pair of group member has perfect correlation (Yule's $Q = 1$). We set this perfectly correlated decision matrix as the initial state to be manipulated.

Member (i)	p_i	Trial (t)											
		1	2	3	4	5	6	7	8	9	10	11	12
Member A	0.6	1	1	1	1	1	1	1	0	0	0	0	0
Member B	0.7	1	1	1	1	1	1	1	1	0	0	0	0
Member C	0.8	1	1	1	1	1	1	1	1	1	0	0	0

Table S1. Decision matrix wherein each pair has perfect correlation

Next, we manipulated correlation by randomly shuffling cells. For each member, we picked one cell with the value 0 and one cell with the value 1, and swapped the values of these two cells. For example, assume that for member A, two cells of trial 4 and trial 9 were randomly selected ($X_{A4} = 1, X_{A9} = 0$). Then, the value of two selected cells were swapped ($X_{A4} = 0, X_{A9} = 1$). The result of this example is shown in Table S2.

Member (<i>i</i>)	p_i	Trial (<i>t</i>)											
		1	2	3	4	5	6	7	8	9	10	11	12
Member A	0.6	1	1	1	0	1	1	1	0	1	0	0	0
Member B	0.7	1	1	1	1	1	1	1	1	0	0	0	0
Member C	0.8	1	1	1	1	1	1	1	1	1	0	0	0

Table S2. Decision matrix after member A's decision cells were swapped. Gray color in cells signifies that these cells were swapped.

We conducted the same process for all members (also Member B and C). We presented the decision matrix and applied this swapping process for all members for one time.

Member (<i>i</i>)	p_i	Trial (<i>t</i>)											
		1	2	3	4	5	6	7	8	9	10	11	12
Member A	0.6	1	1	1	0	1	1	1	0	1	0	0	0
Member B	0.7	1	1	1	1	1	0	1	1	0	0	1	0
Member C	0.8	1	0	1	1	1	1	1	1	1	0	1	0

Table S3. Decision matrix after all members' decision cells were swapped. Gray cells signify cells which have been swapped.

Following this process, correlation among participants decreased. In Figure S3, the correlation among A and B, Q_{AB} is 0.25; and also, Q_{BC} is 0.75; Q_{AC} is 0.6. Thus, average value of correlation among individual judgment is 0.53. Thus, correlation among individual judgment diminished through this shuffling process (Here, the number of trials was set as 12 for an illustration purpose, and thus swapping two cells diminished correlation considerably. However, in our simulation the number of trials was set as 100, so that correlation dropped only subtly by swapping two cells for once). It should be noted that the number of correct decisions made by each member were identical to the initial matrix, while correlation had changed.

If we repeat this shuffling process, the correlation drops further. In this way, we controlled the average value of correlation among individual decisions in a group, and systematically varied correlation of groups while each member's competence was fixed.

Supplementary result of the computer simulations

The results of the computer simulation of the larger group size ($N = 297$) with various committee sizes (M) are shown in Figure S1 and S2. Consistent to the main result, performance of selected majority rule is robust when committee size is fixed to the small numbers (Figure S1) compared when committee size increases proportionally to the whole group (Figure S2).

Figure S1. Results of fixing the number of the committee size (M) and increasing whole group size ($N = 330$). Committee sizes are $M = 3, 5, 6, 8$ and 11 . Each row shows mean of performance of the group members and column shows standard deviation.

Figure S2. The result where committee size (M) increased proportionally to whole group size ($N = 330$) as. Committee sizes (M) are 1/11, 3/11, 5/11, 7/11 or 9/11 of the whole group size (N). Each row shows mean of performance of the group members and column shows standard deviation..