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Property out of conflict: A survey and some new results

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Property Out of Conflict: A Survey and Some New Results*

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Abstract

Property rights often emerge from adversarial interactions in which agents make claims and defend them from the appropriation efforts of others. In this paper, we first offer a survey of the theoretical literature on this issue. We systematize the existing models by classifying them into two families and show that they can explain the emergence of classic types of property rights. We then explore a new model where agents can become the sole owner of a commonly owned production resource through an exclusion contest. We show that if overexploitation under joint property is severe enough relative to the returns to scale of conflict activities, private property emerges out of conflict. Inequality makes common ownership less likely to emerge. Finally, we characterize the set of common ownership regimes which are Pareto efficient and immune to conflict. Results show that proportionality to labour inputs in output sharing makes common ownership more resilient to conflict when inequality is higher.

Keywords: Property rights, Common-pool resource, Open access, Conflict.

JEL codes: D23, D62, D74, O13.

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1 Introduction

History points to an obvious but too often neglected fact: Property rights given by law or custom are not always the fruit of a societal endeavor gently adjusted through "legal and moral experiments".¹ Instead, ownership systems are many times created and altered through a conscious (and sometimes brutal) exercise of force or coercion. The achievement of sufficiently strong control rights by these means was in some occasions the main step to the recognition of the legal ownership regimes we observe today. This process did not -by definition- benefit all participants.²

An example of this phenomenon was the development in 18th century England of private rights to land, traditionally of common property: A rise in the price of wool increased the value of land for sheep farming. This triggered the political initiative of upper classes aimed at establishing private ownership by excluding serfs, often through coercion.³ Other examples include land reform, frontier economies or the discovery of new resources, which most often operated without well-functioning enforcement institutions. This legal vacuum incentivised the use of coercive means to define and maintain property rights. Scarcity only added fuel to the fire. This was the case in Darfur, where despite the existence of a traditional system governing open access, ecological decline spurred a fierce fight over the control of fertile land and water which spawned a massive humanitarian crisis.⁴ The future offers similar prospects. As Lee (2009) argued, intrastate conflicts over the ownership of scarcer resources and interstate conflicts over the new resources made available by climate change will become increasingly frequent.⁵

In this paper, we study the creation of effective property rights by coercive

¹Demsetz (1967).

²In the words of R.H. Tawney, "property is not theft, but a good deal of theft becomes property" (cited in Jordan, 2006).

³"Where enclosure involved significant redistribution of wealth it led to widespread rioting and even open rebellion" (North and Thomas, 1973).

⁴For more examples of past resource capture conflicts worldwide, see Homer-Dixon (1994).

⁵Conflict does not necessarily imply violent behavior. Lobbying and other influence activities are also resource-consuming means to attain property rights: Britain and Norway obtained preferential exploitation rights over the oil and gas found in the North Sea because they were able to diplomatically impose the 'smallest distance to the coast' criterion to other contending nations.

means.⁶ We first revise the fruitful theoretical literature which has explored this issue. We classify the existing models into two families. This allows us to flesh out better their contribution and novelty and to systematize the results of this literature. We highlight that these models can explain the emergence of classic types and theories of property rights such as private property, first occupancy property or the labour theory of property.

In the second part of the paper, we explore a general-equilibrium model of the emergence of property regimes over a production resource. Agents can maintain common ownership, which entails some degree of overexploitation, or engage in a contest whose winner becomes the sole owner of the resource by excluding the loser. We study the incentives of agents to opt for common ownership of the resource or to convert it into private property through confrontation.

Results from the model show that higher returns to scale in the technology of conflict make the emergence of common ownership more likely. This is because a conflict to establish private property becomes fiercer when conflict effort has increased returns. On the other hand, a higher level of inequality makes conflict more likely to take place. We then characterize the set of common ownership regimes which are Pareto efficient and immune to conflict. We show that these regimes give more weight to labour inputs in the distribution of output the more inequality there is.

2 The creation of property rights through conflict

Traditionally, economists embraced the idea that the creation of property rights responds "to the desires of the interacting persons for adjusting to new benefit-cost possibilities".⁷ This is to say that the emergence of property is the outcome of a consensus within a community, a consensus that emerges because the new rights can make everybody better off. This utilitarian approach to property rights (Munzer, 2005) is at the heart of the Coase Theorem, for example.

But the creation of property rights most frequently involves some form

⁶Following Grossman (2001), we say that an agent has effective property rights over an object when the agent controls its allocation and distribution.

⁷Demsetz (1967).

of exclusion. Property is not such unless it is respected by others. That respect often stems from a private exercise of force or, at least, the threat of it. This was the case during most of human history, and it is still the case in places with weak or absent institutions. For years, the economic literature ignored this point and took property rights for granted. One notable exception was De Meza and Gould (1992), who investigated the effects of the creation of individual ownership over common property sites. Enforcing exclusion could be socially inefficient in that model because enclosing a site (by fencing it, for instance) made overexploitation worse in other sites. However, once established, private property was perfectly secure.⁸

Earlier contributions had already acknowledged that ownership is created and maintained through paralegal, often violent, means. Bush and Mayer (1974) explored a model of anarchy where agents use their initial endowments to protect what they have and to appropriate goods from others. Their study of the "natural equilibrium" that emerged out of this setting was pioneering. Another seminal contribution was Umbeck's (1981) theoretical study of the California gold rush of 1848, where contracts had prohibitive transaction costs and property rights were created and maintained through personal violence.

In the mid 90s, a strand of the economic literature studied the allocation of resources between productive and coercive activities. Within this literature, several papers analysed the creation of effective property rights through defence and appropriation. We next revise those seminal papers and the literature they spawned over the following quarter of a century. For the purpose of this review, we focus on environments where institutions are almost completely absent and there is no centralised authority enforcing property rights fully.⁹ Following Grossman (2001), we will first classify these theoretical contributions into two types of models, *common pool models* and *claims models*, highlighting their differences and commonalities. Then, we

⁸Wagner (1995) explored a similar model where groups can enclose a common pool resource at a cost that decreases with the number of insiders. In equilibrium, enclosed and free sites may coexist, with the number of insiders in enclosed sites equating the marginal benefit and cost of adding one more member.

⁹The reader may be surprised to see that we have deliberately left out the family of conflict models à la Skaperdas (1992). In these models, agents clash over a common stock of income or output which they have produced jointly. For us, conflict in these models is distributional rather than about the creation of property rights, and it is thus distinct from the conflict models surveyed here.

conclude this first part of the paper by reviewing extensions of these models and alternative approaches explored in this literature.

2.1 The common pool model

In this family of models, agents make effort to appropriate resources from a common pool or, alternatively, to obtain full control over a resource. This can be land or a natural resource. Common pool models can thus explain the emergence of *first possession property*, the basis for the *first occupancy theory of property*.¹⁰ In what follows, we provide a general common pool model inspired by the canonical model of Hirshleifer (1995), expanded to encompass other models in this strand of the literature (e.g. Skaperdas and Syropoulos, 1995, 1996; Ansink and Weikard, 2009).

Let us assume that there are two agents in the economy (individuals, unitary social groups or countries) indexed by $i = 1, 2$. Each agent possesses E_i units of an endowment which they can use for two purposes, appropriation (a_i) and labour (l_i). Hence $l_i + a_i \leq E_i$. We assume throughout that the endowment is big enough to avoid corner solutions.

The amount of the resource of size R which agent i can capture is

$$r_i = p_i R, \quad (1)$$

where

$$p_i = \frac{a_i^m + \eta_i}{a_1^m + a_2^m + \eta_1 + \eta_2}. \quad (2)$$

We refer to this as the *appropriation technology*. The parameter $m \in (0, 1]$, which Hirshleifer (1995) calls the *effectiveness* of conflict, denotes the returns to scale of appropriation effort. Note that $p_1 + p_2 = 1$ and that $p_i = \eta_i/(\eta_1 + \eta_2)$ when $a_1 = a_2 = 0$. This ratio can be interpreted as the no-conflict division of the resource, which in turn can stem from existing institutions or customs. Observe that η_i can also be interpreted as an earlier investment in arming or influence activities made by agent i (Rai and Sarin, 2009). We will return to this point when reviewing the family of claims models. For the time being, we will assume that η_1 and η_2 are relatively similar so all equilibria are interior.

