



[microreview]

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# A finite cancellative semigroup is a group

Open Mathematics Collaboration<sup>\*†</sup>

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## Abstract

We prove the proposition addressed in the title of this paper.

**keywords:** finite cancellative semigroup, group theory, abstract algebra

*The most updated version of this paper is available at*

<https://osf.io/34vbp/download>

## Notation & Definition

1.  $[1, 2]$
2.  $\mathcal{S}$  = finite semigroup  
(finite set + binary operation + associative)
3.  $\forall x, y, z \in \mathcal{S} : (zx = zy) \rightarrow (x = y)$  left-cancellative
4.  $\forall x, y, z \in \mathcal{S} : (xz = yz) \rightarrow (x = y)$  right-cancellative
5.  $((\mathcal{S} = \text{left-cancellative}) \wedge (\mathcal{S} = \text{right-cancellative})) \rightarrow (\mathcal{S} = \text{cancellative})$

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# Proposition

6. *A finite cancellative semigroup is a group.* [1]

## Proof 1

7. Proposition: All finite semigroups are periodic, i.e., all elements of a finite semigroup are periodic.
8. From (2) and (7),  $\mathcal{S}$  is periodic.
9. Let  $x \in \mathcal{S}$  arbitrary.
10. Since  $x^k = x^\ell$  for  $k < \ell$ , then  $x^{\ell-k} = 1_{\mathcal{S}}$ , so  $\mathcal{S}$  has identity.
11. Let  $y \in \mathcal{S}$  arbitrary, then  $y^n = 1_{\mathcal{S}}$  for some  $n \in \mathbb{N} = \{1, 2, 3, \dots\}$ .
12.  $y^n = y^{n-1}y = yy^{n-1} = 1_{\mathcal{S}}$
13. So,  $y^{n-1}$  is an inverse (left and right).
14. Therefore,  $\mathcal{S}$  is a group. □

## Proof 2

15.  $T(\mathcal{S}) =$  transformation semigroup
16. From Cayley's theorem,  $\mathcal{S} \cong T(\mathcal{S})$ .
17. Since  $\mathcal{S}$  is finite and cancellative, the elements in  $T(\mathcal{S})$  are permutations.
18. Any finite semigroup of permutations is a group.
19. Thus  $\mathcal{S} \cong$  group.
20. Therefore,  $\mathcal{S}$  is a group. □

## Final Remarks

21.  $(\mathcal{S} = \text{finite cancellative semigroup}) \rightarrow (\mathcal{S} = \text{group})$

## Open Invitation

*Review, add content, and **co-author** this paper [3, 4]. Join the **Open Mathematics Collaboration** (<https://bit.ly/ojmp-slack>). Send your contribution to [mplobo@uft.edu.br](mailto:mplobo@uft.edu.br).*

## Open Science

The **latex file** for this paper together with other *supplementary files* are available [5].

## Ethical conduct of research

This original work was pre-registered under the OSF Preprints [6], please cite it accordingly [7]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

## Acknowledgement

+ **Center for Open Science**

<https://www.cos.io>

+ **Open Science Framework**

<https://osf.io>

# References

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