Quantum gravity as the unification of general relativity & quantum mechanics

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Abstract. *A nonstandard viewpoint to quantum gravity is discussed. General relativity and quantum mechanics are to be related as two descriptions of the same, e.g. as Heisenberg's matrix mechanics and Schrödinger's wave mechanics merged in the contemporary quantum mechanics. From the viewpoint of general relativity one can search for that generalization of relativity implying the invariance "within – out of" of the same system.*

Key words: *cyclicity, quantum invariance, conservation of action*

Both general relativity and the Standard model remain the best confirmed experimentally theories correspondingly of gravity and of the rest three interactions known in nature. Quantum mechanics underlies the Standard model and thus quantum gravity should be the unification of general relativity and quantum mechanics, of their principles and equations as well as the physically meaningful conditions, under which it can happen. That unification in three aspects is discussed here: quantum and smooth motion on a trajectory; Lorentz invariance and noninvariance; the Einstein and Schrödinger equation:

The principle of relativity (Einstein 1918) involves the invariance of all physical laws to any smooth mechanical movements. It cannot include quantum leaps, and that unification seems to require a generalization of the cited principle comprising the cases of discrete morphisms.

In fact general relativity solves implicitly the same problem since any discrete leap can be equivalently reformulated as a curvature, i.e. as a ratio between a curved and a straight trajectory. The same ratio can be represented equally well covariantly: as the ratio between the continuous measure and the discrete one between any two points. Consequently that intended generalization is redundant since any discrete motion can be equivalently interpreted as a smooth one.

Anyway quantum mechanics manages to solve the problem otherwise and nontrivially by complementarity: that ratio is forbidden because its members as conjugates cannot be commensurable. Quantum mechanics remains the ratio as a free variable while general relativity assigns to it an exactly determined finite value in any nonsingular space-time point. The additional ("hidden") variable in general relativity in comparison with quantum mechanics is attached by the locally valid conservation of energy-momentum. This means that general relativity projects the quantum picture of the world on the screen of that conservation while quantum mechanics generalizes it as a new though implicit law of conservation of action and describes the world toward any fluctuation of the locally valid conservation of energy-momentum.

Consequently quantum mechanics and general relativity describe one and the same but in different aspects: as a transient process and as a set state accordingly. What fluctuates before to establish is the true space-time together with gravitational field. The unification should be understood as the convergence of a process to its limit as well as the damping of a transient to its set state. However that damping is not a process in time (or space-time), but the true time (or space-time) arises within and by it. What is damping is wave function.

The locally valid conservation of energy-momentum implies Lorentz invariance. As to the smooth pseudo-Riemannian space of general relativity, this means that it is locally “flat”, i.e. locally approximable by Minkowski space in any point. If one identifies gravitational field with pseudo-Riemannian space, gravity converges to the true space-time or to some electromagnetic radiation with almost zero energy. If special relativity unifies space and time in space-time, general relativity advances it to the unification of space-time with energy (mass) in pseudo-Riemannian space. However while the unification of space and time can be “classical” since these quantities are proportional and their differentials as well, the unification of space-time and energy-momentum can be only “quantum” since their differentials are inversely proportional and thus incommensurable implied by the Bekenstein bound (Bekenstein 1973), from which and from the laws of classical thermodynamics general relativity can be deduced (Jacobson 1993).

Electromagnetism is Lorentz invariant, and gravity is not such in general, there existing a smooth transition between them and thus between the global Lorentz noninvariance and the local Lorentz invariance. So electromagnetic radiation without mass at rest can be considered as a zero benchmark in general relativity, to which the global gravity can be read.

The zero benchmark of the Lorentz invariant electromagnetism is a natural bridge from general relativity to the Standard model where it is embedded as the simplest form of symmetry. This is the symmetry of a sphere or ball underlying both Hilbert space of quantum mechanics and pseudo-Riemannian space of general relativity in two quite slightly different ways: There is a well-ordered infinite set of balls in both cases as the parameter of ordering is conjugate to that of the other: e.g. time for pseudo-Riemannian space and frequency for Hilbert space. The second difference is that pseudo-Riemannian space is globally deformed (being smooth, it is locally undeformed) and this deformation toward the “flat” Minkowski space conditions gravity. One can tempt to identify gravity as the deformation of Minkowski space with entanglement as the deformation of Hilbert space being thus “conjugate”. Furthermore that identification requires a process in time (space-time) to be reckoned as a completed whole for Hilbert space involving inevitably actual infinity, which is rather interesting for philosophy.

That zero benchmark allows of a common viewpoint both to general relativity and to quantum mechanics and thus to the Standard model. One and the same space is smoothly deformed in general relativity to explain gravity and the automorphisms of its dual space can represent all quantum symmetries for the Standard model. The physical carrier of that unification is electromagnetism, which manages to be a possible zero benchmark in both cases and to generate that common viewpoint to both branches.

The gauge theories underlying the Standard model “insert” Hilbert space in any space-time point and thus induce the idea for the two “branches” of the Standard model and general relativity and their corresponding spaces, which should be dual as above, to be situated cyclically: Then all space-time is inserted in any point of it. That cyclicity is the dual counterpart of conservation of energy in the framework of a more general law of conservation of action: Indeed as conservation of energy is implied in that framework as that conservation which is “per a unit of time”, as cyclicity is implied within it as the other pole: that conservation which is a “per a unit of energy (mass)” representing a conservation of time (time-period). If the standard conservation

of energy can be thought as a screen, on which the usual “particle” picture of the world is projected, that new conservation projects the world on the dual and curious “wave” screen. Consequently cyclicity is implied by the true fundament of quantum mechanics: wave-particle dualism. The framework to be conserved action is coherent both with general relativity and quantum mechanics as well as with the Standard model.

The last step is the unification of the Schrödinger and Einstein equation on the base built above: One can investigate the conditions, under which each one of them can be reduced to the other and can be formulated as the requirements of invariance to the two equations.

Those conditions are the following:

1. *Cyclicity*:

In mathematical language it means the mutual and equivalent transformation between the operators: $x \leftrightarrow \frac{d}{dx}$, and utilizing Hilbert space, the one-to-one mapping between it and its dual space mutually interpreting wave function and the world line as above.

Cyclicity implies for actual infinity to be accepted as a physical and thus experimental subject since infinity has to be considered not only as a process, but also as a completed whole.

2. *Quantum invariance*:

The Kochen – Specker (1967) theorem excludes any “hidden variable” and thus any well-ordering in a coherent quantum state before measurement, but the results of its measurement are always well-ordered, which implies the well-ordering theorem equivalent to the axiom of choice. The relation of any coherent state before and after measurement requires the invariance to the axiom of choice interpretable physically as quantum invariance. It implies a total “flattening” of a vector space if it is infinitely dimensional as Hilbert space is and of the equations describing processes in it as well. As a result, all, but finitely many mutual curvatures and distortions of pseudo-Riemannian space can be reduced to zero for the infinite dimensions of Hilbert space so that the representations in both spaces are equivalent.

3. *Conservation of action*:

The common consideration of cyclicity (meaning it is a kind of conservation) and the standard conservation of energy (energy-momentum) implies the more general conservation of action. It allows of a variable time instead of the uniformly current time conserving energy to be meant, which is consistent with both equations. It serves technically to equate the different ways of Lorentz noninvariance between the two.

The two equations turn out to be dual counterparts canceling “mutually” both the externality of the universe and the internality of a quantum: Furthermore the universe can be considered equivalently as the externality of any quantum and vice versa: Thus the relativity of the most immense and the tiniest is involved as well as the necessary transition between them happening in any space-time point.

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