Agents have identical payoff functions $u(r_i, l_i)$ satisfying $u_r, u_l > 0$, $u_{rr}, u_{ll} < 0$, $u_{rl} \geq 0$ and $u(r_i, 0) = u(0, l_i) = 0$. This implies that labour effort and the

¹⁰See the illuminating discussion on the theories of property by Munzer (2005).

resource are complementary inputs. Agents simultaneously choose the allocation of their endowment E_i into appropriation and production. Observing that $l_i + a_i = E_i$ at any optimal choice, the first order condition for agent i 's problem is simply

$$\frac{\partial p_i}{\partial a_i} R = \frac{\frac{\partial u}{\partial l_i}}{\frac{\partial u}{\partial r_i}}.$$

In words, the marginal benefit of appropriation activities must equate the marginal rate of substitution between the captured resource and labour effort. Under our assumption on m , p_i is a strictly concave function of a_i so a more abundant resource R induces higher appropriation efforts. In the particular case of Cobb-Douglas utilities, i.e. $u(r_i, l_i) = r_i^{1-\alpha} l_i^\alpha$, it is possible to show that the optimal choice of labour effort satisfies

$$l_i^* = \frac{\alpha}{\alpha + m(1 - p_i)(1 - \alpha)} E_i.$$

Compare that to the labour effort under no conflict, $l_i = E_i$. When property rights over the resource are made effective through appropriation, individuals divert part of their endowment from labour into appropriation activities. This leads to an important implication of common pool models: Conflict over property is necessarily inefficient. If parties could negotiate over how to divide the resource R , Pareto superior outcomes could be attained. It would be natural to expect, for instance, that shares $\eta_1/(\eta_1 + \eta_2)$ and $\eta_2/(\eta_1 + \eta_2)$ would be a focal point in that negotiation.¹¹

2.1.1 Probabilities or shares?

Above, we have assumed that the resource under dispute is divisible and that conflict entails the appropriation of a fraction of that common pool. Alternatively, the conflict may be an *exclusion contest* where the winner obtains full control of the resource (Skaperdas and Syropoulos, 1995, 1996).

In that case, the technology of conflict in (2) should be interpreted as the probability with which agent i obtains the control of the resource. The payoff of agent i would now be $p_i u(R, l_i)$ because agents receive zero in case of losing the exclusion contest since the resource and labour effort are complementary,

¹¹Skaperdas and Syropoulos (1995, 1996) explore other mechanisms agents can use to overcome the inefficiency of a conflict over property.

i.e. $u(0, l_i) = 0$. The first-order condition of the problem for agent i under Cobb-Douglas utilities would then yield the optimal appropriation effort

$$a_i^* = \frac{m(1 - p_i)}{\alpha + m(1 - p_i)} E_i.$$

Following similar arguments to Skaperdas and Syropoulos (1995), it is possible to show that if $\eta_1 = \eta_2$, the richer agent invests more in appropriation so she is more likely to obtain the control of the resource than the poorer agent.

Interestingly, when the resource enters multiplicatively into $u(R, l_i)$, as in the Cobb-Douglas case, the probability version of the common pool model can be reinterpreted to apply to situations where agents compete for *the right to produce*. This may be the case of two agents lobbying to obtain a licence to operate in a market (lawyers, doctors, drivers) or fighting for the right to exploit a resource. The latter is the avenue we explore in our own model in the second part of this paper.

2.1.2 Partially overlapping claims

The common pool models over the full control of the resource can be also understood as models where agents have fully overlapping claims. They claim full ownership of the resource so one agent attaining control of her claim implies the full exclusion of the other from the resource. Ansink and Weikard (2009) explored an interesting model of conflict over water rights where agents hold partially overlapping claims. The addition of claims to the common pool model blurs its differences with the other family of models we will be discussing later.

Formally, agents hold claims C_1 and C_2 over the resource. If $C_1 + C_2 \leq R$ there is actually no rivalry and thus no underlying conflict since agents can take ownership of their claims without interfering with each other. If $C_1 + C_2 > R$ the resource is contested. The intensity of contestation, i.e. the overlap in the claims, is given by $C_1 + C_2 - R$.

Players invest their endowments into appropriation activities in order to make their claims effective so now

$$r_i = p_i C_i + (1 - p_i)(R - C_j). \quad (3)$$

If agent i is successful, she makes her claim C_i effective whereas she receives the "leftover" $R - C_j$ if defeated. Expression (3) can be rewritten

to accommodate the interpretation of p_i as the share of the contested part of the resource $C_1 + C_2 - R$ that agent i can appropriate in addition to the part left unclaimed by the opponent, i.e. $R - C_j$. That is, (3) can be expressed as

$$r_i = R - C_j + p_i(C_1 + C_2 - R).$$

Note that in either case, these formulations encompass the common pool model with fully overlapping claims in (1) when $C_1 = C_2 = R$.

Ansink and Weikard (2009) showed that appropriation efforts are increasing in the size of the claims whereas equilibrium payoffs are decreasing. That means that the case of fully overlapping claims produces maximum aggregate appropriation effort and minimum individual payoffs.

2.1.3 No-conflict equilibria

One commonly leveraged criticism against common pool models is that they cannot feature equilibria where one or both parties expend no effort in appropriation.¹² This is an important criticism because it is very often the case that only one side is aggressive or no conflict over property erupts at all. To some extent, claim models were developed to overcome this drawback. However, as shown by Butler and Gates (2012) in their model of African range wars, it is possible to make no-conflict equilibria compatible with common pool models thanks to the generalized success function in (2).

In the analysis above, we assumed that the parameters η_1 and η_2 were equal and low enough. When p_i is the probability of controlling the resource, $m = 1$ and $u(R, l_i)$ is of the Cobb-Douglas form, it is possible to show that a peaceful equilibrium arises if

$$E_i \geq \alpha \frac{\eta_i}{\eta_j} (\eta_1 + \eta_2) \quad \text{for } i = 1, 2.$$

No agent invests in appropriation if the parameters η_1 and η_2 are relatively equal and high enough. The reason is that the size of $\eta_1 + \eta_2$ is inversely related to the sensitivity of the technology of appropriation to effort. Butler and Gates (2012) interpreted $\eta_1 + \eta_2$ as the strength of pre-existing property rights, which leads to the observation that a conflict over property is less

¹²"[Skaperdas and Hirshleifer] models make no distinction between predation and defense against predation and, hence, cannot consider the possibility of a nonaggressive equilibrium" (Grossman and Kim, 1995, p. 1277).

likely to arise when these rights are stronger. On the other hand, η_i can be interpreted as the head-start of player i in the conflict (coming from an earlier investment in arms, for instance). So when no player has a too strong advantage *and* the technology of appropriation is not very sensitive to effort, i.e. $\eta_1 + \eta_2$ is high enough, universal peace can prevail.

But partially disarmed equilibria can also emerge. When one of the players has a considerable relative advantage, i.e. η_i is high enough relative to η_j , only one player makes appropriative effort in equilibrium. That agent is the disadvantaged one. Actually, if the disadvantaged agent is poor enough, there is a corner equilibrium where she specializes fully in appropriation. i.e. $a_j^* = E_j$. This partially peaceful equilibria thus resembles very strongly the predator-prey models we will revise below where this specialization is imposed exogenously. Here, this division of roles emerges endogenously when one of the agents enjoys a strong head start, either awarded by an institution (formal or informal) or obtained through a prior arming investment.

2.2 The claims model

Let us now review the extensive literature on a second family of models where property rights emerge out of conflict. In these models, agents have initial claims to some property. The security of these claims is endogenous. The extent to which a claim converts into effective property rights depends on the defence effort of the agent who holds this claim and the appropriation efforts of those who challenge it.

Formally, in the claims model, agents can use their endowments as labour (l_i), to defend their claims (d_i) or to appropriate the property claimed by the other agent (a_i). Hence $l_i + d_i + a_i \leq E_i$. As before, we assume endowments are big and only interior solutions exists.

Agents hold *nonoverlapping* claims to property, denoted by C_i . These claims can be over their own endowment E_i , over the output they generate with their labour or over some resource which is complementary to production. For the time being, we will remain as general as possible and entertain these three possibilities.

Denote by p_i the security of i 's claim. This can be understood as the fraction of the claim that i is capable of defending from agent j . If $p_i = 1$, the claim is perfectly secure. Agent i has effective property rights over the amount

$$c_i = p_i C_i + (1 - \beta)(1 - p_j) C_j, \quad (4)$$

where $\beta \in [0, 1]$ denotes the destruction of resources that appropriation induces.

The security of claims is given by a conflict technology axiomatized in Clark and Riis (1998) and of the form

$$p_i = \frac{d_i^m}{d_i^m + \theta a_j^m}, \quad (5)$$

where it is assumed that $p_i = 1$ whenever $a_j = 0$. Note that $p_i \neq 1 - p_j$. The parameter $\theta \in (0, 1]$ denotes the preponderance of appropriation over defence; it is thus assumed that the latter is more effective than the former. In this context, the parameter m is related to the sensitivity of property rights to coercive activities (exclusion or appropriation).

Agents have identical utility functions $u(c_i, l_i)$ satisfying $u_c, u_l > 0$, $u_{cc}, u_{ll} < 0$ and $u_{cl} \geq 0$. As said, agents choose how to allocate their endowment E_i into the three activities, defence, appropriation and production. Assuming for the time being that these choices are simultaneous, and using the fact that $l_i + d_i + a_i = E_i$ in any optimal choice, the first order conditions for the agent i 's problem are such that

$$\frac{\partial p_i}{\partial d_i} C_i = -(1 - \beta) \frac{\partial p_j}{\partial a_i} C_j = \frac{\partial u}{\partial l_i} + p_i \frac{\partial C_i}{\partial l_i}. \quad (6)$$

This expression implies that any interior optimal choice for player i must equate the marginal benefits of defence, appropriation and production activities. The last term includes the possibility of endogenous claims (i.e. $\partial C_i / \partial l_i > 0$) as discussed below.

