Computation and Analysis of Binomial Series

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Abstract: This paper presents computation of binomial series and relation between different series. This idea can enable the scientific researchers to solve the real life problems.

MSC Classification codes: 05A10, 40A05 (65B10)

Keywords: computation**,** binomial coefficient, binomial series, combinatorics

1. Introduction

When the author of this article was trying to develop the multiple summations of geometric series, a new idea stimulated his mind to create a combinatorial geometric series [1-9]. The combinatorial geometric series is a geometric series whose coefficient of each term of the geometric series denotes the binomial coefficient V_n^r. In this article, binomial identities and multinomial theorem is provided using the binomial coefficients for combinatorial geometric series.

2. Combinatorial Geometric Series

The combinatorial geometric series [1-9] is derived from the multiple summations of geometric series[10-19]. The coefficient of each term in the combinatorial refers to the binomial coefficient V_n^r.

$$\sum_{i_1=0}^{n}\sum_{i_2=i_1}^{n}\sum_{i_3=i_2}^{n}\cdots\sum_{i_r=i_{r-1}}^{n} x^{i_r} = \sum_{i=0}^{n} V_i^r x^i \ \ \& \ \ V_n^r = \frac{(n+1)(n+2)(n+3)\cdots(n+r-1)(n+r)}{r!},$$

where $n \geq 0, r \geq 1 \ \ and \ \ n, r \in N = \{0, 1, 2, 3, \cdots\}$.

Here, $\sum_{i=0}^{n} V_i^r x^i$ refers to the combinatorial geometric series and

$V_n^r \ is$ the binomial coefficient for combinatorial geometric series.

3. Analysis of Binomial Series

We know that the general binomial series is $\sum_{i=0}^{n} V_i^{n-i} x^i y^{n-i} = (x+y)^n$.

For examples: by substituting $x = 1$ and $y = 1$ in the binomial series, we get $\sum_{i=0}^{n} V_i^{n-i} = 2^n$.

Also, by substituting $x = 1$ and $y = 2$ in the binomial series, we get $\sum_{i=0}^{n} V_i^{n-i} 2^{n-i} = 3^n$ and

$x = 2$ and $y = 1$ in the binomial series, we get $\sum_{i=0}^{n} V_i^{n-i} 2^i = 3^n$.

Theorem 3.1: $\sum_{i=0}^{n} V_i^{n-i} x^i y^{n-i} = \sum_{i=0}^{n} V_i^{n-i} = 2^n$ if $y = x$.

Proof. Let us substitute $y = x$ in $\sum_{i=0}^{n} V_i^{n-i} x^i y^{n-i} = (x+y)^n$. Then $\sum_{i=0}^{n} V_i^{n-i} x^i x^{n-i} = (2x)^n$, i.e. $\sum_{i=0}^{n} V_i^{n-i} x^n = 2^n x^n \Longrightarrow \sum_{i=0}^{n} V_i^{n-i} = 2^n$.

Hence, the theorem is proved.

4. Conclusion

In this article, a lemma on binomial coefficients was constituted. This idea can enable the scientific researchers to solve the real life problems.

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