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[knowledge base]

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Presheaf (of abelian groups) on a topological space

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Abstract

PRESHEAF and its underlying definitions are presented in this white paper (knowledge base).

keywords: sheaf theory, abelian group, topological space, knowledge base

The most updated version of this white paper is available at

https://osf.io/2y5s4/download

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Definition

1. **Presheaf (of abelian groups) on a topological space X**

$$f : U \to A(U)$$

$$r_{U,V} : A(V) \to A(U)$$

   (a) $r_{U,U} = 1$

   (b) $r_{U,V}\ r_{V,W} = r_{U,W}$ when $U \subset V \subset W$

$f, r_{U,V}$:= functions
U, V := open sets
X := topological space
$U \subset V$
$U \subset X$
A := presheaf
$A(U)$:= abelian group
$r_{U,V} \equiv$ homomorphism (restriction)

[1]

Prerequisites

2. **Ordered pair**

$$(a, b)$$

a := first coordinate
b := second coordinate

[2]

3. **Cartesian product**

$$A \times B = \{(a, b) \mid a \in A,\ b \in B\}$$

A, B := sets
$A \times B$:= Cartesian product

[2]

4. **Binary operation**

$$\star : S \times S \to S$$

S := set
$S \times S$:= Cartesian product

[3]

5. **Group**

$$(G, \star)$$

(a) Associativity: $\forall x, y, z \in G,\ (x \star y) \star z = x \star (y \star z)$

(b) Identity: $\exists e \in G : \forall x \in G,\ e \star x = x \star e = x$

(c) Inverse: $\forall x \in G\ \exists y \in G :\ x \star y = y \star x = e$

G := set
$\star$:= binary operation

[3]

6. **Abelian group**

$$G_b$$

$$\forall g_1, g_2 \in G_b,\ g_1 g_2 = g_2 g_1$$

G_b := group

[4]

7. **Open interval**

$$(a, b) = \{x \in X \mid a < x < b\}$$

X := set

[5]

8. **Open set**

$$X := \text{open set in } Y$$

(a) $X \subseteq Y$

(b) $\forall x \in X,\ \exists (a, b) :\ x \in (a, b),\ (a, b) \subseteq X$

X, Y := sets
(a, b) := open interval

[5]

9. **Arbitrary Union**

$$\bigcup X$$

X := collection of sets
$\bigcup X := \{y \mid \exists Y \in X,\ y \in Y\}$

[5]

10. **Arbitrary Intersection**

$$\bigcap X$$

$X :=$ collection of sets
$\bigcap X := \{y \mid \forall Y \in X,\ y \in Y\}$

[5]

11. **Topology** on S

$$\mathcal{T}$$

$S :=$ set
$\mathcal{T} :=$ collection of open subsets of S
$X, Y :=$ collection of sets
$\bigcup X :=$ arbitrary union
$\bigcap Y :=$ arbitrary intersection

(a) $\varnothing, S \in \mathcal{T}$

(b) $(X \subseteq \mathcal{T}) \to (\bigcup X \in \mathcal{T})$
[$\mathcal{T}$ is closed under arbitrary unions]

(c) $(Y \subseteq \mathcal{T},\ Y \text{ finite}) \to (\bigcap Y \in \mathcal{T})$
[$\mathcal{T}$ is closed under finite intersections]

[5, 6]

12. **Topological space**

$$(S, \mathcal{T})$$

$S :=$ set
$\mathcal{T} :=$ topology on S

[5, 6]

13. **Homomorphism**

$$f^h$$

(a) $f^h : G \to H$
$\forall x, y \in G : f^h(x * y) = f^h(x) \circ f^h(y)$

$f^h :=$ function
$G, H :=$ sets
$*, \circ :=$ binary operations
$(G, *), (H, \circ) :=$ groups

[3, 4, 7]

Open Invitation

Review, *add* content, and **co-author** this *white paper* [8, 9].
Join the **Open Mathematics Collaboration**.
Send your contribution to `mplobo@uft.edu.br`.

Open Science

The **latex file** for this *white paper* together with other *supplementary files* are available in [10].

Ethical conduct of research

This original work was pre-registered under the OSF Preprints [11], please cite it accordingly [12]. This will ensure that researches are conducted with integrity and intellectual honesty at all times and by all means.

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