2.2.1 The nature of claims

It is at this point where it becomes useful to distinguish between the three possible types of claims considered in the literature.

Endowments: Grossman and Kim (1995) interpreted the endowment E_i as the claim subject to appropriation. Because these endowments are initially owned by the agents, this can be thought as a model on the emergence of *private property*.

When claims are over endowments, expression (4) becomes

$$c_i = p_i E_i + (1 - \beta)(1 - p_j) E_j.$$

Assuming that $u(c_i, l_i)$ is separable and of the form $u(c_i, l_i) = c_i + \alpha_i l_i$ where $\alpha_i > 0$ is agent i 's marginal product of labour, the optimal interior choice of agent i in (6) boils down to

$$\frac{\partial p_i}{\partial d_i} E_i = -(1 - \beta) \frac{\partial p_j}{\partial a_i} E_j = \alpha_i.$$

Given the assumptions on p_i , more productive agents devote, *ceteris paribus*, less effort to defence and appropriation. They tolerate more appropriation because they can produce more output. The opposite happens for richer agents, who devote more effort to protection because they have a bigger claim to shield from appropriation.

Resource claims: Suppose that the claims C_1 and C_2 are still exogenous but are defined now over some resource which complements labour. These claims may be entitlements awarded by a state with weak or no enforcement capacity, or may be the result of agents having seized a fraction of the resource provisionally (Umbeck, 1981). This version of the claim model is close to the set up of common pool models, although recall that here claims remain nonoverlapping.

Assuming that $u(c_i, l_i)$ is of the Cobb-Douglas form $u(c_i, l_i) = c_i^{1-\alpha} l_i^\alpha$, the optimal interior choice of agent i in (6) is characterized by

$$\frac{\partial p_i}{\partial d_i} C_i = -(1 - \beta) \frac{\partial p_j}{\partial a_i} C_j = \frac{\alpha}{1 - \alpha} \frac{c_i}{l_i},$$

where c_i is as in (4). If we further assume that $C_i = C_j$ (Grossman, 2001), it is possible to show that the optimal choice of appropriation and defence satisfies $a^* = (1 - \beta)d^*$ and the security of agent i 's claim in equilibrium is

$$p_i^* = \frac{1}{1 + \theta(1 - \beta)^m}. \quad (7)$$

The security of claims to property decreases with the preponderance of appropriation relative to defence θ and increases with its destructiveness β and the returns to scale of conflict activities m .

Output: A third possibility is that agents claim the property of the output they have produced with their labour (Grossman, 1996a). This version of the

claims model captures the emergence of *the right to the fruits of one's labour*, often referred to as the *labour theory of property* (Locke, 1689[1976], section 10). Ownership of output is only sustained to the extent agents can defend it from the appropriation effort of the other agent.

Formally, the claim of agent i is now given by $C_i = \alpha_i l_i$. Assuming that $u(c_i, l_i) = c_i(l_i)$, the first order condition (6) implies that at any interior solution

$$l_i^* = \frac{d_i^*}{m(1 - p_i^*)}.$$

Naturally, labour input increases with the security of property claims p_i . After some manipulation it is possible to show that when $m = \theta = 1$ the shares of output agents can secure in equilibrium satisfy

$$\frac{p_i^*}{p_j^*} = \sqrt{\frac{\alpha_j}{\alpha_i}}.$$

This implies that the property of the more productive agent is less secure in equilibrium than the output of the less productive agent. The reason lies in the difference in the opportunity cost of conflict activities for the two agents. The less productive agent has more incentives to invest in appropriation than the more productive agent has to invest in defending her own.

2.2.2 Predator-prey models

One popular variation of the model described above is the predator and prey model (Grossman, 1996a,b; Noh, 2002; Kolmar, 2008; Denter and Sisak, 2011; Jennings and Sanchez-Pages, 2017). One of the agents, the prey, can only invest in defence whereas the other agent, the predator, can only invest in appropriation. The claim of the predator to her own output is fully secure. The prey however must fight for her claim. The security of this claim thus depends on how successful the prey is in defending from the predator.

Predator-prey models are meant to represent historical or state-of-nature contexts where some agents specialize in aggression. This might have been the case of tribes like the Mongols or the Vikings but would also fit with the decision of modern individuals to specialize in theft or extortion. Because they only feature one conflict input per agent, these models are closer to common pool models. Actually, recall we saw in section 2.1.3 that common pool models can generate an endogenous specialization into production and aggression. Apart from the endogeneity of the choice, the differences are that

claims are nonoverlapping in the predator-prey model and that the prey still invests in defence, whereas in the model described in section 2.1.3 the prey only invests in production and both agents are contesting the same resource.

Formally, consider a variation of the output claim model just reviewed above where agent 1 is the prey and agent 2 is the predator. Impose $a_1 = 0$ and thus $d_2 = 0$, which in turn implies $p_2 = 1$. This means that now

$$\begin{aligned} c_1 &= p_1 E_1, \\ c_2 &= E_2 + (1 - \beta)(1 - p_1)E_1. \end{aligned}$$

The first-order conditions for the problem of the predator yield that her optimal interior appropriation effort satisfies

$$a_2^* = \frac{\alpha_1}{\alpha_2} E_1 (1 - \beta) p_1^* \frac{m(1 - p_1^*)}{1 + m(1 - p_1^*)},$$

where p_i^* is as in (7). The predator becomes more aggressive the richer and the relatively more productive the prey is. This is simply because the wealth to be appropriated increases with E_1 and α_1 . The predator also intensifies her appropriation effort when the opportunity cost of appropriation, given by her own labour productivity α_2 , goes down. Potentially, the predator may specialize completely in theft, i.e. $a_2^* = E_2$. This is more likely to happen the poorer the predator is.

Grossman and Kim (1996b) and Kolmar (2008) assumed from the start that the predator fully specializes in appropriation and imposed that the predator makes no productive effort, i.e. $l_2 = 0$. Regardless of the specific modelling choices, all these models yield imperfect secure claims under simultaneous choices.

2.2.3 Sequential choices

In all the models reviewed so far, either common pool or claims models, property was always insecure in equilibrium. We have discussed equilibria with one or both agents being non-aggressive but even in those cases property was not fully secure. This is unsatisfactory because we commonly observe that property rights are respected even when the threat of appropriation is real. The reason is that agents, while remaining peaceful, spend sufficient resources to deter predation. In our daily lives, for instance, we invest into alarm systems and security doors to protect our property from burglars. Few

people specialize in theft precisely because of the deterrence effect of these investments.

One way out of the insecurity of property in equilibrium is to assume success functions different from the ones we have considered so far. Grossman (1996a) and Jennings and Sanchez-Pages (2017) used weakly-monotonic conflict technologies where there is a threshold in the defensive effort of the prey contingent on the appropriation effort of the predator $\bar{d}(a_j)$ such that $p_i = 1$ for any $d_i \geq \bar{d}(a_j)$. These functional forms allows the emergence of equilibria under simultaneous choices where the prey enjoys fully secure property by investing just enough in defence activities as to deter the predator from engaging in appropriation.

An alternative often considered in the literature is to retain the conflict technology in (5) but to make choices sequential. In the first stage, defensive effort is chosen, either by the two agents or by the prey, depending on the model. In the second stage, players allocate their remaining endowments between appropriation and production (if the claim is endogenous).

Going back to Grossman and Kim (1995), it is straightforward to show that the best response appropriation effort of agent j in the second stage given that agent i has made effort d_i in the first one is $a_j = 0$ if and only if

$$d_i \geq \frac{\theta(1 - \beta)}{\alpha_j} E_i. \quad (8)$$

Agents in the first stage then face a trade off between making a high enough investment in defence that deters predation or diverting part of that investment to production and tolerate appropriation in return. Grossman and Kim (1995) showed that agent i opts for full security when $\theta(1 - \beta) \leq 1/2$, that is, when the preponderance of appropriation is low enough and its destructiveness is sufficiently high.

Later sequential models extended the results of Grossman and Kim (1995) to other settings. Grossman and Kim (1996b) assumed a predator-prey structure where the prey makes her defence and production choice in the first stage and the predator chooses her appropriation effort in the second. In addition to condition (8), a non-aggressive equilibrium with fully secure property requires the prey not to be too much richer than the predator, i.e. E_1/E_2 should not be too high.

Kolmar (2008) studied the effect of sequentiality in an output claims model. The additional insights he obtained stem from the disincentive effects

that arise when the fruits of one's labour are insecure. The main result is that, even if an equilibrium with fully secure property arises, that equilibrium is socially inefficient. The reason is that the prey holds back productive effort to avoid becoming too attractive for the predator and trigger a conflict over property.

2.2.4 Further extensions

Transfers: The results in Grossman and Kim (1996b) and Kolmar (2008) show that a wealthy or productive prey is less likely to enjoy fully secure claims to property. This suggests that the prey may be interested in making a transfer to the predator in order to avoid being attacked. Noh (2002) studied that possibility by exploring a model where the prey can transfer part of her output to the predator in the second stage conditional on the predator remaining non-aggressive.¹³ He showed that an equilibrium with transfers exists and that it Pareto dominates the equilibrium without transfers. The reason is that the predator becomes less interested in appropriating the prey's output and redirects all her endowment to production, whilst the prey now needs a lower defence effort to fully deter the predator.

Uncertainty Denter and Sisak (2011) studied an interesting extension of the predator-prey model where there is asymmetric information about the value of the claim of the predator. Before players move, the claim of the predator C_2 is drawn from some differentiable density function $G(C_2)$ with compact support $[\underline{C}_2, \overline{C}_2]$. Then the prey and the predator move sequentially.

Whereas the utility for the predator is the same as in the standard predator-prey model with exogenous claims, the utility for the prey is now

$$\int_{\underline{C}_2}^{\overline{C}_2} u(c_1(C_2), l_1) dG(C_2),$$

where $c_1(C_2) = p_1(d_1, a_2(d_1, C_2))C_1$ and $a_2(d_1, C_2)$ is the best response appropriation effort of the predator with claim C_2 to agent 1's defensive effort d_1 .

¹³The transfer thus entails a form of commitment unlike transfers in the contest model by Bevia and Corchón (2010).

Denter and Sisak (2011) show that property rights are always insecure in this setup. Moreover, an *imperfect property* equilibrium may arise where property is insecure ex-ante but may be fully secure ex-post. The reason is that the defence effort of the prey aims to deter the average predator so predators with low enough claims are fully deterred.

2.3 Other static models

2.3.1 Conflict over land and common-pool resources

Many of the models discussed above aimed to capture conflicts about the creation of effective property rights over a resource. These models simplified on the production front by assuming that the resource was of fixed value or its exploitation followed a simple linear technology. They thus left out scenarios such as conflicts over land tenure or resources whose exploitation is subject to decreasing returns. Next we review a few models with a richer description of the resource whose property is subject to contestation.

In two very similar papers, Alston, Libecap and Mueller (1999, 2000), presented a model of conflict over land. They were inspired by the land reform which took place in Brazil in the late 80s. The reform expropriated farmers to settle landless peasants. Because the process was slow, a legal vacuum emerged. Landless peasants invaded farmland meant to be expropriated and claimed possession rights. In response, farmers tried to evict the squatters often using violence in addition to legal means.

These authors proposed a predator-prey resource claims model where farmers invest in evicting squatters from their land and squatters spend resources into making their claim stand. The novelty here is that the result of the conflict is a compound lottery. With some endogenous probability π , the farmers evict the squatters. If unsuccessful, squatters expropriate the farmer with some probability. These probabilities depend on the investments of farmers and squatter and on exogenous parameters such as the strength of the rule of law and of the farmer's claim.

Expression (4) can be rewritten in a simplified version of this model as

$$\begin{aligned} c_1 &= \pi R + (1 - \pi)(1 - p_2)R \\ c_2 &= (1 - \pi)p_2 R, \end{aligned}$$

where the farmers are assumed to be the prey and the squatters, the predator. Unfortunately, comparative statics are too complex to produce clean analytic

results. This is because conflict efforts are strategic complements for farmers but strategic substitutes for squatters.

In Hotte (2001), the owner of a natural resource site must decide how much to invest in enforcing her right. This is because the site is assumed to be located at a frontier region where property rights are not fully enforceable and potential competitors may contest ownership of the resource. The question for the owner is whether to protect the property and enjoy a long-term use of the site or to deplete the resource immediately and make it worthless. Although the model is set in continuous time, the owner of the site and the competitor make their investments in defence once and at the start, albeit sequentially. There is an instantaneous probability of the site changing hands. This together with the fact that the resource produces a constant flow of payoffs implies that the problem is essentially static. Agents maximize the discounted sum of flow of payoffs and this probability follows an inverse exponential distribution.

Given all these assumptions, the results of the model are in line with those of a predator-prey model with sequential moves: Fully secure ownership of the resource can emerge. A novel insight is that depletion of the resource is more likely to happen when its value is higher. This is because the potential competitor invests more heavily in appropriation so defending the site becomes too costly. To avoid this, the owner decides to exhaust the resource immediately.

Baker (2003) studied a static model on the emergence of land tenure regimes. This model had two novel features. One is that claims are endogenous; the two agents stay at the extremes of a unit segment and decide how much of it they want to claim and defend. Claims can thus overlap or not. The other feature is that the preponderance parameter θ in (5) is endogenous and decreases with the size of the claim to be defended. Bigger parcels are more difficult to defend. After claims are made, agents decide how much to invest in defending theirs and in appropriating the land claimed by their rival. These decisions are sequential as in Kim and Grossman (1995), implying that claims can be fully secure in equilibrium.

Another interesting aspect of Baker (2003) is that the different regimes that can emerge in equilibrium fit the various property arrangements observed since the times of hunter-gatherers. Land may be owned jointly, land may be left unclaimed and undefended; full private property rights arise when all land is claimed and claims are fully secure; and open access emerges when no land is claimed nor defended. Which of these tenure regimes arises in

equilibrium depends on how resource rich land is and how the preponderance parameter changes with the size of claims.

Finally, Sanchez-Pages (2006) explored a model of conflict over a common pool resource that serves as the basis of the model we present in the second part of this paper. Agents can engage in an exclusion contest whose winner becomes the sole owner of the resource. On the other hand, agents can opt for free access, but that involves overexploitation due to 'the tragedy of the commons' (Hardin, 1968). The property regime, private or open, emerges out of the agents' choice. In this set up, conflict Pareto dominates free access when the decisiveness of conflict parameter m is not too high. In that case, the prospect of monopolistic access to the resource makes agents choose unanimously to engage in a costly exclusion contest.

2.3.2 Coalition formation

All the models discussed so far preclude the possibility of agents forming groups in order to defend their property or to jointly appropriate an asset. Although we have admitted the possibility of agents being groups, they were assumed to be unitary. But individuals often join together to be more effective in appropriation at the cost of more diluted property rights. Common ownership may emerge if the grand coalition encompassing all individuals forms.

In their pioneering contribution, Bush and Mayer (1974) looked at coalitional deviations from "orderly anarchy", a situation with no appropriation investments, and studied whether the core of this game is non-empty. In this exercise, the authors encountered one of the main difficulties in the analysis of coalition formation in conflict: The presence of spillovers. What outsiders do as a response to the formation of a group has an impact on the payoff of members. Bush and Mayer (1974) opted for the α core, which assumes that outsiders minimize the payoff of the members of a newly created group. Given the harshness of the assumed punishment, the α core is non-empty. However, one problem with the core in general is that it is not a farsighted concept, so these core allocations are not immune to individual deviations from agents who decide to become aggressive.

Jordan (2006) followed up on this effort by looking at a new family of coalitional games, pillage games. There is no production or appropriation, only a fixed wealth to be distributed. A power function determines how powerful a group is. This function establishes a dominance relation by which a

group can appropriate the entire wealth of a less powerful group. An allocation is said to be in the core of this game when it is immune to pillage from more powerful groups. Spillovers are still present because what a coalition can plunder depends on the power of the coalitions formed by outsiders.

The author explores several power functions. When the power function is based on total wealth, allocations in the core are tyrannical, that is, with an agent owning all wealth, or they feature two agents owning half of the wealth each. When the power function also factors in the size of the group, tyrannical allocations are no longer immune to pillage and the core may be empty.

Finally, Sanchez-Pages (2007) studied a model of coalition formation where groups compete for the right to control a common-pool resource by excluding others. The formation of the grand coalition means that there is no conflict and there is free access to the resource. When free access implies the joint exploitation of the resource, the grand coalition is socially efficient but agents have strong incentives to break up into two rival coalitions. As in Sanchez-Pages (2006), a conflict among coalitions of the same size Pareto dominates non-cooperative exploitation under free access when the returns to scale of the conflict technology m are not too high.

2.4 Dynamic models

All the models we have reviewed so far have been static. This leaves out important questions that require a dynamic analysis, such as the long term implications of conflict over property on capital accumulation or the property regimes that emerge in the steady state. We next study contributions which expanded on the claims and common pool models in order to address these questions.

2.4.1 The emergence and stability of property regimes

Repeated interactions Repeated games are well-suited to study the emergence and stability of property regimes. When agents interact repeatedly, they may condition their strategies on past outcomes. The shadow of the future may curb their incentives to engage in appropriation and peaceful equilibria may be more easily attained. In addition, infinite horizon games may help us understand who will acquire property and how secure it will be by characterizing the convergence to steady state property regimes.

The first contribution in this line of enquiry was Muthoo (2004), who examined the origin of the right to enjoy the fruits of one's labor. The author looked at an infinitely repeated common pool model but where investments in arms are exogenous, so the shares agents can appropriate are fixed. Each period, agents decide how much to work and then whether to trigger conflict or not. If at least one of them does, i appropriates the property of j with probability p_i and viceversa. It is assumed that with probability $1 - p_1 - p_2$ a stalemate happens and no property exchanges hands. Agents differ in their strength, described by p_i , and in how productive they are.

Results of the model highlight that heterogeneity undermines the emergence of secure property. Provided that agents are sufficiently patient, property rights over output are less likely to emerge when one agent is strong but unproductive whereas the other is weak but productive. Another result, in line with Noh (2002), is that output transfers can increase the set of parameters under which secure property rights emerge.¹⁴

Hafer (2006) proposed a novel infinitely repeated game where a large population of players meet in pairs every period and fight for the control of the parcel of land one of them owns. This fight takes the form of a war of attrition. Agents differ in their productivity of labour so those with a higher productivity value the land more and are less willing to surrender. However, productivity is private information. That implies that when a landless agent meets an owner, the former knows that the latter is likely to be a high productivity agent because high productivity agents persist longer in the war of attrition. By the same token, an owner knows that a landless challenger is likely to be a low productivity agent who will surrender earlier. This creates a dynamic whereby owners tend to consolidate their rights over the land they occupy, lending support to first occupancy property rights. Moreover, conflict becomes absent at some point in time, because landless agents do not challenge owners anymore, and property becomes fully secure.

The dynamic exploitation of natural resources Several models we discussed in sections 2.1 to 2.3 dealt with the appropriation and exploitation of resources. But they were static, so they could only apply to situations where resource assets were non-durable and non-renewable. However, many important conflicts have emerged over the property of durable resources.

¹⁴One difference with respect to Noh (2002) is that repeated interactions dispense with the need to assume ex-post commitment.

This includes land, fishing stocks or oil fields.

Sekeris (2014) studied a model where agents choose each period whether to peacefully exploit a durable and renewable common pool of resources or to appropriate whatever stock is left. In the absence of conflict, the Folk Theorem of repeated games ensures that the resource will be cooperatively exploited if players are sufficiently patient. However, this possibility is ruined by the threat of conflict. The author showed that joint exploitation of the resource stock breaks down when it gets sufficiently depleted so the cooperative equilibrium is destroyed. By the same token, the threat of conflict also eliminates the equilibrium where agents exploit the resource non-cooperatively and inefficiently until it is fully depleted. Again, this is because confrontation takes place when the resource becomes sufficiently scarce; the winner of the conflict enjoys monopolistic control and overexploitation stops from that point onwards.

2.4.2 Evolutionary models

By simplifying on the strategic interactions front, evolutionary models are well-suited to the study of the emergence of property rights out of a state of anarchy. Warneryd (1993) was an early contribution in that direction. The basic interaction is a 2x2 game where players must decide whether to become armed at a cost or not. As in Hafner (2006), a random fraction of players receive a resource and players meet in pairs. Arming means that the agent can appropriate the resource of any unarmed owner she meets, but it is wasteful if meeting a non-owner. The equilibrium depends on the fraction of owners. When it is low or high, an equilibrium where no one arms is more likely to emerge. At intermediate levels, a mixed equilibrium appears where agents arm with some positive probability and property is no longer secure. Because this equilibrium is evolutionary stable, that means that a division of the population between aggressive and peaceful agents is asymptotically stable.

More recently, Eswaran and Neary (2014) have studied whether the first occupancy principle and Locke's labour theory of property can emerge in an evolutionary setting. One-period lived agents are randomly allocated a productive asset which they can use by investing labour. Unlucky agents can fall back to a subsistence activity or try to appropriate the output of an asset owner by investing resources. Owners also invest in defence and manage to retain a fraction of output as in (5) with $\theta = 1$. The output

obtained determines a person's fitness (or offspring), but agents's valuation of output may differ from the fitness it grants. The question is which set of preferences over output is evolutionary stable.

The results of the model show that nature can hardwire preferences for private property. In other words, the set of evolutionary stable preferences is such that owners value output more than non-owners. In that sense, the right to enjoy one's fruits of labor emerges through natural selection; owners who do not value their property enough get wiped out. The authors consider a slightly simplified model with no production but where the preponderance of appropriation θ is allowed to vary. When $\theta < 1$ it is again the case that owners value their property more than predators. Due to the absence of labour in this version of the model, this result can be interpreted as an evolutionary basis for the first occupancy doctrine. The advantage defenders enjoy in the technology of conflict leads nature to select a higher value of property for those who get to own the asset.

2.4.3 Growth with insecure property rights

So far we have seen that the possibility of appropriation makes agents invest in defence and also incentivises them to engage in appropriation themselves. This has important implications for the allocation of resources away from productive uses. It is to expect that the combination of resource diversion and insecure property rights should have strong deleterious effects on capital accumulation and growth. As Gonzalez (2012) pointed out, "the creation of wealth and the creation of effective property rights are competing uses of scarce resources."

The seminal paper in this line of enquiry was Grossman and Kim (1996a), who embedded the predator-prey model in a standard growth model. Each period, a generation of the predator and prey dynasties must decide how to allocate their endowment between productive capital and appropriation/defence. Their endowment is the wealth they inherited from the previous generation of their dynasty. The model analyses the possible trajectories of wealth accumulation that emerge from this set up.

As we know from section 2.2.2, the decision of the prey to tolerate predation or deter the predator depends on the size of the wealth of the predator relative to the wealth of the prey. The dynamics of the model in Grossman and Kim (1996a) imply that the inherited wealth of the predator grows faster than that of the prey. This is because predation makes the wealth the prey

inherits dwindle with each generation. At some point, the prey generation is so poor that it decides to fully deter the predator by investing in defence. Property becomes fully secure from that point onwards.

Chan and Laffargue (2016) also used a predator-prey model but integrated it in a Malthusian growth model. Population of the prey and predator dynasties grows depending on their per capita income. Because moves are sequential, fully secure property emerges in the steady state if and only if defence is sufficiently preponderant (θ low) and appropriation is destructive enough (β high). One novel insight of this model is that more patient predators are less aggressive and thus worse off when appropriation is not deterred. This is because plundering the prey makes her grow at a lower rate. Patient predators refrain from being too aggressive and the prey can shield a disproportionately bigger share of their claim. As Noh (2002), Chan and Laffargue (2016) explored the possibility of transfers from the prey to the predator. The commitment problem that emerged in the one-shot case can be solved now because the prey may threaten to stop making transfers in the future if the predator attacks in the present period.

In the spirit of Grossman and Kim (1995), Gonzalez (2007) assumed away the distinction between predator and prey and considered instead a continuum of infinitely-lived and identical agents. These assumptions reduce the strategic component of the model to a minimum as players now best respond against the average allocation of resources. This allows a clean characterization of the laws of motion of individual investment and consumption.

Formally, each period t , player i chooses how much to consume (c_i^t), how much to invest in capital (k_i^t), with productivity $\alpha > 0$, and how much in defence (d_i^t) and appropriation (a_i^t) under the constraint,

$$c_i^t + k_i^{t+1} + d_i^{t+1} + a_i^{t+1} = p_i^t \alpha k_i^t + q_i^t \alpha k^t, \quad (9)$$

where q_i^t is how much i appropriates from the average output αk^t at time t . Agent i plays against the average defence and appropriation efforts d^t and a^t . Capital fully depreciates after one period.

One main result of this model is that the threat of predation increases current consumption at the expense of investment. This is due to the insecurity of property, which increases with θ in (5), the preponderance of appropriation over defence. This leads to inefficiently low growth. However, more secure property rights do not necessarily lead to higher social welfare. The intuition for this surprising result is that when defence efforts become more effective,

conflict intensity increases. A lower θ diverts resources from consumption to investment so future wealth increases. This increases the incentive to engage in appropriation by diverting further resources away from consumption.¹⁵

In Gonzalez (2007), security of property remains constant over time. Individuals manage to shield a fraction $p_i^* = 1/(1 + \theta)$ of their endowment from appropriation each period. In contrast, in Grossman and Kim (1996a) property becomes less secure with time until the prey decides to deter the predator from that point onwards, although this is at the cost of imposing a predator-prey structure. Kumar (2008) offered an alternative approach by studying a differential game where both agents can invest in defence and appropriation and the security of property rights evolves over time. The dynamics stem from making the appropriable share of output endogenous.¹⁶ Modifying (9) accordingly, the constraint under which agents now operate is

$$c_i^t + k_i^{t+1} + g_i^{t+1} + a_i^{t+1} = \lambda^t \alpha k_i^t + q_i^t (1 - \lambda^t) \alpha k^t,$$

where $\lambda^{t+1} = (1 - \delta)\lambda^t + \sum_{i=1,2} g_i^t$ is the share of secure output, which depreciates at rate δ . Unfortunately, the model becomes very complex. No analytical results can be obtained beyond the existence of a unique equilibrium under open loop strategies and multiple equilibria under feedback strategies. Goel and Sen (2019) got a bit more mileage from this setting by assuming instead that λ^t is the result of a negotiation between the agents. This makes possible to show that property rights are insecure in any steady state because the poorer agent can always benefit from that insecurity.

3 A model of property rights in the shadow of conflict

In this second part of the paper, we present a new model of the emergence of different regimes of property over a production resource (a technology, a pasture, a fishery). This model builds on the literature revised in section

¹⁵This results relies however on a very specific set of assumptions. It no longer holds if more general utility functions and partial capital depreciation are assumed (Alpetkin and Levine, 2009) or if a fraction of the agents' endowments is free from appropriation (Yoo, 2013).

¹⁶This is similar to Boyce and Bruner (2012), who engodenize the security of property rights in a one-shot sequential game where agents voluntarily contribute to increase the size of $\eta_1 + \eta_2$ in (2).

2.3.1, and especially on Sanchez-Pages (2006). The novelty here is that we expand on the description of production under common ownership. Rather than assuming that agents exploit the resource non-cooperatively, we consider alternative arrangements such as binding agreements.¹⁷ We study when these arrangements are immune to an exclusion contest aimed at establishing private property over the resource.

Consider two risk-neutral individuals or unitary groups who jointly own the production resource. These agents can attempt to impose their sole ownership over the resource through a contest whose winner obtains monopolistic control by excluding the loser. We assume that agents first simultaneously decide whether or not to challenge common ownership. If at least one of them decides to do so, the exclusion contest takes place.

3.1 Non-cooperative exploitation under common ownership

Agreeing to commonly own the resource is a basic commitment. Agents commit to allocate their initial endowments to labour effort (labour henceforth) only. Let us $l_i \in [0, E_i]$ denote the amount $i = 1, 2$ decides to invest in the exploitation of the resource where E_i is her initial endowment.

For the sake of exposition, let us follow Sanchez-Pages (2006) and assume at first that agents exploit the resource non-cooperatively when they own it jointly. Later we will consider alternative arrangements. In that case, agents do not internalize the negative externality associated with their decisions and over-exploitation arises (the well-known 'tragedy of the commons'). Our formalization of this problem follows a simplified version of the canonical model by Cornes and Sandler (1996): The amount of output produced is given by a strictly concave and twice-differentiable production function $f(\cdot)$ which depends only on the total labor input $L = l_1 + l_2$ and satisfies $f(0) = 0 < f'(0)$.

In addition, we assume that $f(\cdot)$ attains a unique maximum at $L^* \in (0, E_1 + E_2)$. In the canonical formulation of the commons' problem, the production function is increasing everywhere but a unit cost of labor generates a strictly concave payoff function with a unique maximum. However, with a monotonic production function and a unit cost of labour, the comparison

¹⁷See Bevia and Corchon (2017) for a survey of the literature on production under common ownership with fully secure property.

with the outcome of the exclusion contest becomes too cumbersome. The assumption of a non-monotonic production function keeps the model tractable and transparent whilst still equivalent to the canonical one.

As it is standard in the literature, output under non-cooperative production is assumed to be distributed proportionally to individual labor contributions. Under this assumption, payoffs with common ownership are

$$u_i^F = \frac{l_i}{l_1 + l_2} f(l_1 + l_2), \quad \text{for } i = 1, 2.$$

Both agents choose how much labour to invest in the resource by taking as given the labour input of the other agent. It is straightforward to show that at the symmetric Nash equilibrium of this non-cooperative game, the total level of labor denoted by L^F satisfies

$$f'(L^F) = -\frac{f(L^F)}{L^F},$$

provided that $L^F \leq 2 \min\{E_1, E_2\}$. We will assume this for the rest of the paper.

Common ownership with non-cooperative exploitation leads to "the tragedy of the commons". Labour is oversupplied with respect to the efficient level, i.e. $L^F > L^*$. This is because agents do not take into account the reduction in marginal productivity of the resource they bring by increasing their own labour input.

3.2 The exclusion contest

The alternative to common ownership is coercive exclusion. If one of the agents challenges joint ownership, they engage in confrontation in order to become the sole owner of the resource at the risk of being excluded.

The outcome of the exclusion contest is probabilistic and depends on the appropriation effort (effort henceforth) agents invest into the contest. Denote by $r_i \in [0, E_i]$ the effort invested by agent $i = 1, 2$. Given r_1 and r_2 , a conflict technology determines each's player probability of winning the contest. We adopt the functional form in (2) with $\eta_1 = \eta_2 = 0$, so the resource becomes the private property of agent i with probability

$$p_i = \frac{r_i^m}{r_1^m + r_2^m}, \quad \text{for } i = 1, 2.$$

When the exclusion contest takes place, agents choose the amount r_i they invest in excluding the other agent and the amount l_i they will invest into labour if they become the owners of the resource.¹⁸ Payoffs are then

$$u_i^C = p_i f(l_i) = \frac{r_i^m}{r_1^m + r_2^m} f(l_i), \quad \text{for } i = 1, 2.$$

Players maximize the above expression subject to $0 \leq r_i + l_i \leq E_i$ taking the choices of the other agent as given. Note that the cost of conflict effort are the foregone production possibilities of diverting part of the endowment away from production.

It can be shown that when agents have different endowments, their effort choices differ and the wealthier agent is more likely to win the contest. Prior differences in wealth would thus explain conflict over property and inequality in the access to production resources on the basis of the superior strength wealthier individuals enjoy in confrontation. To avoid that, we will assume throughout the rest of the section that agents have the same initial endowments, i.e. $E_1 = E_2 = E$.

The next proposition shows that there exists a unique equilibrium for the Exclusion game under these assumptions.

Proposition 1 *With identical endowments, the exclusion contest admits a unique Nash Equilibrium characterized by the interior first order condition*

$$f'(l^C) = \frac{m}{2(E - l^C)} f(l^C).$$

Proofs are contained in the Appendix. In contrast with the common ownership regime, the resource is underexploited under private property, i.e. $l^C < L^*$. The reason is that agents divert so much of their endowment to effort in the exclusion contest that the labour input the winner is left with is below the efficient level. Agents now oversupply effort instead of labor.

3.3 When is common ownership immune to conflict?

Next, we derive the conditions under which common ownership of the production resource can emerge. Differences in the returns of the conflict and

¹⁸It is immaterial whether each agent makes their choice of r_i and l_i sequentially or simultaneously.

production technologies will determine whether the resource remains in a joint ownership regime or becomes private property.

Agents compare the equilibrium payoff under common ownership and the equilibrium payoff from the exclusion contest. The incentives to engage in confrontation come from the fact that victory in the contest awards exclusive property rights, whereas common ownership entails overexploitation of the resource. However, if the exclusion contest is too fierce, there is little advantage from confrontation. Agents would be better off by sharing the resource.

Formally, let us denote by

$$l^F = \begin{cases} E & \text{if } E < L^* \\ L^* & \text{otherwise,} \end{cases}$$

the optimal labour input when only one agent owns the resource. It is easy to see that if $f(L^F) \geq f(l^F)$, agents are worse off by triggering the exclusion contest: In the latter case, they would obtain output $f(l^C) (< f(l^F))$ with probability $1/2$, whereas they get half of $f(L^F)$ under common ownership. Hence, a necessary condition for private property to emerge out of conflict is that the resource should be sufficiently overexploited under common ownership, i.e. $f(l^F) > f(L^F)$. We will maintain this assumption for the rest of the analysis.

Proposition 2 *Suppose that $f(l^F) > f(L^F)$. Then there exists a threshold $M > 0$ on the decisiveness of the conflict technology such that both agents prefer common ownership to conflict if and only if $m \geq M$.*

Even when the inefficiency associated with common ownership is severe, the room for a conflict over the control of the resource is limited by its eventual intensity: If the conflict technology is very decisive (for instance, if the quality of weapons or the tolerance of society towards influence activities is high), agents invest a big share of their endowments in the exclusion contest. Confrontation becomes then so fierce that the resource is left virtually unexploited when it is made private. Common ownership can then prevail.¹⁹

The following example illustrates this point.

¹⁹Our condition on m resembles the condition on this parameter ($m < 1$) in Hirshleifer (1995) ensuring the stability of anarchy, a situation where agents fight but retain viable shares of output.

Example 1: Let the production function take the quadratic form $f(L) = L - bL^2$. Assume that $E \geq 1/(3b)$ in order to ensure the existence of an interior solution for the common ownership problem. Under this regime, the equilibrium payoff is $u^F = 1/(9b)$. It is easy to check that the condition $f(l^F) > f(L^F)$ in Proposition 2 holds. Although the closed form for agents' payoff in the exclusion contest is rather involved, straightforward calculus shows that the critical threshold in the decisiveness of the conflict technology is $M = 3Eb - 1$. The range of parameter values under which conflict takes place increases with b , the proxy for the concavity of the production function (output elasticity is decreasing in b), and on the size of the initial endowments. Overexploitation is more severe as b increases, making common ownership less attractive. Although, the level of overexploitation does not depend on E , the equilibrium effort in the exclusion contest does; a very fierce conflict is less likely to affect the production capabilities of richer agents if they become the sole owner of the resource.

To summarize, we should expect private property to emerge out of conflict when the inefficiency associated with shared ownership is strong enough and conflict is not too resource consuming. In that case, both agents want to engage in conflict and the common ownership regime cannot emerge. As the returns to scale of conflict increase, it is more likely that the conflict over the resource will be too fierce and the resource will remain jointly owned even though it will be overexploited.

3.4 Alternative common ownership arrangements

3.4.1 Sharing rules

The proportional sharing rule employed so far formalizes the case where individual labor inputs are completely disagreeable and agents exploit the resource in a fully noncooperative fashion under common ownership. However, this possibility does not exhaust the set of all the regimes under which a production resource can be exploited. Ostrom (1990) presents persuasive evidence showing that users of common property resources frequently develop governance systems based on sanctions and monitoring that help them to overrule opportunistic behavior.

One can consider alternative "rights and duties" regimes where owners write contracts on their labour input. These contracts can be conveniently

modelled as *sharing rules* specifying how output is shared as a function of input contributions (Moulin, 1990). These rules alter the incentives of the agents exploiting the resource.

In particular, we will consider the family of mixed sharing rules studied in Cornes and Hartley (2002) which consists of a convex combination between proportional and exogenous sharing. The fraction of the output agent i receives is

$$\alpha_i = (1 - \lambda)\gamma_i + \lambda \frac{l_i}{l_1 + l_2}, \quad \text{for } i = 1, 2, \quad (10)$$

where $\lambda \in [0, 1]$ is the proportionality of the rule and γ_i is an exogenous weight such that $\gamma_1 + \gamma_2 = 1$. When $\lambda = 1$, we return to the case of fully non-cooperative exploitation of the resource. When $\lambda = 0$, agents fully commit to an exogenous division of output. One frequently studied case is the egalitarian rule $\gamma_1 = \gamma_2 = 1/2$. When $\gamma_i \neq 1/2$, the weight reflect a preexisting inequality in the distribution of commonly produced output, coming from bargaining power or tradition, or can be seen as the sharing of the revenue levied by a proportional tax on output at rate $1 - \lambda$.

In this case, payoffs under common ownership become:

$$u_i^F = \alpha_i f(l_1 + l_2) = ((1 - \lambda)\gamma_i + \lambda \frac{l_i}{l_1 + l_2}) f(l_1 + l_2), \quad \text{for } i = 1, 2.$$

Intuitively, as output sharing becomes less proportional, i.e. λ decreases, we approach a cooperative production scenario in which agents are just interested on maximizing total production. This in turn limits overexploitation and should make conflict over the resource less likely to arise. On the other hand, asymmetries in the exogenous sharing imply that the agent with a lower weight receives less output as the rule become less proportional. This should make her more prone to challenge common ownership.

Cornes and Hartley (2002) show that a unique equilibrium under common ownership exists when the sharing rule is as in (10). Moreover, agent i 's share of total labour input in equilibrium is

$$\frac{l_i^F}{L^F} = \frac{1 - (1 - \lambda)\gamma_i}{1 + \lambda}, \quad (11)$$

implying that the agent with the lower exogenous weight inputs more labour. Without loss of generality we will assume from now on that $\gamma_1 = \gamma > 1/2$. Hence, the parameter γ measures the inequality in exogenous weights.

Equilibrium uniqueness allows us again to compare the payoff under common ownership and under the exclusion contest in order to establish conditions under which common ownership is immune to conflict. As before, this threshold depends on the decisiveness of conflict effort m but also on the inequality of exogenous weights given a sharing rule with proportionality λ .

Proposition 3 *Assume that $f(l^F) > f(L^F)$. Then there exists a threshold $M(\gamma) > 0$ such that both agents prefer common ownership to conflict if and only if $m \geq M(\gamma)$. This threshold is decreasing in γ .*

The interplay between the conflict technology and the underlying inequality in the sharing rule agents employ under common ownership determines which property regime emerges. Private property emerges if the inequality in exogenous weights is high relative to the decisiveness of conflict effort. The player with more incentives to trigger conflict is agent 2 because her weight is the lowest. When her weight is very low, provided that conflict is not too fierce, she prefers to challenge common ownership.

Example 2: Consider again the data from Example 1. Straightforward calculations show that the payoff under common ownership, which now depends on λ and γ_i , it is equal to

$$u_i^F = \frac{\lambda + (1 - \lambda)\gamma_i}{b(2 + \lambda)^2}, \quad \text{for } i = 1, 2. \quad (12)$$

This implies that $u_2^F < u_1^F$. The threshold $M(\gamma)$ is such that $u_2^F = u^C$. Note that for $m = 0$, the contest becomes a fair ‘lottery, agents invest nothing in conflict and $u^C = 1/8b$. It is easy to show that in this case $u_2^F < u^C$, so by continuity of u^C with respect to m , private property emerges if the returns to scale of conflict effort are low enough. The following figure depicts this threshold $M(\gamma)$ when $E = b = 1$ for different values of λ .

Figure 1: Equilibrium property regimes
when $E = b = 1$.

The steeper curve in Figure 1 represents $M(\gamma)$ when $\lambda = 0$. Above that curve, common ownership can prevail because conflict is too fierce. The same happens for the other two curves, which depict, from north to south, $M(\gamma)$ when $\lambda = 0.5$ and $\lambda = 0.8$. As expected, all three curves are increasing in γ , showing that more inequality shrinks the range of m where common ownership prevails. This effect is stronger the lower λ . This is because more inequality in weights lowers agent 2's share of output in equilibrium when the sharing rule is predominantly exogenous. Note also that given a high enough level of inequality γ , more proportional rules, i.e. with a higher λ , increase the likelihood of common ownership. This is because proportionality can compensate agent 2 for her lower exogenous weight.

3.4.2 Endogenizing the sharing rule

The above discussion has assumed that the proportionality of the sharing rule λ was fixed. Given the technology of conflict m and the underlying level of inequality γ , common ownership may be inferior to a conflict to establish private property. To avoid this, agents can agree on a level of proportionality that will improve upon confrontation. We know this cannot be the case when m is very low, but the question remains if m is sufficiently high: which types of input contracts are more likely to be immune to conflict?

To answer this question, we need to explore how equilibrium payoffs under common property change with λ . Using (11), it is straightforward to show

that equilibrium payoffs are

$$u_i^F = \frac{\lambda + (1 - \lambda)\gamma_i}{1 + \lambda} f(L^F).$$

Again, note that the assumption $\gamma > 1/2$ implies $u_2^F < u_1^F$. The first term in the expression above is decreasing (increasing) in λ for agent 1 (2). The second term is decreasing in λ as more proportionality only worsens overexploitation. That implies that agent 1's payoff attains a maximum when the sharing rule is fixed, i.e. $\lambda = 0$.

As hinted by Figure 1, the effect of λ on u_2^F is more complex. Less proportional rules reduce overexploitation but also decrease agents 2's share of output as sharing relies more on the exogenous weights. The overall effect depends on the underlying level of inequality γ and on how overexploitation intensifies as the sharing rule becomes more proportional. For that reason, we need to assume a functional form for $f(\cdot)$ if we want to proceed further. We will assume from now on that the production function takes the quadratic form in examples 1 and 2.

Under that assumption, it is possible to show that the utility of agent 2 under common ownership attains a maximum at the level of proportionality

$$\lambda_2^* = \min\{2(2 - \frac{1}{\gamma}), 1\}.$$

This implies that agents' preferences over λ diverge as inequality increases to the extent that $\lambda_2^* = 1$ when $\gamma \geq 2/3$. That is the reason why, as seen in Figure 1, more proportionality in the input contract can make conflict more likely when inequality in weights is low.

We would like now to characterize the range of sharing rules agents can bargain over. This range is described by the set of λ that are Pareto efficient and immune to conflict. This implies that, because agent 1 prefers the fully fixed rule, this set must lie between $\lambda = 0$ and λ_2^* .

Before stating formally this result we need some additional notation. Let us denote by $\underline{\lambda}_2 < \lambda_2^*$ the level of proportionality such that $u_2^F = u^C$ for a given m . This threshold, if exists, is unique. Finally denote by $M(\lambda_2^*, \gamma)$ the threshold in the returns to conflict effort for a given γ such that $u_2^F \geq u^C$ for any $m \geq M(\lambda_2^*, \gamma)$. Because u^C is decreasing in m and u_2^F does not depend on it, this threshold exists, it is unique and increases with γ as we learnt in Proposition 3. With all this notation at hand, we can now state the following proposition.

Proposition 4 *Assume that $f(l^F) > f(L^F)$ and that the production function is quadratic. No sharing rule is immune to conflict if $m < M(\lambda_2^*, \gamma)$. Otherwise, the set of Pareto efficient and immune to conflict sharing rules are those with $\lambda \in [\max\{\underline{\lambda}_2, 0\}, \lambda_2^*]$.*

When inequality in exogenous weights is high, i.e. $\gamma > 2/3$, agent 2 is unambiguously better off with more proportional rules because they grant her a higher share of output. She will thus challenge contracts that are not proportional enough. In contrast, only intermediate sharing rules can emerge when inequality is low, i.e. $\gamma \leq 2/3$. The reason is that the overexploitation proportionality brings outweighs the gain in the share of output agent 2 receives since now the endogenous weights already grant her a relatively equal share. She is thus less willing to allow more overexploitation in order to obtain a bigger share of output.

4 Conclusion

In this paper, we have revised the economic literature studying the emergence of property rights out of coercive and influence activities. In contrast with the traditional view in Economics, which takes property rights as granted, we have presented a number of models where agents establish property rights by appropriation and exclusion and by defending their claims from others. We hope to have rekindled the interest of researchers in this strand of the economic literature which, we believe, can offer useful insights to many real-world phenomena.

We have also explored a new model highlighting that ownership regimes over production resources emerge in the shadow of conflict. Private property is more likely to appear when conflict is not too consuming and when initial inequalities in ownership rights are high. The common ownership regimes more likely to be immune to conflict should reward hard work more the bigger initial inequalities are.

Although simple, the model presented in the second part of this paper is robust to straightforward modifications. The introduction of a substitute for the output good (e.g. leisure) would increase the payoff under common ownership but at the same time would reduce the intensity of conflict, generating additional incentives to initiate one. Similarly, having more than two agents would make confrontation fiercer and the set of conflict technologies giving

rise to a contest over property would shrink. At the same time, common ownership would become less attractive since overexploitation would worsen. Having more agents would also open the door to voting (Corchon and Puy, 1998) or some other mechanisms aimed at selecting a sharing rule. All these remain open avenues for future research.

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Proof of Proposition 1. Since agents do not derive utility from the endowments they do not use it is clear that $l_i = E - r_i$. The first order condition for agent i is

$$m(1 - p_i) \frac{f(E - r_i)}{r_i} = f'(E - r_i), \quad \text{for } i = 1, 2. \quad (13)$$

This condition yields a unique best response for any level of effort by the opponent. Some manipulation shows that in any equilibrium the choice made by both agents must be identical, i.e. $r_1 = r_2$.

Now take the function

$$\varphi(r) = \frac{m}{2r}f(E-r) - f'(E-r).$$

This function is defined over $[0, E]$. The values of r that make $\varphi(r)$ equal to zero are the Nash Equilibria of our Exclusion Game. Now we will show that an equilibrium exists indeed and it is unique. First, one can show that if $\varphi(r) \leq 0$ then $\varphi(r)$ is strictly decreasing in r : The derivative of $\varphi(r)$ with respect to r is:

$$\frac{\partial \varphi(r)}{\partial r} = -\frac{m}{2r^2}f(E-r) - \frac{m}{2r}f'(E-r) + f''(E-r) \leq 0,$$

where the last inequality holds from the fact that when $\varphi(r) \leq 0$ it is clear from (13) that $f'(E-r) \geq 0$.

Finally, note that $\varphi(r) \rightarrow \infty$ when $r \rightarrow 0$ whereas $\varphi(r) \rightarrow -f'(0) < 0$ when $r \rightarrow E$. Since $\varphi(r)$ is continuous, the previous result ensures that there exists a unique value for r^* that makes $\varphi(r^*) = 0$. Such r^* is the unique (and symmetric) Nash Equilibrium of the Exclusion Game. ■

Proof of Proposition 2. First we show the equilibrium level of effort is increasing in m . From expression (13) we know that r^* is implicitly defined by

$$r^* = \frac{mf(E-r^*)}{2f'(E-r^*)}.$$

By the Implicit Function Theorem

$$\frac{\partial r^*}{\partial m} = \frac{1}{m} \frac{f(E-r^*)f'(E-r^*)}{f'(E-r^*)^2 - f''(E-r^*)f(E-r^*)} > 0.$$

Hence, the equilibrium level of effort is increasing in m . When $m = 0$, the exclusion contest becomes a fair lottery and payoffs are simply $f(l^F)/2$, that is greater than u_i^F by assumption. As m increases, agents' labor input decrease but the winning probabilities are still $p_1 = p_2 = 1/2$ by symmetry. Since the production function is continuous and $f(0) = 0$, there must exist a threshold $M > 0$ such that $u_i^F = f(L^F)/2 = f(l^C)/2 = u_i^C$. Below that

threshold, the output under conflict is still greater than the output under common ownership and both agents prefer to engage in the exclusion contest. ■

Proof of Proposition 3. The proof of this Proposition follows the same procedure as the previous one. The first order condition of the common ownership problem under the mixed sharing rule in (10) is now

$$(1 - \lambda)\gamma_i f'(L) + \lambda \frac{L - l_i}{L^2} f(L) = 0.$$

Again, this implicitly defines a unique best response labour input l_i to each l_j . Solving for l_i yields that the optimal labour input for agent i satisfies

$$l_i = L \frac{\lambda + (1 - \lambda)\gamma_i \varepsilon(L)}{\lambda(1 - \varepsilon(L))},$$

where $\varepsilon(L) = f'(L)L/f(L)$ is the elasticity of output. Adding up across the two agents yields that the total labour input in equilibrium under the mixed sharing rule satisfies

$$\varepsilon(L^F) = -\lambda.$$

This implies that overexploitation vanishes as λ decreases. Efficiency is attained when $\lambda = 0$. The equilibrium payoff u^M thus decreases with λ . The equilibrium share of labour inputs is thus

$$\frac{l_i^F}{L^F} = \frac{1 - (1 - \lambda)\gamma_i}{1 + \lambda}, \quad \text{for } i = 1, 2.$$

which implies that the equilibrium payoff is

$$u_i^F = \frac{\lambda + (1 - \lambda)\gamma_i}{1 + \lambda} f(L^F), \quad \text{for } i = 1, 2. \quad (14)$$

Given the assumption $\gamma_1 > 1/2$, agent 2 is the one with highest incentive to challenge common ownership for a given λ . The threshold $M(\gamma)$ is hence one that satisfies $u_2^F = u^C$. Therefore

$$\frac{\partial M(\gamma)}{\partial \gamma} = -\frac{\partial u_2^F / \partial \gamma}{\partial u^C / \partial m} = \frac{1 - \lambda}{1 + \lambda} \frac{f(L^F)}{\partial u^C / \partial m} < 0,$$

where the second equality comes from differentiating (14) with respect $\gamma = 1 - \gamma_2$ for agent 2 and the inequality comes from noting that $\partial u^C / \partial m < 0$ as shown in Proposition 1. ■

Proof of Proposition 4. The first part of the proposition is obvious. When $m = 0$, we already know that $u^C > u_2^F(\lambda = \lambda_2^*)$. So there is a unique threshold $M(\lambda_2^*, \gamma) > 0$ such that $u_2^F(\lambda = \lambda_2^*) \geq u^C$ for any $m \geq M(\lambda_2^*, \gamma)$. For $m < M(\lambda_2^*, \gamma)$, no immune to conflict sharing rules exist.

For the second part, assume $m \geq M(\lambda_2^*, \gamma)$, and let us first rewrite (12) for agent 2 as

$$u_2^F = \frac{1 - (1 - \lambda)\gamma}{b(2 + \lambda)^2}.$$

Simple calculations show that u_2^F attains a critical point at $\lambda = 2(2 - \frac{1}{\gamma})$. Further derivations show that

$$\frac{\partial u_2^F}{\partial^2 \lambda} = 2 \frac{3 - (7 - \lambda)\gamma}{b(2 + \lambda)^4},$$

implying that λ_2^* is indeed a maximum. This implies that u_2^F is everywhere increasing in λ when $\gamma > 2/3$ and $\lambda_2^* = 1$. Agents have opposed preferences over proportionality and the range of Pareto efficient and immune to conflict rules is between $\max\{\underline{\lambda}_2, 0\}$ and 1.

When $\gamma \leq 2/3$, u_2^F attains an interior maximum. Denote by $\bar{\lambda}_2$ the level of proportionality such that $u_2^F = u^C$ where $\bar{\lambda}_2 > \lambda_2^*$. By continuity of u^C , for m just above $M(\lambda_2^*, \gamma)$ it holds that $0 < \underline{\lambda}_2 < \bar{\lambda}_2 < 1$. Only sharing rules with intermediate levels of proportionality, that is, those with $\lambda \in [\underline{\lambda}_2, \bar{\lambda}_2]$, are immune to conflict. But note that the rules with proportionality parameter $\lambda \in (\lambda_2^*, \bar{\lambda}_2]$ are not Pareto efficient as both agents would be better off by selecting $\lambda = \lambda_2^*$. When m is high enough, it is the case that $\underline{\lambda}_2 < 0$. Hence, the lower bound of the range of Pareto efficient and immune to conflict sharing rules is $\max\{0, \underline{\lambda}_2\}$. ■